Quantifier Processing: Approaches to Natural Language Quantification

Kevin Paterson
University of Leicester
Lecture 1 topics:

- Approaches in formal semantics.
- Psychological approaches.
  1. Psychometric approaches
  2. Truth verification procedures
  3. Quantifiers in human reasoning
  4. Quantifiers and discourse comprehension
  5. Acquisition of quantification
What are Quantifiers?

- everything, nothing, three books, the ten professors, John, John and Mary, only John, firemen, every, at least five, most, all but ten, less than half of the, John's, some student's, more male than female, usually, never, each other, few, some, many, not all, seven, exactly three, a significant number.
Syllogisms & Logical Quantifiers

- Provides account of reasoning with premises that include the quantifiers all, some, none, not all.
  
  **Major premise:** All mortals die.
  
  **Minor premise:** All men are mortals.
  
  **Conclusion:** All men die.

- Research on syllogistic reasoning remains a dominant approach in study of human reasoning. *(e.g., Johnson-Laird & Byrne, 1991)*
Syllogisms & Logical Quantifiers

- First-order logic uses universal quantifier \( \forall \) and existential quantifier \( \exists \) as operators that bind individual variables.

- Following Piaget, there has been considerable interest in developmental psycholinguistics in how quantifiers (particularly universals) are acquired - see Lecture 5.

- Also interest in understanding how children and adults deal with order problems and ambiguities introduced by quantifiers in the predicate logic (and higher order logics) - see Lectures 4 & 5.
Quantifier Order

- Meaning differs depending on the order of quantifiers.
  - Everybody likes some food.
    \[ \forall X \exists F \text{ food}(F) \land \text{ likes } (X,F) \]
  - There is a food that everyone likes.
    \[ \exists F \forall X \text{ food}(F) \land \text{ likes } (X,F) \]

- In natural language, there is often ambiguity over the order of quantifiers, which must be resolved.
  
  Every boy kissed a girl.
  
  \[ \forall \text{ boy} \exists \text{ girl} \, \text{kiss}(\text{boy, girl}) \]
  
  \[ \exists \text{ girl} \forall \text{ boy} \, \text{kiss}(\text{boy, girl}) \]
Generalised Quantifier Theory

- Provides account of natural language quantifiers.
- Quantifiers denote sets of sets.
- “Every man runs.”
  - Every man is the set of subsets that include men
  - (i.e., properties associated with set of all men).
  - “Every man runs” denotes set of all men who run.
- Solution enables a set-theoretic approach to quantification that is compositional.

Generalised Quantifier Theory

- To verify truth of “Every man runs.” it is necessary to:
  - Identify set of all men.
  - Identify set of all entities that run.
  - Identify set of sets that intersect with set of all men.
  - Verify that set of runners belongs to that set.

- This is intractable in processing terms, so B&C argue that verification involves the identification of a “witness set” - i.e., set of men who run.

Key axioms

- Conservativity
- Extensionality
- Isomorphism
- Symmetry
- Monotonicity

Symmetry

- Symmetrical quantifiers permit the conversion of statements as follows:
  - If some lawyers are crooks then some crooks are lawyers.
  - If no lawyers are crooks then no crooks are lawyers.
- Other quantifiers don’t:
  - If all lawyers are crooks then all crooks are lawyers.
  - If most lawyers are crooks then most crooks are lawyers.
- Some, all, three are symmetric. All, most, every, each are non-symmetrical.
Symmetry

- Existential there-sentences admit only symmetrical quantifiers. (see also Lecture 5)

There are some dolphins in the bath.
There are no dolphins in the bath.
There are three dolphins in the bath.
There are all dolphins in the bath.
There are most dolphins in the bath.
There are every dolphins in the bath.
Monotonicity

- Many quantifiers give rise to directional patterns of inference.

- Monotone-increasing (upwards entailing):
  - Permits inferences from subset to set.
    Every man runs fast -> Every man runs

- Monotone-decreasing (upwards entailing):
  - Permits inferences from set to subset.
    No man runs -> No man runs fast
  - (see Lecture 2 for discussion of processing consequences)
Monotonicity

- Classify the following in terms of monotonicity:
  - Few
  - A few
  - Many
  - Some
  - Not many
  - None
  - Most
  - Only
  - At least five
  - Exactly three
Monotonicity

- Non-monotone quantifiers do not permit directional inferences
  Exactly three men run fast $\Rightarrow$ Exactly three men run
  Exactly three men run $\Rightarrow$ Exactly three men run fast

- Monotone-decreasing licence negative polarity items
  Some men ever run backwards.
  Not many men ever run backwards.
  Every boy has any friends.
  No boy has any friends.

(see Lecture 2 for further discussion)
Psychological Approaches
Psychometric Approaches

- How many is many? How much is a lot?
- General observation that people prefer to respond to questionnaires using vague quantifiers rather than specific amounts.
- Consequently, large effort put into mapping quantifiers onto amounts or proportions, or at least establishing the rank ordering of quantifiers.
- For instance, Bass et al. (1974) found in a psychophysical estimation study that “many” was approximately twice as much as “some”.

For discussion, see Moxey & Sanford (1993)
Hormann (1983) showed that interpretation of quantifiers einige (some), mehrere (several), and ein paar (a few) is a function of size of objects and spatial situation of objects.

A few people standing...before a hut.

- ...before the house.
- ...before the city hall.
- ...before the building.

Quantifiers receive higher estimates for situations farther done the list.

Denotations of non-logical quantifiers is context dependent.

For discussion, see Moxey & Sanford (1993)
Context and Estimation

- Moxey & Sanford (1993) showed that denotation of a quantifier depends on the listeners expectations.

  Number of people who might enjoy a party.
  Proportion of audience influenced by a political speech.
  Proportion of female doctors in a local hospital.

- Collected base-line estimations, then evaluated amount denoted by quantifiers.
- Baseline estimations affected number attributed to high-denoting qs (quite a few, many, a lot) only - but no differences for low-denoting qs (very few, few).

For discussion, see Moxey & Sanford (1993)
Truth Verification Approaches

- Considerable interest in 1970s in using sentence-picture comparison task to uncover component processes of truth verification. (for overview, see Just & Carpenter, 1975, 1976; Tanenhaus, Carroll, & Bever, 1976; Wason, 1959)

- Many studies compared evaluation of affirmative and negative sentences.
  - The star is above the square. ★ □
  - The star is not above the square. □ ★

- (Negative sentences typically have longer verification times).

For discussion, see Moxey & Sanford (1993)
Truth Verification Approaches

- Just & Carpenter (1971) also examined explicit and implicit negative quantificational expressions (compared to affirmatives & semantic negatives).
  - Explicit: None of the dots are red.
  - Implicit: Scarcely any / Hardly any / Few of the dots are red.
  - Semantic: A minority of the dots are red.
  - Affirmatives: Many of the dots are red.
- All 3 negatives took longer to verify than affirmatives.

For discussion, see Moxey & Sanford (1993)
Many of the dots are red.
Few of the dots are red.
Truth Verification Approaches

- Just & Carpenter (1972) examined eye movements when verifying affirmative and negative statements.
  - Syntactic Affirmative: Some of the dots are red.
  - Semantic Affirmative: The majority of the dots are red.
  - Syntactic Negative: Few of the dots are red.
  - Negative: The minority of the dots are red.
- For affirmatives, fixation made on red dots first.
- For negatives, fixations made on black dots first.
- Evidence that negatives focus attention onto different parts of information (see Lecture 2 for further discussion).
Reasoning with Quantifiers

- Key explanations of the causes of error.
  - Conversion errors (Chapman & Chapman, 1959; Newstead & Griggs, 1983)

  Some A are B
  Therefore some B are A

  No A are B
  Therefore no B are A

  All A are B
  Therefore all B are A

  Some A are not B
  Therefore some B are not A

Johnson-Laird, Byrne, Tabossi (1989)
- Key explanations of the causes of error.
  - Figural Effects
  - Order of mention influences performance

A – B
B – C
Therefore A – C

B – A
C – D
Therefore C – A
Approaches to syllogistic reasoning

1. Mental models (e.g., Johnson-Laird, 1983)
   - People create mental tableaux representing instances of premises
   - “Read-off” putative conclusion from initial model.
   - Attempt to create alternative models that falsify this conclusion
   - Accept as valid conclusions that cannot be falsified
   - Errors occur due to parsimonious representation of initial model and failures to construct alternative models.
All of the athletes are bakers.

[A] B

[A] B

[A] B

[¬A] [B]

[¬A] [¬B]

(where [ ] indicates exhaustive representation and ... indicates that the model can be further elaborated).

Fully explicit model:
All of the athletes are bakers.

All of the bakers are canoeists

Invites conclusion: All of the athletes are canoeists.

Could check validity of this conclusion by elaborating the model representation.
Reasoning with Quantifiers

1. Mental Models approach also provides an explanation to reasoning with multiple quantifiers, which in rule-based accounts is based on predicate logic.

Premises

None of the painters is in the same place as any of the musicians.
All of the musicians are in the same place as all of the authors.

Conclusion:

None of the painters is in the same place as any of the authors.

Johnson-Laird, Byrne, Tabossi (1989)
None of the painters is in the same place as any of the musicians.

[ | P | P | P | | M | M | M | ]

(Where | demarcates locations)

All of the musicians are in the same place as all of the authors.

[ | P | P | P | | M | M | M | | A | A | A | ]

Invites conclusion:

None of the painters is in the same place as any of the authors.
Reasoning with Quantifiers

- Some criticisms of the approach
  - Generally criticised because it’s not clear how much of the work is achieved by the notation, and whether this is psychologically real.
  - Also concerns about whether approach can be extended to incorporate non-logical quantifiers.
  - For instance, how would it represent most, or 2, or 60? (Geurts, 2003).
   - Reasoners have mental rules akin to those found in formal logic.

1. **Heuristic/Analytic Approaches** (Evans, 2006).
   - Dual process account based on processing heuristics.

   - Humans designed for reasoning with uncertainty
   - Can recast premises in terms of probabilities (i.e., All A are B is $P(B \mid A) = 1$).
   - Conditional probabilities will often restrict possible conclusions, leading to “logical” inference.

1. **Monotonicity of quantifiers** (Geurts, 2003a,b, but see also Newstead, 2003).
Reasoning with Quantifiers

- Little research on reasoning with non-logical quantifiers.
- Johnson-Laird & Byrne (1989) examined “only”.
  - Argued that only is implicitly negative and that this can affect reasoning.
  - In particular, because this negation is overtly represented in model, reasoning should be easier with A than B.

A
Only bankers are authors.
Mark is not a banker.
Therefore, Mark is not an author.

B
All authors are bankers.
Mark is not a banker.
Therefore, Mark is not an author.
Reasoning with Quantifiers

- Geurts (2003) reports studies using ‘at least n’ & ‘at most n’.
  - Surprisingly easy to reason with many syllogisms (see example below).

At least 3 reporters played against more than 2 foresters.
All foresters were communists.
Therefore, at least 3 reporters played against more than 2 communists.
Language Comprehension

- Growing interest in understanding process of assigning a semantic interpretation to a sentence.
- That is, how do readers and listeners combine lexical semantic information and syntactic information (and context) to determine the meaning of a sentence.
  - What aspects of sentence meaning are computed during comprehension?
  - What mechanisms underlie ambiguity resolution?
- Processing of quantifiers is a central issue in this research - See Lectures 2, 3, and 4.
Acquisition

- Longstanding interest in the acquisition of quantification, originating in Piaget’s groundbreaking research on children’s comprehension & reasoning.

- Piaget observed non-adult performance in children’s reasoning with quantifiers.
  - What are the nature of this non-adult performance?
  - Can accounts of language acquisition explain these differences?

- Continuing interest in acquisition of quantification reveals substantial differences between child and adult performance - see Lecture 5.
Challenges

- Generalized Quantifier Theory provides a powerful framework for the formal analysis of natural language semantics.
  - To what extend does its approach and principles provide a framework for understanding natural language processing?
  - Similarly, can this formal framework provide insights into human reasoning?
- Following lectures focus on psychological / psycholinguistic research on the acquisition and processing of natural language quantifiers.
References

- Hörmann, H. (1983). The calculating listener or how many are einige, mehrere, and ein paar (some, several, and a few)? In R. Baurle, C. Shwarze, & A. Van Stechow (Eds.), *Meaning, use, and interpretation of language*. Walter de Gruyter.
References

References