

## SOME EXAMPLES OF STABILITY MODE PERSISTENCE AND CHANGE IN VORTEX STRUCTURE FORMATION

Aldo Rona<sup>1</sup>, Stephen J. Garrett<sup>2</sup> & J. Paul Gostelow<sup>1</sup>  
<sup>1</sup>*Department of Engineering, University of Leicester, Leicester, UK*  
<sup>2</sup>*Department of Mathematics, University of Leicester, Leicester, UK*

**Abstract** Three rather different physical cases have been studied. All represent very practical cases for which the modal behaviour of vortical structures is not completely understood. The work on these problems is on-going and it is hoped that physical confirmation, enhanced understanding and predictive capability for the vortical structures encountered will eventuate.

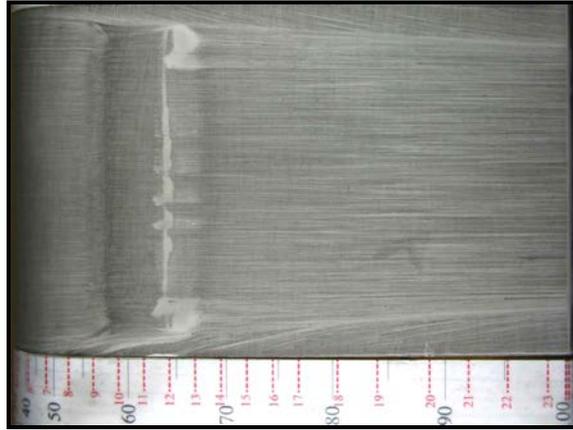
### EXAMPLE 1: SURFACE FLOWS OVER ROTATING CONES

Experimental studies have shown that the boundary-layer flow over a rotating cone is susceptible cross-flow and centrifugal instability modes of spiral nature, depending on the cone sharpness. For half-angles ( $\psi$ ) ranging from propeller nose cones to rotating disks ( $\psi \geq 40^\circ$ ), the instability triggers co-rotating vortices, whereas for sharp spinning missiles ( $\psi < 40^\circ$ ), counter-rotating vortices are observed. A mathematical description is provided of the onset of co-rotating vortices for a family of cones rotating in quiescent fluid, with a view towards explaining the effect of  $\psi$  on the underlying using numerical and asymptotic methods transition of dominant instability. The stability of inviscid cross-flow modes (type I) is investigated, as well as modes arising from a viscous–Coriolis force balance (type II). The influence of  $\psi$  on the number and orientation of the spiral vortices is examined, with comparisons drawn between the two distinct methods as well as with previous experimental studies. The results indicate that increasing  $\psi$  has a stabilizing effect on both the type I and type II modes. Favourable agreement is obtained between the numerical and asymptotic methods presented here and existing experimental results for  $\psi > 40^\circ$ . Below this half-angle it appears that an alternative instability mechanism is at work, which is not amenable to investigation using the broad-cone formulation. An alternative formulation has been developed and properties of the counter-rotating vortices successfully predicted.

### EXAMPLE 2: TURBINE BLADES AND SWEEPED CYLINDERS

Suction surface flow visualization on turbine blades at subsonic and transonic speeds showed robust streamwise streaks on a lengthy time-average basis (Figure 1). The flow on the suction surface, under the influence of a strong favourable pressure gradient, was initially laminar. Further downstream laminar separation and transition to turbulence were encountered. The turbulent boundary layer region then persisted to the trailing edge. In this journey, the streamwise streaks were unaffected regardless of the boundary layer state. Similar behaviour has been observed by Halstead [1], who had surface film confirmation of the boundary layer state throughout. The streamwise vortical structures, whilst not particularly strong, are persistent and would seem to exert a stabilizing influence. The lateral spacing between streaks on convex surfaces had been predicted by Kestin and Wood [2].

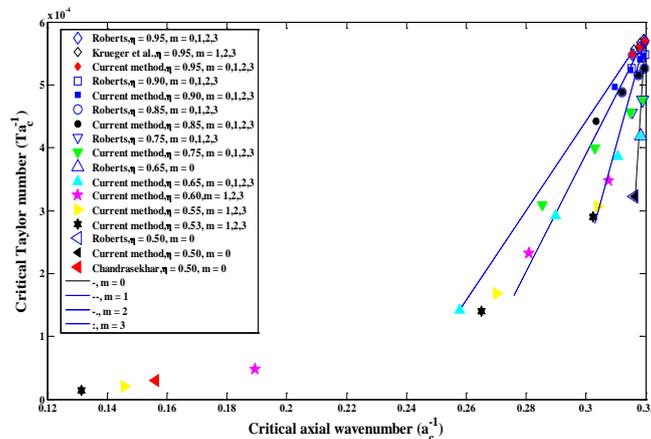
Observations of streaks on turbine blades and unswept cylinders were to provide a firm basis for referencing the influence of sweep. High speed testing was undertaken on a 38 mm diameter cylinder and low-speed testing on an unswept 152 mm diameter cylinder. The results are in excellent agreement with the Kestin and Wood theory, providing confirmation of the theory for circular cylinders. Although the Kestin and Wood work [2] is related to un-swept circular cylinders it also provides an excellent benchmark for sweep effects on turbomachinery blading. The introduction of sweep brings consideration of a wide range of instabilities. Cross-flow instability results from the inflectional behaviour of a three-dimensional boundary layer and is thought to be prominent and aggressive. Experimental work on a circular cylinder has been undertaken by the authors over a range of sweep angles from zero to 610 giving results for lateral spacing and angular orientation of the vortical streaks. At high-sweep angles the results are consistent with those of Poll [3] and of Takagi *et al.* [4]. It is demonstrated that streamwise and cross-flow structures are present on the suction surface of swept and un-swept turbine blades. The available information on fine structures comes from surface flow visualization. Work is now progressing on hot wire measurements away from the surface. The aim is to demonstrate the relationship between the structures and the surface traces. It is also hoped to provide information on the changing behaviour of the vorticity as the sweep angle is increased. The streamline curvature disturbance has been found to be stationary in nature and to be resilient, often persisting from leading edge to trailing edge. The cross-flow instability becomes more significant as sweep is increased. It grows aggressively and rapidly, being predominantly of a travelling nature, and has a major role to play in the transition process.



**Figure 1.** Surface flow visualization of the suction surface of a turbine blade at a discharge Mach number of unity.

### EXAMPLE 3: TAYLOR VORTICES BETWEEN ROTATING CYLINDERS WITH WIDE GAP

Taylor vortices develop in the gap between concentric rotating cylinders when the Taylor number  $Ta$  exceeds the first critical value. Concentric rotating cylinders with radius ratios  $\eta = 0.53$ , aspect ratios  $\Gamma = 11.36$  and  $\mu = 0$  have been investigated. The test case is characterized by a larger annular gap width  $d$  than classical journal bearing test cases and by a Taylor number of  $2.36 \times 10^6$ , which is beyond the first critical Taylor number at which Taylor vortices develop. Some interesting flow features are observed in this region of flow parameters. Analytical results at the azimuthal wave-number  $m = 3$  show a pattern that deviates from the linear trend reported in previous literature as the radius ratio  $\eta \leq 0.6$  (Figure 2). This amounts to the onset of a new behaviour at this radius ratio, as further indicated by a step change in the gradient of the predicted variation of the axial wave-number and of the azimuthal wave velocity with the radius ratio at  $\eta = 0.65$ . The flow pattern and flow variables from Particle Image Velocimetry at  $\eta = 0.53$  show wavy vortex flow between the concentric cylinders with low aspect ratio  $\Gamma < 25$  and at a high Taylor number, well beyond the published Taylor number for transition to turbulent flow. This regime is characterised by an enhanced waviness near the end-walls as compared to the wavy flow in the central region, without the flow breaking up to a turbulent state, for this test case. At the tested Taylor number, the experimental result suggests that the azimuthal wave-number  $m = 3$  does not persist and the flow displays  $m = 1$  characteristics.



**Figure 2.** Variation of  $Ta_c^{-1}$  with  $a_c^{-1}$  for  $\mu = 0$ ,  $0.50 \leq \eta \leq 0.95$ , and  $0 \leq m \leq 3$ . Current method (full symbols), Roberts [5], Krueger *et al.* [6] and Chandrasekhar (open symbols).

### References

- [1] D. Halstead. The use of surface-mounted hot-film sensors to detect turbine-blade boundary-layer transition and separation. *MS Thesis*, Iowa State University, 1989.
- [2] J. Kestin and R. Wood. On the stability of two-dimensional stagnation flow. *Journal of Fluid Mechanics* **44**: 461–479, 1970.
- [3] D. Poll. Some observations of the transition process on the windward face of long yawed cylinders. *Journal of Fluid Mechanics* **150**: 329–356, 1985.
- [4] S. Takagi, N. Tokugawa & N. Itoh. Characteristics of unsteady disturbances due to streamline curvature instability in a three-dimensional boundary layer. *Proceedings of the 6<sup>th</sup> IUTAM Symposium on Laminar-Turbulent Transitions*, R. Govindarajam, ed., Springer, New York: 369–374, 2006.
- [5] R. Donnelly and K. Schwarz. Experiments on the stability of viscous flow between rotating cylinders. VI. Finite-amplitude experiments. *Proceedings of the Royal Society of London, A* **283**: 531–556, 1965.
- [6] E. Krueger, A. Gross & R. Di Prima. On the relative importance of Taylor-vortex and non-axisymmetric modes in flow between rotating cylinders. *Journal of Fluid Mechanics* **24**: 521–538, 1966.
- [7] S. Chandrasekhar. The stability of viscous flow between rotating cylinders. *Proceedings of the Royal Society of London, A* **246**: 301–311, 1958.