Mass and spin
of the galactic center black hole Sgr A*

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IF $M_{\text{BH}} = 3.6 \times 10^6 M_\odot$ and IF 16.8 min $= P_{\text{Kepler}}$ then $a = 0.5$
members of the same group have $\Delta P/P \leq \frac{1}{n-1}$

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<th>State/</th>
<th>Instrum.</th>
<th>$f$ (mHz)</th>
<th>$n$ (s)</th>
<th>$P$ (s)</th>
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The Equations

\[ v^{(\Phi)} = \frac{\bar{\omega}}{\alpha} (\Omega - \omega), \]
with the Boyer-Lindquist functions:

\[ \alpha = \frac{\Omega}{\Sigma}, \]
\[ \Delta = r^2 - 2Mr + a^2, \]
\[ \rho^2 = r^2 + a^2 \cos^2 \theta, \]
\[ \Sigma^2 = (r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta, \]
\[ \bar{\omega} = \frac{\Sigma}{\rho} \sin \theta, \]
\[ \omega = \frac{2a M r}{\Sigma^2}. \]

\[ v^{(\Phi)}(r, a) \] is solved numerically, setting \( \Omega = \Omega_K \).

Results

\[ v^{(\Phi)}(r, a) \] decreases monotonically with increasing \( r \)
for \( a \leq 0.9953, \)
\[ v^{(\Phi)}(r, a) \] shows a local minimum and a local maximum
close to the marginally stable orbit
for \( a > 0.9953! \)
Figure 1: Orbital velocity $v^{(\Phi)}$ versus orbital radius for $a = 0.99616$. Outside the radial range shown $v^{(\Phi)}$ decreases monotonically with radius.

N.B. $\partial v^{(\Phi)}/\partial r \geq 0$; it represents the velocity change per unit length in radial direction and is measured in $(\text{time})^{-1}$. One can define a sort of related angular frequency $\Omega_c = 2\pi \frac{\partial v^{(\Phi)}}{\partial r} \bigg|_{\text{max}}$ derived for $r_{\text{min}} < r < r_{\text{max}}$. 
Figure 4: As a function of \((1 - a)\) are shown the radial position \(r_{\text{max}}\) of the local maximum of \(v^{(\Phi)}\) (curved dashed line), the radial position \(r_{\text{min}}\) of the local minimum of \(v^{(\Phi)}\) (curved dotted line). The vertical dashed line is at \(a = a_c = 0.9953\). The cross marks \(r_{31}\) and \(a = a_f = 0.99616\), which is the solution of the parametric resonance model with commensurable orbits. The solid line represents the innermost marginally stable orbit \(r_{ms}\) defined by \(\Omega_R = 0\).
Characteristic accretion disk frequencies

from Nowak & Lehr, 1998

\[ \Omega_K = (r^{3/2} + a)^{-1} \]

Kepler

\[ \Omega_V^2 = \Omega_K^2 \left(1 - \frac{4a}{r^{3/2}} + \frac{3a^2}{r^2}\right) \]

vertical epicyclic

\[ \Omega_R^2 = \Omega_K^2 \left(1 - \frac{6}{r} + \frac{8a}{r^{3/2}} - \frac{3a^2}{r^2}\right) \]

radial epicyclic

\[ \Omega_{LT} = \frac{2a}{r^3} \]

Lense-Thirring

c = G = 1,

\[ [r] = \frac{GM}{c^2}, \]

\[ r = 1 = r_g, \]

\[ [\Omega] = \frac{c^3}{GM}, \]

\[ a = \frac{cJ}{GM^2}. \]

\[ r = r_{ms} \quad \text{if} \quad \Omega_R = 0 \]

\[ \Rightarrow \Omega_K > \Omega_V > \Omega_R, \quad \text{for} \ a \to 1: \Omega_{LT} > \Omega_K \]
R: radial epicyclic frequency for $r_{max}$ and $r_{min}$.

V: vertical epicyclic frequency.

$c = \Omega_c = 2\pi \frac{\partial \omega^{(e)}}{\partial r} \bigg|_{max}$.

For the crossing of $[c]$ and $[V/R = 3:1]$ harmonic resonances exist:

$$\Omega_c = 2\pi \frac{\partial \omega^{(e)}}{\partial r} \bigg|_{max} = R \text{ und } V = 3 \times R.$$ 

The solution fixes the spin at $a = 0.99616$

and the distance to the central black hole at $r = 1.546$.

A simple relation between the mass of the black hole and any of the epicyclic frequencies, e.g. $V$, emerges:

$$M = \text{const.} / V$$
$M_{\text{BH}} = 2.6 \pm 0.2 \times 10^6 M_\odot$

(Genzel et al., 1996)

$M_{\text{BH}} = 3.7 \pm 1.5 \times 10^6 M_\odot$

(Schödel et al., 2002)

$M_{\text{BH}} = 4.07 \pm 0.62 \times 10^6 M_\odot \ (R_0/8 \ \text{kpc})^3$

(Ghez et al., 2003)

$M_{\text{BH}} = 3.59 \pm 0.59 \times 10^6 M_\odot$

(Eisenhauer et al., 2003)

$M_{\text{BH}} = 3.7 \pm 0.4 \times 10^6 M_\odot \ (R_0/8 \ \text{kpc})^3$

(Ghez, 2004)

$M_{\text{BH}} = 3.61 \pm 0.32 \times 10^6 M_\odot$

(Eisenhauer et al., 2005)

$M_{\text{BH}} = 3.28 \pm 0.13 \times 10^6 M_\odot$

(Aschenbach, 2004)
The Model

Frequency ratios are due to resonances between radial ($\Omega_R$) and vertical ($\Omega_V$) epicyclic oscillations enforced by Kepler ($\Omega_K$) commensurable orbits:

i.e., with $r =$ orbit radius, $a =$ BH angular momentum there exist 2 radii $r_{31}$ and $r_{32}$ such that:

(a) $\Omega_V(r = r_{31}, a)/\Omega_R(r = r_{31}, a) = 3 : 1$,

(b) $\Omega_V(r = r_{32}, a)/\Omega_R(r = r_{32}, a) = 3 : 2$,

(c) $\Omega_K(r = r_{31}, a) = n \times \Omega_K(r = r_{32}, a)$, \hspace{0.5cm} (n a natural number).

The only solution of Equ. (a) - (c) is:

$n = 3, \hspace{0.5cm} a = 0.99616, \hspace{0.5cm} r_{31} = 1.546, \hspace{0.5cm} r_{32} = 3.919$

r in GR units

Conclusion: There are two possible radii, $r_{31}$ or $r_{32}$, where $\Omega_R$ and $\Omega_V$ are in resonance; the angular momentum $a$ is identical.

The mass of the black hole $M_{BH}$ is given by

$M_{BH,31}/M_{\odot} = 4603.3/\nu_{up}$ \hspace{0.5cm} for $r = r_{31}$, or

$M_{BH,32}/M_{\odot} = 3046.2/\nu_{up}$ \hspace{0.5cm} for $r = r_{32}$.

$\nu_{up}$ [Hz] is the highest frequency observed in the twin or triplet.
High-Frequency Quasi-Periodic Oscillations in Galactic Black Hole Binaries

GRO J1655-40: 450 Hz, 300 Hz | ratio = 3:2

XTE J1550-564: 276 Hz, 184 Hz | ratio = 3:2

GRS 1915+105: 168 Hz, 113 Hz | ratio = 3:2

(relative frequency error ≈ 1.5%)
Comparison with Observations

Masses of the microquasars and Sgr A* have been measured dynamically, using stars orbiting the black hole. These masses \( M_{BH,\text{dyn}} \) are compared with the predictions of the model based on the measured QPOs \( M_{BH,\text{model}} \).

\[
\begin{align*}
\text{GRO J1655-40:} & \quad M_{BH,\text{dyn}} = 7.02 \pm 0.22 M_\odot \quad & M_{BH,\text{model}} = 6.76 \pm 0.1 M_\odot \\
& \quad M_{BH,\text{dyn}} = 5.8 - 6.8 M_\odot \\
\text{XTE J1550-564:} & \quad M_{BH,\text{dyn}} = 8.4 - 11.6 M_\odot \quad & M_{BH,\text{model}} = 11.04 \pm 0.2 M_\odot \\
\text{GRS 1915+105:} & \quad M_{BH,\text{dyn}} = 14.0 \pm 4.0 M_\odot \quad & M_{BH,\text{model}} = 18.13 \pm 0.36 M_\odot \\
& \quad \text{(} i = 66^\circ \text{)} \\
& \quad \text{(} M_{BH,\text{dyn}} = 17.8 \pm 5.5 M_\odot \text{)} \quad \text{for} \ i = 53^\circ \\
\text{Sgr A* [10}^6 M_\odot\text{:} & \quad M_{BH,\text{dyn}} = 3.7 \pm 1.5 \\
& \quad M_{BH,\text{dyn}} = 4.07 \pm 0.62 \\
& \quad M_{BH,\text{dyn}} = 3.59 \pm 0.59 \\
\end{align*}
\]

N.B., \( r = r_{32} \) for microquasars, \( r = r_{31} \) for Sgr A*!
• 3 microquasars and Sgr A* have \( a = 0.99616 \); does this indicate that \( a = 0.99616 \) is the highest value it can grow to, thus replacing the 'Kip Thorne' limit?

• 3 microquasars and Sgr A* have in common twin (3:2) or triplet (3:2:1) QPOs, have the same \( a \) and have each a jet; does the existence of any of these 3 properties imply the existence of the other two?

• Are the vertical epicyclic oscillations at \( r = 1.546 \) the seed for the creating of jets? If they are,
  - the width \( d \) of a jet at its base would be \( d = 1.5 \times 10^{12} \) cm for Sgr A*. Bower et al. (2004) have recently measured \( d < 2.4 \times 10^{12} \);
  - there would be the possibility that jets could be created by strong gravity effects rather than electromagnetic effects (the Blandford-Znajek mechanism).
A scenario and some ideas

1. If \( a \) has reached 0.99616 radial epicyclic oscillations are excited at \( r = 1.546 \) because of \( \Omega_c = \Omega_R \). \( \Omega_V \) is excited as well because of the resonance with \( \Omega_R \). This is valid for a test particle and for very low accretion rates \( \dot{M} \).

2. At higher \( \dot{M} \) the amplitudes of \( \Omega_R \) and \( \Omega_V \) grow and energy is stored.

3. At even higher \( \dot{M} \) the vertical oscillations become unstable because they can’t store the energy anymore. The source flares (the state of Sgr A*; timescales: rise time \( \sim \) one cycle of \( \Omega_V \), flare duration \( \sim \) a few cycles of \( \Omega_R \)) at most.

4. With increasing \( \dot{M} \) the amplitude of \( \Omega_R(r_{31}) \) grows and a radial wave is running back and forth through the disk, until it reaches \( r = 3.919 \) starting to excite \( \Omega_R(r_{32}) \) and \( \Omega_V(r_{32}) \). This is the state of the 3 microquasars.