The category of convergences has several nice properties, like Cartesian-closedness, that its subcategory of topologies does not have. Therefore, numerous questions formulated in topological terms admit more luminous formulations in the framework of convergences. The relation between topologies and convergences is analogous to that between real and complex numbers.

Compact families constitute a common generalization of compact sets and convergent filters. Their applications are multiple. For instance, open sets in Scott hyperspace topology have been characterized as compact families. A generalization of the Tikhonov product theorem says that a filter in a product space is compactoid (every finer ultrafilter is convergent) if and only if its every projection is compactoid.

It is interesting that compactness is not a topological notion, but a pseudotopological one, like countable compactness is a paratopological concept (pretopologies, paratopologies and pretopologies are reflective classes of convergences which include topologies). In fact, the evoked generalization of the Tikhonov theorem is an immediate consequence of the fact that the pseudotopologizer S has a (remarkable) property of commuting with arbitrary products

\[ S(\prod \xi) = \prod_{\xi \in \Xi} S\xi. \]

Completeness of a convergence is a notion relative to that of fundamental filter with respect to a collection of (convergence) covers. If \( \mathcal{P} \) is such a collection, then a filter \( \mathcal{F} \) is called \( \mathcal{P} \)-fundamental if \( \mathcal{F} \cap \mathcal{P} \neq \emptyset \) for each \( \mathcal{P} \in \mathcal{P} \); a convergence is \( \mathcal{P} \)-complete if each \( \mathcal{P} \)-fundamental filter is adherent. The completeness number \( \text{compl}(\xi) \) of a convergence \( \xi \) is the least cardinal such that there exists a collection of covers \( \mathcal{P} \) of \( \xi \) of that cardinality, for which \( \xi \) is-complete. In this setting, a convergence \( \xi \) is countably complete if \( \text{compl}(\xi) \leq \aleph_0 \) (Cech-complete if \( \xi \) is a Tikhonov topology), locally relatively compact whenever \( \text{compl}(\xi) < \aleph_0 \), thus \( \text{compl}(\xi) = 1 \), compact if \( \text{compl}(\xi) = 0 \).

This perspective enables one to unify the study of various variants of completeness and compactness, in particular, that of their preservation under operations on convergences. For example, the Tikhonov product theorem is also an immediate consequence of the following formula for the completeness number:

\[ \text{compl}(\prod \Xi) = \sum_{\xi \in \Xi} \text{compl}(\xi). \]