Transition mechanisms within the boundary-layer flow over slender vs. broad rotating cones

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Aims

• Motivate the hypothesis of an alternative mode of instability within the rotating-cone boundary layer.

• Briefly summarise the preliminary mathematical analysis.
Background: rotating disk and cone

Flow vis due to Kohoma, Kobayashi et al.
Transitional region over rotating cones

- Disks and broad cones ($\Psi > 40$ degs)
  - Co-rotating stationary cross flow vortices.
  - Disk analyses show that related to type I and II convective modes.

- Slender cones
  - Pairs of counter-rotating stationary Gortler-type vortices, indicating centrifugal instability.
Onset of turbulence over rotating cones

• Nickels’ experiments on a family of cones
  - Broadest cone onset at local Re=2.5x10^5, independent of rotation rate
  - Slender cones onset in advance of this and dependent on rotation rate
• Consistent with Kobayashi & Izumi (1982)
Theoretical studies of the transitional region

- Garrett, Hussain & Stephen (JFM, 2009)
  - Formulation consistent with previous disk literature
  - Stationary modes
  - Numerical and asymptotic analyses in agreement
  - Identify type I and II convective modes
Experimental comparisons

- Critical Re close to experimental observations for broad cones only
- ...suggests an alternative centrifugal mode
Growth rates of type I and II modes


- Type I an II growth rates decrease with half-angle
Hypothesis:

There exists an alternative Gortler-type mode that dominates for small half-angles.

- We have been unable to find this using a formulation consistent with that used on a rotating disk.
- Need an alternative formulation...
Formulation

- Axes aligned with spiral vortices with particular angle $\Phi$
- Scale *all* spatial variables on boundary-layer thickness.
Asymptotic analysis (short wavelength)

- Perturbing quantities: \( \{ \tilde{u}(\eta) \}, \tilde{v}(\eta), \tilde{w}(\eta) \} \exp(iax + iby) \)
- Taylor number: \( T = \frac{2 \cot \psi \cos \phi}{\sin^5 \psi} \)
- Expansions,

\[
\begin{align*}
 a & = \epsilon^{-1} \\
 \tilde{u} & = E(u_0(\eta) + \epsilon u_1(\eta) + \epsilon^2 u_2(\eta) + \ldots), \\
 \tilde{v} & = \epsilon^2 E(v_0(\eta) + \epsilon v_1(\eta) + \epsilon^2 v_2(\eta) + \ldots), \\
 \tilde{w} & = E(w_0(\eta) + \epsilon w_1(\eta) + \epsilon^2 w_2(\eta) + \ldots), \\
 T & = \epsilon^{-4}(\lambda_0 + \lambda_1 \epsilon + \lambda_2 \epsilon^2 + \ldots),
\end{align*}
\]

- i.e. for a particular assumed value of Re (high), look for a solution in the short wavelength limit.
Eigenrelations:

• Leading order:

\[
(K^2 + 1)^3 = -\lambda_0 \tilde{h}_1^4 (1 + \tilde{x} \cos \phi - \tilde{y} \sin \phi) \left( \tilde{V} \cos \phi + 1 \right) \frac{\partial \tilde{V}}{\partial \eta}.
\]

• Next order:

\[
\frac{\partial^2 w_0}{\partial \xi^2} + \zeta \left( \frac{\tilde{V}''(0)}{\tilde{V}'(0)} + \tilde{V}'(0) \cos \phi \right) w_0 - \lambda_1 \tilde{h}_1^4 (1 + \tilde{x} \cos \phi - \tilde{y} \sin \phi) 3^{-\frac{1}{3}} \tilde{V}'(0) w_0 = 0,
\]

• Where

\[
\tilde{h}_1 = 1 + \tilde{x} \cos \phi - \tilde{y} \sin \phi + \eta \cos \psi \sin^2 \phi,
\]
Solution

- Identified neutrally stable centrifugal mode with Taylor number

\[
\begin{align*}
\bar{T} &= T h_1^4 (1 + \tilde{x} \cos \phi - \tilde{y} \sin \phi) \\
&= \epsilon^{-4} \left( \frac{1}{|\tilde{V}'(0)|} + \frac{2.3381 \times 3^{\frac{1}{3}} \epsilon^{\frac{3}{2}}}{|\tilde{V}'(0)|} \left[ \frac{\tilde{V}''(0)}{\tilde{V}'(0)} + \tilde{V}'(0) \cos \phi \right]^2 + \ldots \right). 
\end{align*}
\]

- The Taylor number is given as a function of wavenumber and the steady-flow profile at the wall.
Neutral curve

- Interpretation?
Job done?

- Experiments show *circular modes* for very slender cones, i.e. zero waveangle...
- ... likely that Gortler modes most dangerous for low waveangle and low half angle.
- Need knowledge of critical Reynolds numbers and growth rates.
- Numerical analysis about to commence with a view to obtaining complementary results and further info on measurables.
- Also, aim to theoretically predict the changeover from “slender” to “broad” half-angles.
Further work

- Numerics (linear, incompressible)
- Experimental comparisons with KTH team
- Look at non-linear extensions

- Compressibility (cone[broad/slender], sphere)
- Controlling transition?
  - Smooth and/or distributed roughness over disk
  - Surface mass flux
- Global modes
Thank you for listening.

Questions?