Periodicity of finite-dimensional algebras

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Let $A$ be a finite-dimensional $K$-algebra over an algebraically closed field $K$. Denote by $\Omega_A$ the syzygy operator on the category mod\,$A$ of finite-dimensional right $A$-modules, which assigns to a module $M$ in mod\,$A$ the kernel $\Omega_A(M)$ of a minimal projective cover $P_A(M) \to M$ of $M$ in mod\,$A$. A module $M$ in mod\,$A$ is said to be periodic if $\Omega^n_A(M) \cong M$ for some $n \geq 1$. Then $A$ is said to be a periodic algebra if $A$ is periodic in the module category mod\,$A^e$ of the enveloping algebra $A^e = A^{\text{op}} \otimes_K A$. The periodic algebras $A$ are self-injective and their module categories mod\,$A$ are periodic (all modules in mod\,$A$ without projective direct summands are periodic). The periodicity of an algebra $A$ is related with periodicity of its Hochschild cohomology algebra $HH^*(A)$ and is invariant under equivalences of the derived categories $D^b(\text{mod}\,A)$ of bounded complexes over mod\,$A$. One of the exciting open problems in the representation theory of self-injective algebras is to determine the Morita equivalence classes of periodic algebras. We will present the current stage of the solution of this problem and exhibit prominent classes of periodic algebras.