Climate change, catastrophic risk and the pure rate of time preference

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Background

- Economists insist on discounting future costs and benefits in dynamic analysis. This includes deterministic Integrated Assessment Models (IAMs) of climate change.
- Many scientists and concerned citizens of the world are inclined to thinks that if a 1%-point increase in the discount rate leads to large increases in the probability of severely catastrophic events, then something is seriously wrong with economic analysis.
- This has lead to a strained discourse between economists and other participants.

More background

- Debate on discounting have spilled into the economics profession.
- Stern uses ”low” discounting to “get” results palatable to scientists.
- Dasgupta, Nordhaus and Weitzman have aggressively challenged these results.
- Nævdal and Vislie will here make the claim that if we are concerned about serious threshold effects with catastrophic outcomes, the role of discounting is minor. Focus on costs, risk and uncertainty.

Other views on discounting of catastrophic risk

- See Weitzman, Gollier and others

A Very Stylized Model Emissions

- There is an emission activity u, which generates instantaneous economic benefits.
- The cost of reducing u from its unregulated level, u^0, is given by $(c/2)(u^0 - u)^2$
- Emissions of u leads to the accumulation of stock pollutants driven by $\dot{x} = u - \delta x$
- The only environmental damage in the model comes from a threshold effect
- Constant marginal damage from stock, $-a$.

The threshold and the Threshold Effect

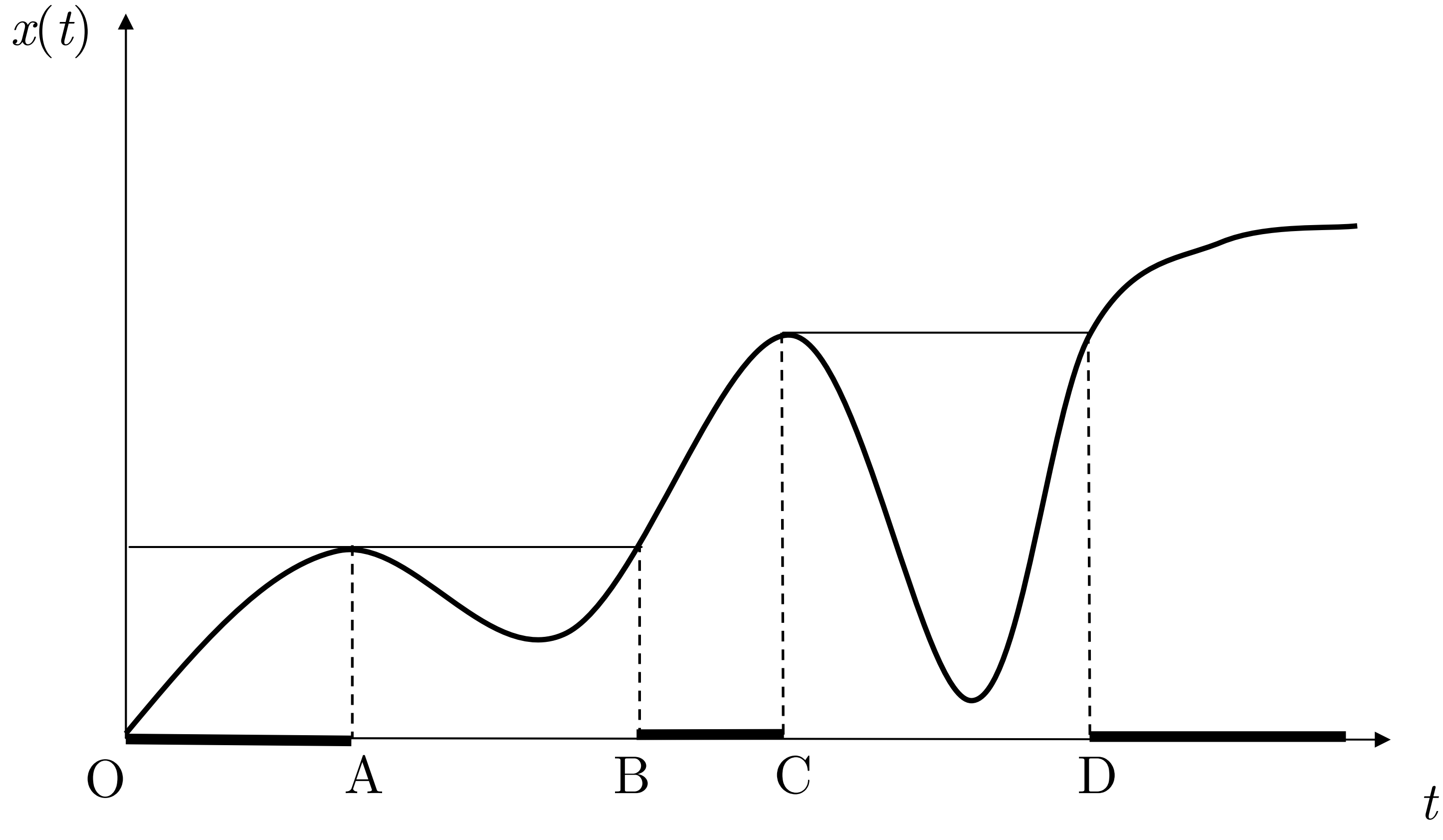
- There is a threshold $\underline{x}$ such that if there is some τ where $x(\tau) = \underline{x}$ then this triggers a catastrophic event.
- For simplicity $\underline{x}$ is assumed to be exponentially distributed over $[x(0), \infty)$ with intensity λ.
- The catastrophic event is assumed to be given by a constant stream of disutility given by G.
- Interpretation of G. Solves the equation:

$$\int_{\tau}^{\infty} (Damages) e^{-rt} dt = e^{-r\tau} \frac{G}{r}$$

Intuitively…

- You are running towards the edge of a cliff with a blindfold.
- The faster you run, the more money you earn
- The faster you run the higher is the probability that you go over in the next unit of time.
- If you stop or go backwards this probability becomes zero.

Threshold risk – Arbitrary $x(t)$



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The Stochastics of Thresholds

- As shown, threshold effects have potentially complicated risk structures.
- One way to incorporate risk into optimal control models is to use the hazard rate

  $\lim_{dt \to 0} \Pr(\text{disaster in } [t,d+dt] | t>t)/dt$

- For our problem the hazard rate is:

$$\lambda \times \max(\dot{x}, 0) = \lambda \times \max(u - \delta x, 0)$$

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The Model

- Solved with backwards induction
- First solve the optimization problem conditional on the catastrophe having happened.

$$J(\tau, x(\tau)) = e^{r\tau} \max_{u(s)} \left(\int_{\tau}^{\infty} \left(-G - ax - \frac{c}{2}\left(u^0 - u\right)^2 \right) e^{-rs} ds \right)$$

- Here this is real easy. After the catastrophe *G* is a constant. Does not matter.

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Solution after disaster:

$$u\left(s\mid\tau,x\left(\tau\right)\right)=u^{0}-\frac{a}{c\left(r+\delta\right)}$$

$$\mu\left(s\mid\tau,x\left(\tau\right)\right)=-\frac{a}{r+\delta}$$

$$x\left(s\mid\tau,x\left(\tau\right)\right)=\frac{\left((cu^{0}(r+\delta)-a)+e^{\delta(s-t)}(a+c(r+\delta)(x\left(\tau\right)\delta-u^{0}))\right)}{c\delta(r+\delta)}$$

$$J\left(\tau,x\left(\tau\right)\right)=-\frac{a}{r+\delta}x\left(\tau\right)+\frac{a^{2}-2acu^{0}(r+\delta)}{2cr(r+\delta)^{2}}-\frac{G}{r}$$

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The pre catastrophe problem

- $\gamma(t)$ is the damage variable. = 0 before the catastrophe and $-G$ after.

$$\max_{u(t)} E\left(\int_0^\infty \left(\gamma(t) - ax - \frac{c}{2}\left(u^0 - u\right)^2\right) e^{-rt} dt\right)$$

- Form the Hamiltonian:

$$H = \gamma - ax - \frac{c}{2}\left(u^0 - u\right)^2 + \mu(u - \delta x) + \lambda_\tau(t)\left(J\left(t, x(t)\right) - z(t)\right)$$

Optimality conditions (simplified)

- The standard stuff...

$$u = \arg\max H = u^0 + \frac{\mu}{c} + \frac{\lambda}{c}\left(J\left(t, x\left(t\right)\right) - z\right)$$

$$\dot{\mu} = r\mu - \frac{\partial H}{\partial x}$$

- Except this bit:

$$\dot{z} = rz + \frac{c}{2}\left(u^0 - u\right)^2 + \lambda\left(u - \delta x\right)\left(-\frac{G}{r} - z\left(t\right)\right)$$

What is z?

- When incorporating catastrophic risk into an optimal control problem, part of the problem is that we need to spell out the *net* consequences of a disaster from the time it occurs.

$$\left(J\left(t,x\left(t\right)\right)-z\left(t\right)\right)$$

Expected value of ”remaining” utility at time t.

Current value of disaster happening At time t.

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Revisiting the Hamiltonian

- The Hamiltonian is the derivative of the objective function at time *t*. NNP adjusted for risk.

$$H = \underbrace{\gamma - \frac{c}{2}\left(u^0 - u\right)^2}_{\text{Todays utility}} + \underbrace{\mu(u - \delta x)}_{\text{Value of savings}} + \underbrace{\lambda\left(u - \delta x\right)\left(-\frac{G}{r} - z\left(t\right)\right)}_{\substack{\text{Probability of catatrophe at time t} \\ \text{multiplied by cost of disaster}}}$$

Steady state quantities

$$u_{ss} = \lim_{t\to\infty} u(t) = u^0 - \frac{a}{c(r+\delta)} + \frac{1}{\lambda}\left((r+\delta) - \sqrt{(r+\delta)^2 + 2G\frac{\lambda^2}{c}} \right)$$

$$x_{ss} = \lim_{t\to\infty} x(t) = \frac{u^0}{\delta} - \frac{a}{c\delta(r+\delta)} + \frac{1}{\lambda\delta}\left((r+\delta) - \sqrt{(r+\delta)^2 + 2G\frac{\lambda^2}{c}} \right)$$

Also depends on *r*, but how strong is the interaction?

Let's check the steady state stock

$$x_{ss} = \lim_{t \to \infty} x(t) = \frac{u^0}{\delta} + \underbrace{\frac{-a}{c\delta(r+\delta)}}_{\Delta x_a^{ss}} + \underbrace{\frac{1}{\lambda\delta}\left((r+\delta) - \sqrt{(r+\delta)^2 + 2G\frac{\lambda^2}{c}}\right)}_{\Delta x_G^{ss}}$$

Δx_a^{ss} = Steady state reduction in stock caused by the deterministic stock pollutant effect

Δx_G^{ss} = Steady state reduction in stock caused by the threshold effects

- Note how *r* affects these two entities differently

Let's check the parenthesis

$$x_G^{ss} = \left((r+\delta) - \sqrt{(r+\delta)^2 + 2G\frac{\lambda^2}{c}} \right)$$

- Clearly negative for G and $\lambda > 0$. Is on the form:

$$A - \sqrt{A^2 + B}$$

- Not likely to be very dependent on *r* for two reasons
  - $r+\delta = A$ is a very small number compared to *B* if we are talking about a serious catastrophe
  - The way *A* enters the expression is close to $A - \sqrt{A^2}$

To make the point clear!

- δ is no larger than 1/200 and probably smaller so let us set it to zero.

$$\mathrm{EI}_r \Delta x_a^{ss} = -1,$$

$$\mathrm{EI}_r \Delta x_G^{ss} = -\frac{r}{\sqrt{r^2 + \frac{2G\lambda^2}{c}}}$$

- Increasing *r* from 5% to 5.05% has a huge impact on the deterministic part but next to nothing on the threshold part

Steady State Solutions – Prices and values ($a = 0$ for clarity)

$$\mu = \frac{-\delta\left(c(r+\delta) - c\sqrt{(r+\delta)^2 + 2\frac{G}{c}\lambda^2}\right)}{r\lambda} > 0$$

$$z = \frac{-c\left(\left((r+\delta)^2 + \frac{G}{c}\lambda^2\right) + \sqrt{(r+\delta)^2 + 2\frac{G}{c}\lambda^2}\right)}{r\lambda^2} < 0$$

- Value function is negative in steady state.
- Shadow price of *x* is positive!?
  - Explanation the shadow price is the value of a risk free marginal increase in *x*
- Obviously highly dependent on *r*

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Extensions 1

- The climate model here is perhaps too simplistic. Athmospheric CO2 causes radiative forcing which affects the rate of change in temperature. We thus write our climate model as:

$$\dot{x} = u - \delta x$$

$$\dot{F} = \alpha x - \beta F$$

- *F* is temperature. α and β are positive parameters. I expect β to be a rather small number.

The role of discounting in a more complex climate model - result

- The steady state reductions in atmospheric CO_2 may now be written:

$$\Delta x_F^{ss} = \frac{1}{\alpha\lambda\delta}\left((r+\delta)(r+\beta) - \sqrt{(r+\delta)^2(r+\beta)^2 + 2G\frac{\alpha^2\lambda^2}{c}}\right)$$

- Same algebraic structure as before. *r* enters additively in the numerator. If β is a small number, *r* will have an negligible effect on the stabilization target.

Extensions 2

- The model had a very stylized representation of costs and benefits in instantanuous utility.
- May be generalized to a utility function of the form:

$$\int_0^\infty \Phi(u,\gamma)e^{-rt}dt$$

- I havn't completed the proof yet, but it looks like…

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More general utility function

- … the effect of the discount rate on marginal utility will in steady state look something like:

$$\Phi'_u(u,\gamma) = \text{Constant} \times \left((r+\delta) - \sqrt{(r+\delta)^2 + 2G\frac{\lambda^2}{c}} \right)$$

- This in contrast to the case where we are concerned with a deterministic stock pollutant

$$\Phi'_u(u,x) = \frac{Expression}{r+\delta}$$

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What it means

- In both cases, the changes in the discount rate will affect steady state emissions by forcing changes in marginal utility.
- However, the effect is much stronger with a deterministic stock pollutant, than with a a threshold effect because changes in small numbers in numerators have a smaller effect than changes in small numbers in the denominator.

Message!

- It does not appear that the discount rate is very important for at what level one wants to stabilize the stock pollutant if one is mostly concerned about threshold effects.
- This does not imply that the discount rate is unimportant for how to reach the steady state
- Further it can be shown that the discount rate is important for other questions such as when is it optimal to let steady state $x = 0$.

Some more

- Note that the hazard rate affects the problem in a way that is inverse to the way the discount rate works.
  - In many problems with *exogenous* risk, the hazard rate and the discount rate appear as a sum in the denominator.
  - Here the risk is endogenous which flips the relationship around
- Economists who look at IAM models and scientists concerned about threshold risk are not looking at the same problem, which is why they have trouble talking to one another
- Thank you for your attention