Workshop on Topology in Dynamics

University of Leicester
Leicester, United Kingdom

April 3–7, 2017

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Quasicrystal formation in soft matter

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Complex quasiperiodic structures are observed in a variety of soft matter systems, including dendrimers. We investigate the formation and stability of such quasicrystals in both two [1, 2] and three [3] dimensions using density functional theory and a phase field crystal model. The molecular interactions result in two length scales (in the golden ratio) being favoured, which in the model can be controlled independently. In addition to regular crystals, one-, two- and three-dimensional quasicrystals can be found. We compute the minima of the free energy of the competing structures to determine the phase diagram. In three dimensions, the icosahedral quasicrystal can be the global minimum free energy state. We find that strong nonlinear coupling between density waves at the two length scales are responsible for stabilizing the icosahedral quasicrystal. Density functional theory and Brownian dynamics simulations reveal that the two-dimensional system has a fluid phase and two crystalline phases with different lattice spacing. Of these the larger lattice spacing phase can form an exotic periodic state with a sizeable fraction of highly mobile particles: a Crystal-Liquid. Near the transition between this phase and the smaller lattice spacing phase, quasicrystalline structures may be created by a competition between linear instability at one scale and nonlinear selection of the other. This dynamic mechanism for forming quasicrystals is qualitatively different from mechanisms observed previously. The system first forms a small length scale crystal. Only when this phase is almost fully formed (i.e., the dynamics is far into the nonlinear regime) does the longer length scale start to appear, leading to the formation of quasicrystals.

[1] A.J. Archer, A.M. Rucklidge, and E. Knobloch, Phys. Rev. Lett. 111, 165501 (2013).

[2] A.J. Archer, A.M. Rucklidge and E. Knobloch, Phys. Rev. E 92, 012324 (2015).

[3] P. Subramanian, A.J. Archer, E. Knobloch and A.M. Rucklidge, Phys. Rev. Lett. 117, 075501 (2016).

Reversing and extended symmetries of shift spaces

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The reversing symmetry group is considered in the setting of symbolic dynamics. While this group is generally too big to be analysed in detail, there are

interesting cases with some form of rigidity where one can determine all symmetries and reversing symmetries explicitly. They include Sturmian shifts as well as classic examples such as the Thue-Morse system with various generalisations or the Rudin-Shapiro system. We also look at generalisations of the reversing symmetry group to higher-dimensional shift spaces, then called the group of extended symmetries. We develop their basic theory for faithful $\mathbf{Z}^d$-actions, and determine the extended symmetry group of the chair tiling shift, which can be described as a model set, of Ledrappier's shift, which is an example of algebraic origin, and of the visible lattice points, which is a recent example in the class of weak model sets.

This is joint work with John Roberts and Reem Yassawi.

Minimal compacta with nonminimal squares

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In this talk I will sketch a method of constructing compact spaces that admit minimal homeomorphisms, but whose Cartesian square do not admit such homeomorphisms. The talk is based on [1].

References

[1] Boronski J.P., Clark A., Oprocha P. *A compact minimal space Y such that its square $Y \times Y$ is not minimal*, arXiv:1612.09179

Matching in a one-parameter family of piecewise affine maps.

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I will discuss the phenomenon of matching (i,e., that the left and right orbit of a discontinuity will synchronize after a number of iterates) for a family of piecewise affine interval maps. Matching causes invariant measures to be piecewise constant, and entropy to behave tamely as function of the parameter. In particular cases, matching occurs prevalently in parameter space, yet the non-matching parameter set becomes interesting, with various self-similarity properties.

Graph maps, inverse limits and measurable pseudo-Anosov maps

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In this talk we present a construction which applies to graph maps in general and yields interesting surface homeomorphisms as follows:

1) a pseudo-Anosov map if the graph map is a train track map; 2) a generalized pseudo-Anosov map if the graph map is post-critically finite and has an irreducible aperiodic transition matrix; 3) an interesting type of surface homeomorphisms - which we call measurable pseudo-Anosov maps - which generalizes both the previous classes otherwise.

In particular, this produces a unified construction of surface homeomorphisms whose dynamics mimics that of the tent family of interval endomorphisms, completing an earlier construction of unimodal generalized pseudo-Anosov maps in the post-critically finite case.

Pseudo-suspensions

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In this talk we will explain the technique of pseudo-suspension and show how it allows us to construct hereditarily indecomposable continua of intermediate complexity. Along the way we will review some of the exotic properties such spaces have and show that nonetheless they occur as minimal sets of diffeomorphisms.

References

[1] Boronski J.P., Clark A., Oprocha P. *New exotic minimal sets from pseudo-suspensions of Cantor systems*, arXiv:1609.09121

Extended symmetries of algebraic subshifts
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Baake, Roberts and Yassawi defined extended symmetries of zero-dimensional shift spaces. They proved that the full shift has a full extended symmetry group, but the Ledrappier shift has a finite extended symmetry group. It is natural to suspect that, more generally, algebraic subshifts will have a finite extended symmetry group if they are mixing. But this is not easy to prove. In this talk I will explain why it is natural to expect this, and why it is not easy to prove. This is work in progress with Reem Yassawi and others.

Shadowing, chain transitivity and omega limit sets
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Let $f : X \to X$ be a continuous map on a compact metric space. A subset A of X is internally chain transitive provided for any $\delta > 0$ and any two points x and y in A, there is a δ-pseudo orbit of points in A starting at x and ending at y. It is well known that omega limit sets are internally chain transitive, and for some systems such as Shifts of Finite Type, a set is internally chain transitive if an only if it is an omega limit set. Moreover if f has shadowing and the set of omega limit sets is closed in the Hausdorff metric, (for example any map of the closed interval with shadowing), then again a set is internally chain transitive if and only if it is an omega limit set.

In this talk we consider under what conditions omega limit sets are completely characterized by internal chain transitivity. It turns out that variants of Pilyugins notion of orbital shadowing are what is required.

Inverse limits of unimodal maps
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I will present three recent results about inverse limits of unimodal maps:

- The maximum admissible (backwards) itinerary mode locks.

- Natural extensions of unimodal maps are semi-conjugate to sphere homeomorphisms, and in the cases of tent maps and quadratic maps the semi-conjugacy has mild point preimages.
- A description of the prime ends of Barge-Martin embeddings into the disk.

A common theme is that the statements and proofs of these results rely heavily on the *height* $q(f) \in [0, 1/2]$ of the unimodal map.

Homotopy observations on substitution tiling spaces

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Since work of Kellendonk and of Anderson & Putnam at the turn of the century, the main approach to the computation of cohomology for a substitution tiling space has been through the use of various associated finite cell complexes, *approximants*. A number of possible models have been developed, in large part driven by the aim of constructing approximants as simple as possible so as to facilitate computation. An observation, implicit in the work on this by Barge & Diamond, is that for cohomology one only needs a model which is shape equivalent to the tiling space, not homeomorphic to it. This talk, part of a joint project with Greg Maloney, looks at how some systematic homotopy theory can use this observation and in many cases can simplify the approximants usually considered. As an illustration, we give an conceptual proof why the "half-collaring" technique of Gaehler and Maloney works.

Lyapunov exponents and absence of absolutely continuous diffraction

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Aside from the Rudin-shapiro substitution and its extensions (via Hadamard matrices by Frank, and via Lax and Grimm's construction), no other one-dimensional substitution has been found to contain an absolutely continuous diffraction component. In this talk, I will present a method of confirming this absence via the positivity of the Lyapunov exponents which are computable from the matrix cocyles associated to the substitution, which are intimately related to a renormalization scheme that can be extended from letter frequencies to measures. Results for which this is affirmative will be tackled, particularly for aperiodic constant length substitutions (binary and n-letter bijective abelian).

Shadowing property, entropy and ergodic subsystems.

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When we investigate the space of invariant measures from ergodic theory point of view, we are usually not that much interested in the topological structure of underlying space. By famous Jewett-Krieger theorem, we can view invariant measures as supported on minimal systems and numerous further generalizations allow to add even more topological (dynamical) properties to the underlying system. On the other hand, there are examples of systems with quite rich dynamical structure (e.g. topologically mixing) but not that much interesting invariant measures (e.g. only trivial measure, only atomic measures, etc.). In other words, connections between topology and ergodic theory (on compact metric spaces) is not that tight.

In this talk we will provide some characterizations of invariant measures in the case when a dynamical system (X, T) has the shadowing property. We will show that often invariant measures can be approximated by a special class of minimal dynamical systems. We will also comment on possibilities of approximation of entropy.

Shadowing is strong in tent maps

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The shadowing property (SP), introduced in 1972 by Sinai, can be roughly described as the property shared by those systems whose long term behaviour, up to a given degree of accuracy, can be efficiently computed. Many variations on this classical notion have been introduced since. One, which we find interesting because of its connection to ω-limit sets, is so called limit (or asymptotic) shadowing (LmSP). Although demonstrably disparate properties, there are indications that in the class of interval maps (SP) implies (LmSP).

In this talk we show that this is indeed the case for tent maps and, more generally, for piecewise linear interval maps with constant slopes.

Dynamics of stochastic substitution subshifts

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Stochastic substitutions are a natural generalisation of their classical 'deterministic' counterpart, whereby at every step of iterating the substitution, instead of replacing a letter with a predetermined word, every letter is independently replaced by a word from a finite set of possible words according to a probability distribution.

Previous work has focused on particular examples or classes of such substitutions but until now, very little has been done in order to lay the groundwork for a general framework. We'll discuss the subshifts which are associated to such substitutions and, via plenty of examples, explore the dynamical properties of these systems. Throughout, we'll compare results from the deterministic setting to the corresponding results from the stochastic setting.

Around the Furstenberg Topology

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Let T be a topological group acting on a compact metric space X. For an idempotent u in the enveloping semigroup E, one may define a special subset of X, denoted by Xu. This set is of fundamental importance for phenomena related to distality. In general Xu is not closed, hence not compact. However, one may weaken the topology on Xu to obtain a compact space; the new topology is called the Furstenberg topology after its creator.

This talk will provide an introduction to the theory behind the Furstenberg topology, leading up to an interesting result by Veech concerning a link between its separation properties and the maximal equicontinuous factor of the system. Finally, we will mention a partial generalisation of Veech's theorem. In it, we consider some recently-introduced topological separation properties which until now have mostly been studied in the context of general topology, and link them to the extension of the maximal equicontinuous factor. In this way, we make one direction of Veech's theorem applicable in the case of more general systems, such as the Thue-Morse or Rudin-Shapiro.

Topological horseshoes for surface homeomorphisms

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We have recently introduced a new orbit forcing theory for surface homeomorphisms, based on the concepts of maximal isotopies and Brouwer-Le Calvez foliations. In this recent work we present a simple topological criteria for the existence of a topological horseshoe. We will also describe some of the applications, including descriptions of transitive sets for zero-entropy homeomorphisms of the sphere, and the existence and continuity of rotation numbers for nonwandering points of zero-entropy homeomorphisms of the Annulus.

Cohomology computations for the Euclidean hull of a tiling

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The Euclidean hull of Penrose tilings is the space of all tilings of rigid motions of kite and dart tiles, meeting according to their matching rules at their boundaries. I shall outline two general methods of computing the cohomology of the Euclidean hull of a two-dimensional tiling. These ideas are based on the work of [1], in which the cohomology of the Euclidean hull of Penrose tilings is determined. As time permits, I shall discuss some recent joint work with John Hunton in generalising these ideas to higher dimensions.

[1] J.J. Walton, Cohomology of rotational tiling spaces, arXiv:1609.06606, Sep. 2016

C*-algebras associated with aperiodic tilings and their classification

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I will introduce Kellendonk's C*-algebra associated with an aperiodic tiling and show that these algebras are classifiable by the Elliott invariant (essentially K-theory). Classification follows from showing that Kellendonk's tiling groupoids have finite dynamic asymptotic dimension and the proof is mainly topological. This talk will not assume a background in C*-algebras and I will introduce all the relevant notions.