SOME CANONICAL EXAMPLES OF STREAMWISE VORTEX STRUCTURE FOR ROTATING COMPONENTS

J. P. Gostelow*
University of Leicester
Leicester, LE1 7RH, U.K.

S. J. Garrett
University of Leicester
Leicester, LE1 7RH, U.K.

A. Rona
University of Leicester
Leicester, LE1 7RH, U.K.

D. S. Adebayo
University of Leicester
Leicester, LE1 7RH, U.K.

ABSTRACT
Nose cones, turbine blades and bearings have rotating components and represent very practical geometries for which the modal behavior of vortex structures is not completely understood. These three rather different physical cases are being studied. A common theme of competition between modes and vortex types, whether counter-rotating or co-rotating, emerges. The objective of ongoing work is to obtain physical confirmation, enhanced understanding and predictive capability for the vortex structures encountered in rotating machines.

INTRODUCTION
Stable vortex structures have been observed in a range of flows of industrial relevance, such as flows over conical aerodynamic fairings, turbine blades with cylindrical leading edges and between co-axial cylinders. The persistence of the vortical structures, over a defined duty or flight envelope appears to be a common theme among these applications.

Investigations of experimental results over the envelope of geometrical and flow parameters have guided the development of predictive methods for these structures, based on asymptotic perturbation methods of inviscid baseline flow models. Three examples of flows generating vortical structures indicate a possible common driver, involving stationary streamwise vorticity. This encourages the development of the current models to give a generalized representation for this class of flows.

SURFACE FLOWS OVER ROTATING CONES
Evidence for the existence of a hitherto unidentified instability mode in boundary-layer flows over rotating cones exists in the literature. This new mode is in addition to the crossflow (type I) and streamline curvature (type II) modes that are already known to exist [1] on rotating cones, disks and spheres. It is important to realize that throughout this paper the word mode refers to a physical mechanism rather than a spatial entity.

Evidence set 1: experimental observations
The visualization studies by Kobayashi and co-workers [2, 3] of rotating cones with slender half-angles show the existence of pairs of counter-rotating Görtler-type vortices prior to the appearance of turbulence. However, as the half-angle \( \psi \) is increased beyond 40°, the visualizations clearly show that these vortices change to...
co-rotating vortices as are usually reported on rotating disks and spheres. One might suppose that the counter-rotating vortices are expected to arise from a dynamic instability induced by the centrifugal force of the flow field, and are in contrast to the co-rotating vortices, attributed to an underlying crossflow instability.

**Evidence set 2: experimental measurements of the onset of turbulence**

Further evidence is obtained by considering experimental measurements for the onset of turbulence by [3] and Nickels & Bertényi, University of Cambridge (personal communication, 2007) compared to the onset of local absolute instability predicted by Garrett & Peake [4]. The exact role of local absolute instability in transition over the rotating disk is less clear than originally proposed by Lingwood [5,6], see [7] for example. Nevertheless the theoretical onset of local absolute instability is extremely close to numerous consistent measurements of the onset of turbulence over the rotating disk and this provides a useful of means of comparison. In particular, Garrett & Peake demonstrate that the critical local Reynolds number for local absolute instability over rotating cones is independent of half-angle, with \( Re_x \approx 2.5 \times 10^5 \).

Figure 1 shows the comparison with experimental measurements for the onset of turbulence reported by [3]. For cones with \( \psi > 60^\circ \), we see that transition occurs at a local Reynolds number independent of the half-angle and reasonably close to the predicted onset of local absolute instability.

![Figure 1. Critical \( Re_x \) for the Onset of Local Absolute Instability [4] and Measured Transitional Values [3].](image)

**Evidence set 3: analytical predictions of convective instability**

Garrett et al. [1] present mathematical studies of the rotating-cone boundary layers using a formulation that is

\[ \text{Critical } Re_x \]


\[ \text{Rotational velocity} \]

\[ \text{Critical } Re_x \]

\[ \text{Rotational velocity} \]

\[ \text{Critical } Re_x \]

\[ \text{Rotational velocity} \]

\[ \text{Critical } Re_x \]

\[ \text{Rotational velocity} \]

1 This close agreement suggests that local absolute instability may well be involved in the transition over broad rotating cones, consistent with the rotating-disk flow. For more slender cones, the measured critical Reynolds numbers decrease sharply with decreased half-angle and occur in advance of the predicted onset of local absolute instability.

Figure 2 shows Nickels & Bertényi’s measurements for the onset of turbulence over three cones, each with distinct half-angle, at different rotation rates. We see that the measured critical Reynolds number over the broadest cone (\( \psi = 60^\circ \)) is in good agreement with the predicted onset of local absolute instability. It is independent of rotation rate (which suggests that the dashed line in Fig. 1 can be extended to at least this half-angle). However, the onset of turbulence over the slender cones with \( \psi = 30^\circ \) and \( \psi = 15^\circ \) is again well in advance of the predicted onset of local absolute instability and dependent on the rotation rate. Furthermore, they reported different behavior in the turbulent intensity through transition in the case of the most slender cone.

**Figure 2. Experimental Data Due to Nickels & Bertényi for the Onset of Turbulence, Uppermost Plot is \( \psi = 60^\circ \). (Cone Angle = 2\( \psi \)).**

1 Note that the precise definition of turbulent flow is somewhat subjective in experimental terms and Kobayashi & Izumi’s measurements are subject to some flexibility. This is clearly demonstrated by comparing their measurements for half-angles close to 90° with Lingwood’s measurement on a rotating disk [6] (the horizontal dashed line at 90°).
consistent with other rotating-disk studies ([6], for example) and demonstrate that convective modes of type I and II exist for all \( \psi \). The onset of convective instability is then associated with the onset of the spiral vortices; the critical Reynolds numbers and other measurable quantities of the spiral vortices (number, angle of orientation) compare well with experimental observations of [3] for \( \psi \geq 40^{\circ} \). However, an increasing discrepancy is found for \( \psi < 40^{\circ} \). This is clearly seen in Fig. 3, where the theoretical predictions of the orientation angle for vortices arising from the type I mode are compared to the experimental observations at each half-angle. Such comparisons suggest that the vortices found on slender cones cannot be attributed to the type I and II modes. (Note that the type II predictions are of much higher wave angles.)

The behavior of the type I and II modes was further elaborated by Garrett [8] who extended the numerical results of [1] to consider the amplification rates of the modes through the convectively-unstable region as a function of half-angle. He found that the amplification rates of both mode types reduce with decreased half-angle. This finding is consistent with the hypothesis of a centrifugal mode that dominates at slender half-angles.

These three distinct sets of evidence clearly suggest an instability mode, arising from centrifugal effects, that exists in addition to the well-known type I and II modes. We envisage all three modes existing within the flow over rotating cones of any half-angle, but with a relative dominance that depends on the particular half-angle. It is therefore likely that a critical half-angle exists for the switch from a predominantly centrifugal instability (manifesting as counter-rotating vortices) to the cross-flow instability (manifesting as co-rotating vortices) as the half-angle is increased. Although experiments have only been conducted at a small number of distinct half-angles, the evidence suggests this critical half-angle to be around \( 40^{\circ} \). From an engineering perspective, this means that the transition mechanics at work over rotating propeller and fan nose cones are distinct from the mechanics over sharp spinning missiles, for example. Further study is warranted.

An analytical study into the hypothesized centrifugal mode is ongoing and we have found that a fundamentally different approach is required to existing broad-cone studies in order to make progress. In particular, the centrifugal analysis requires greater attention to small-scale effects (at a scale comparable to the boundary-layer thickness) in all directions. A preliminary asymptotic analysis has been completed, based on the assumptions of short wavelength and high spin rates, that clearly shows the existence of the centrifugal mode. The analysis is complicated and it is inappropriate to detail it here; the reader is however referred to [9, 10] for full details. However, the preliminary results of a complementary numerical analysis of the same governing equations can be seen in Fig 3. For slender cones the numerical predictions of the centrifugal mode clearly show better alignment with experiments.

**TURBINE BLADES AND SWEPT CYLINDERS**

Suction surface flow visualization on turbine blades at subsonic and transonic speeds showed robust streamwise streaks on a lengthy time-average basis [11] (Fig. 4). The flow on the suction surface, under the influence of a strong favorable pressure gradient, was initially laminar but further downstream, laminar separation and transition to turbulence were encountered. The turbulent layer then persisted to the trailing edge, as did the streaks which were unaffected by the boundary layer state. Similar behavior was observed by Halstead [12], who had surface film confirmation of the boundary layer state throughout. The streamwise vortical structures, whilst not particularly strong, are persistent and would seem to exert a stabilizing influence on the flow. Observations of streaks on turbine blades and unswept cylinders were to provide a firm basis for referencing the influence of sweep.
The lateral spacing between streaks on convex surfaces was predicted by Kestin & Wood [13]. That work used an inviscid flow model to predict the spanwise wavelength ($\lambda$) of the streamwise vortices, normalized by the cylinder diameter for different Reynolds number and free stream turbulence levels ($Tu$) at zero sweep. The resulting prediction is given by Eq. (1):

$$\lambda = 1.79\pi \frac{D}{Re^{0.5}}.$$  

A broad agreement is found between the predicted and measured $\lambda/D$ from differing test facilities. Streaks on turbine blades and unswept cylinders provided a firm basis for referencing the influence of sweep. Low and high speed experiments gave excellent agreement with theory [14].

The normal flow past a circular cylinder is a canonical case and testing was undertaken at high speeds on a 37.3 mm diameter cylinder and at low speeds on a 152 mm diameter cylinder. The lateral spacing between streaks on cylinders had been predicted by Kestin & Wood [13] and the present tests gave excellent agreement with their theory. Their work on unswept circular cylinders provides a good benchmark for understanding and predicting sweep effects on cylinders and turbomachinery blading. In earlier work on a normal cylinder, Kestin & Wood [15] published two-point hot wire measurements at a 60° azimuth from the leading edge stagnation line on the circular cylinder. The observed periodicity demonstrated that flow structures were present away from the surface and were compatible with the observed surface streaks.

Most of the available information on fine structures has come from surface flow visualization. Work is now in progress on hot wire measurements away from the surface. The aim is to demonstrate the relationship between the structures and the surface traces. Insight into the streamwise structures that print the surface traces is sought by hot-wire measurements at a Reynolds number of 150,000 on an unswept circular cylinder. The wind tunnel centerline velocity is measured by a combined head Pitot tube placed below the cylinder and three diameters upstream of its leading edge.

Two TSI constant temperature anemometers power the hot-wire probes with a 1.8 over-heat on their balanced resistance. The output is sampled at 2 kHz by LabView through a National Instruments acquisition card. Averages and standard deviations from 800 samples per position, repeated 30 times, are recorded. Each hot wire is calibrated using a DISA Type 55D41 with a TT320S digital pressure transducer. Two Dantec P15 single hot wires are located 1 mm above the windward cylinder surface, at 60 degrees azimuth from the leading edge stagnation line. The probe wires are parallel to the unswept cylinder axis. The ratio of the velocity from the moving hot wire, $u_m$, to that from the fixed hot wire, $u_r$, shows significant scatter with high velocity ratio clusters at 2 mm intervals. This ratio compensates for wind tunnel velocity set point drift effects.

Figure 5 shows, by the central line with circles, the velocity effects. The traverse velocity, $u_m$, is given by the moving hot wire and is normalized by the reference velocity, $u_r$. This is synchronously recorded by the fixed
hot wire, and is plotted against the cylinder axial direction \( z \). This ratio compensates for wind tunnel velocity set point drift.

The outer lines show the \( t \)-distribution 95% confidence interval band of this ratio from the hot wire standard deviation records. These hot wire measurements indicate that, within the limits of the instrumentation, a spanwise velocity distribution pattern with a spanwise wavelength of 2 mm is present. This compares with a flow visualization deduced wavelength in the range \( 2.14 \text{ mm} < \lambda < 2.31 \text{ mm} \) and with \( \lambda = 2.2 \text{ mm} \) from Eq. (1). These results appear to support the presence of near-surface streamwise vorticity over the cylinder surface.

One outcome of these investigations is to establish that organized streamwise vorticity may occur more frequently on convex surfaces, such as turbine blade suction surfaces, than was previously appreciated. Investigations and predictions of flow behavior should be extended to encompass this possibility. These applications often also have an appreciable degree of sweep and it is appropriate to enquire how sweep affects the instabilities. The question of sweep is addressed in the next paragraphs. It is hoped to provide information on the changing behavior of the spanwise velocity field as the sweep angle is increased. The streamline curvature disturbance has been found to be stationary in nature and to be resilient. The crossflow instability becomes more significant as sweep is increased. It grows aggressively and rapidly, being predominantly of a traveling nature, and has a major role to play in the transition process.

Experimental work on a circular cylinder was undertaken by the authors over a range of sweep angles, \( \Lambda^\circ \), from zero sweep to 61\(^\circ\), giving surface visualization results for lateral spacing and angular orientation of vortical streaks.

Figures 6 and 7 demonstrate that at high-sweep angles the results are consistent with those of Poll [16] and of Takagi et al. [17]. Experimental work, confirming the zero-sweep results, gave a reference, \( \lambda_o \), for subsequent work over a wide range of sweep angles. No data had been published on streamwise and crossflow vortices in the useful sweep range of up to 50\(^\circ\). Testing has been undertaken over a range of sweep angles from zero to 61\(^\circ\), giving results for the lateral spacing (Figs. 6 and 7) and angular orientation of the streaks. At high-sweep angles, the results are consistent with those of Poll [16]. At low Reynolds numbers first order-theories for circular cylinders predict the effects of sweep quite well. The approach of Takagi et al. [17] using hot wire techniques, offers an opportunity to identify both stationary and traveling instability modes.

The introduction of sweep brings consideration of a wide range of instabilities. Crossflow instability results from the inflectional behavior of a three-dimensional boundary layer. Streamwise and crossflow structures are present on the suction surface of swept and unswept turbine blades. Crossflow instability becomes more significant at high sweep angles. It grows aggressively and rapidly, being mainly of a traveling nature. The resulting observed streaks could be of concern for the thermal design of turbine blades.
Care was taken to check that the wind tunnel results and the results of Poll were quoted and normalized in the same way, using the unswept results as a reference. Kestin & Wood’s theoretical result (1) is accessible and is in agreement with a regression line fit through the experimental results for unswept cylinders. Different first order theoretical approaches to generalizing the Kestin & Wood’s prediction of vortex spacing to non-zero sweep angle resulted in the same simple modification:

\[ \lambda = 1.79\pi \frac{D}{Re^{0.5}} \cos(\alpha). \]  

(2)

This is the traditional Cosine Rule used to predict sweep effects on airfoils. This approach had been found to be valid only for subcritical flows with a critical Reynolds number that decreased with increasing sweep [18]. It will be seen that \( \lambda/\lambda_{0} = 1/\cos(\lambda) \) is a reasonable descriptor of the measurements over the sweep angle range 0° to 60.1°. Equation (2) is used to plot the lateral vortex spacing, normalized by the unswept case, in Fig. 6. These results are self-consistent and also compatible with Poll’s results, which were obtained at higher Reynolds numbers. The \( \lambda/\lambda_{0} = 1/\cos(\lambda) \) theoretical curve is plotted and demonstrates reasonable agreement with both the Poll data and the new data. At 50° sweep, the theoretical and experimental points of Takagi et al. [17] involved ingenious use of theory and hot wire data to discriminate between stability modes. Takagi discovered that, at 50° sweep, the crossflow mode dominated; this is the same mode identified by Poll and it is the lateral spacing from the crossflow mode that is plotted in Fig. 6. Takagi also examined the mode caused by streamline curvature from the upstream free stream. This appears to be mostly a result of the local concave streamline curvature in the flow region, delimited by the mean stagnation streamline to the cylinder front stagnation point and the windward surface of the cylinder. This local concave streamline curvature moderates the stabilizing effect of the cylinder’s convex surface. At these low Reynolds numbers, Takagi found that the streamline curvature mode persisted much longer than the crossflow mode. Takagi’s results are broadly consistent with our results and those of Poll.

Full data sets were obtained at 55° sweep, by Poll and from the current tests. These are of particular value in determining consistency; the lateral sweep spacings are plotted in the form of 1000/\( D \) as a function of 1000/(\( Re \))\(^{0.5} \) in Fig. 7. The present results, from an unswept cylinder, and that of Ackerman [19], are also quite well represented by a straight line. Despite the large gap in the intermediate Reynolds number range, it is instructive to compare the experimental results with those from the equivalent cosine rule lines. This comparison confirms that the present results are reasonably compatible with those of Poll, although the Poll results do indicate a change toward higher spanwise spacings at high Reynolds numbers. This could be evidence of the conditions associated with the change in mode from the counter-rotating streamwise vorticity at zero sweep to the co-rotating vorticity usually associated with high sweep angles. The inclusion of a point from Takagi’s experiment is also of interest. Although the sweep angle for the Takagi result was 50° instead of the 55° of the other experiments, it falls on the 55° sweep line. This therefore reinforces the observations of wider spacings encountered at high sweep. Since Takagi was able to clearly identify this point as driven by crossflow instability, the evidence supports a qualitative difference between the streamwise instability at zero sweep and low Reynolds numbers and the crossflow instability at high sweep and high Reynolds numbers. The streamwise disturbance has been found to be resilient, often persisting from leading edge to trailing edge. Crossflow instability becomes more significant as sweep is increased. The strength increases aggressively and rapidly with sweep. This instability appears to be predominantly of a traveling nature, and has a major role to play in the transition process.

A good summary of the difference between the two modes is given by Tokugawa et al. [20]: “Detailed observations, however, show that the crossflow mode decays with the distance from the source much faster than the streamline curvature mode and allows the latter to be dominant in a region further downstream.” Essentially, the crossflow instability may have a major role to play in the transition process but it is the streamline curvature mode that is still present, and seemingly unchanged, when the boundary layer becomes turbulent.

Turbine blades may exhibit extremes of surface curvature, both convex and concave, and of pressure gradient, both favorable and adverse. Leading edge bluntness, temperature, Reynolds number and Mach number are all quite challenging. As a consequence, turbine blades are susceptible to the different modes and it should not have come as such a surprise that these
instability modes exist. Given their potential role in boundary layer transition and its modeling, in heat transfer and in blade sweep, it seems important to be fully aware of the modes and their incorporation into the blade design process. The observed streaks of the various modes, both stationary and traveling, could be of particular relevance for the thermal design of turbine blades. It is hoped to give designers confidence about the flow regimes they might anticipate for a given sweep angle and particularly of when and how the vigorous crossflow instability mode is likely to be encountered.

WIDE GAP ROTATING CYLINDERS

Taylor vortices develop in the gap between concentric rotating cylinders when the Taylor number $Ta$ exceeds the first critical value. These vortical structures can be regarded as a type of streamwise vorticity developing in an enclosed flow, in which the reference streamwise direction is azimuthal. Observations of these structures are obtained on the meridional plane of coaxial cylinders characterized by a wide annular gap, as sketched in Fig. 8. The flow structure in this closed system is tested for changes with cylinder radius ratio, $\eta$, aspect ratio, $\Gamma$, and Taylor number, $Ta$. Specifically, two Perspex coaxial cylinders of $\eta = 0.44$ and 0.53, $\Gamma = 7.81$ and 11.36, and outer to inner cylinder angular speed ratio $\mu = 0$ are tested over the Taylor number range $1.18 \times 10^6 \leq Ta \leq 10.93 \times 10^6$. Optical access to the meridional plane by conventional Particle Image Velocimetry (PIV) gives an un-obstructed view of the secondary flow dynamics. The motion in the flow is visualized by a fine mist of 20% polyethylene glycol (PEG600) and 80% water that is atomized using a Dantec Dynamics Studio seeding generator, model 10F03. This resulted in rich and well-diffused PIV seeding, as shown in Fig. 9. The flow is illuminated by a two-cavity double-pulsed Nd:YAG Litron L Nano laser triggered at 4 Hz. The flow is imaged by a Dantec Dynamic Studio FlowSense 4M CCD camera with a resolution of 2048 x 2048 pixels with a 60 mm AF Micro Nikon image lens, operated in double-frame acquisition mode at a frequency of 4 Hz with an f/4 aperture.

The image pairs obtained from the CCD camera give both qualitative flow visualization, when considered individually, and a quantitative velocity vector map, once processed in pairs by PIV. Dantec-Dynamics Studio PIV software version 2.30 was used, starting from a 32 x 32 pixel interrogation area and ending with a 16 x 16 pixel interrogation area with a 50% overlap. The lower portion of the meridional plane is shown in Fig. 9 and Fig. 10, which is the area between the inner and the outer cylinder indicated by the arrow of the ‘Test section’ label in Fig. 8. As such, the inner rotating cylinder forms the top boundary of each image whereas the fixed outer cylinder delimits the bottom of each image. The images from the upper portion of the meridional plane displayed similar, symmetrical, trends.

The test cases are characterized by a larger annular gap width $d$ than classical journal bearing test cases and by a Taylor number range $1.18 \times 10^6 \leq Ta \leq 10.93 \times 10^6$, that is beyond the first critical Taylor number at which Taylor vortices develop. Some interesting flow features are observed in this region of flow parameters.

At the inner cylinder angular speed $\Omega = 100$ rpm, the flow visualization of Fig. 9(a) shows typical features of Taylor vortex flow, in which an axial stack of well-defined and ordered Taylor vortex cells develops. The cells appear as regularly spaced, with well-defined boundaries, resulting
Figure 10. Meridional Plane PIV Vector Maps between Rotating Cylinders at Different Angular Speeds

\( \eta = 0.44, \Gamma = 7.81 \)

in the agglomeration of the PIV seeding towards the cell-centers. The cell boundaries are neatly displayed by thin regions of rarefied PIV seeding. Figure 9(a) also suggests that the locations of these boundaries may be stationary, since their motion would have promoted local mixing of the local flow with the neighboring seeding-rich flow.

At the higher angular speed of 214 rpm, Fig. 9(b) suggests a change in the flow regime. The boundary between neighboring vortex pairs is less clearly defined. The faint variation in seeding particle concentration enables the identification of about four Taylor vortex cells in the mid-span region \( 2.5 \leq X/R_i \leq 7.0 \) but their identification is more difficult than in Fig. 9(a), as the PIV seeding is more uniformly distributed along the cylinder span. This is evidence of a more intense axial flow mixing taking place at 214 rpm, which can be explained by the onset of time-dependent changes in the axial position and size of the Taylor vortex cells. This is consistent with a change in instability mode from Taylor Vortex to Wavy Taylor Vortex flow, in which the flow axial-symmetry is broken up by travelling azimuthal waves [21]. Above \( \Omega = 214 \) rpm, the PIV seeding between the cylinders is fully mixed and individual vortex cells are no longer identifiable with the flow visualization technique of Fig. 9.

A different view of the flow in the meridional plane between the rotating cylinders is obtained from the velocity vector maps, computed from the processing of image pairs of the seeded flow by adaptive spatial cross-correlation. Figure 10 shows a sample of the velocity vector maps obtained at increasing angular speeds from the same \( \eta = 0.44 \) and \( \Gamma = 7.81 \) geometry of Fig. 9. The sample is representative of a trend observed throughout sets of 100 vector maps taken at different angular speeds. The velocity vector maps reinforce the inference from flow visualization that a change in flow regime takes place with increasing angular speed.

The instantaneous flow field at \( \Omega = 100 \) rpm in Fig. 10(a) shows four pairs of counter-rotating Taylor vortices in the annulus between the rotating inner cylinder and the stationary outer cylinder. The flow pattern is characterized by regularly spaced vortices in the central region of the annulus, with the vortices at either end being elongated due to end-wall solid boundary effects. The velocity at the boundary between neighboring vortices is essentially radial in Fig. 10(a), indicating that there is no significant exchange of fluid between vortices. The analysis of the entire set of instantaneous 100 vector maps at \( \Omega = 100 \) rpm has shown that the position and orientation of the boundaries between adjacent vortices as well as of vortex centers do not change with time. The steady radial flow at the boundary between neighboring vortices supports the observation from Fig. 9(a) that boundaries between neighboring vortices are flat and perpendicular to the cylinder axis. This is a characteristic of the axisymmetric Taylor instability regime.

Figure 9 suggested a transition in the flow regime, from Taylor vortex flow to Wavy vortex flow, between \( \Omega = 100 \) rpm and \( \Omega = 240 \) rpm. The velocity vectors of Fig. 10 (b-d) indicate the Wavy vortex flow regime is maintained over the range \( \Omega = 214 \) rpm to \( \Omega = 500 \) rpm. The four pairs of counter-rotating Taylor vortices at \( \Omega = 100 \) rpm are still observable over this range. This pattern is modulated by the waviness of the flow and the boundaries between neighboring vortices are neither flat nor perpendicular to the cylinder axis.

By \( \Omega = 650 \) rpm, Fig. 10(e) shows that the flow regime has developed turbulent flow features, as the flow field
appears to be characterized by randomness and distorted regular Taylor vortices can be clearly seen. There is breakdown of axial periodicity in the flow pattern. The flow field at $\Omega = 650$ rpm shows large-scale motion with many small-scale vortices embedded within it. This is a recognizable feature of turbulent flow in which the time evolution is stochastic rather than deterministic. The flow field is changing so that it was not possible to observe the recurrence of any one pattern of vortices, or vector field image, in all the 100 PIV snapshots. In some snapshots, Taylor vortices are not identifiable in the fully irregular turbulent flow and, at other times, distorted Taylor vortices are present that cannot be easily distinguished in the background turbulent flow. This flow pattern can no longer be described by well-defined Taylor vortices, although structures associated with the Taylor vortices remain. This is indicative of a further change in the flow regime to Turbulent Taylor vortex flow.

The flow generated by wide gap circular cylinders tested at high Taylor numbers can be interpreted as an occurrence of organized streamwise structures in a channel in which the motion is azimuthal. The structures are paired, counter-rotating, and display a regular spacing within the limits of the cylinder length to outer radius ratio $\Gamma$ available from the experimental set-up. These structures are persistent over time, with their location being either substantially time-invariant, at the lower Taylor numbers, or exhibiting time-dependent variations that still enable their identification as streamwise vortices running in the azimuthal direction at higher Taylor numbers. This trend is also found in the $\eta = 0.53$, $\Gamma = 11.36$ results, which are reported in Adebayo [22].

The flow pattern from Particle Image Velocimetry at $\eta = 0.44$ shows wavy vortical structure in the flow between the concentric cylinders with low aspect ratio $\Gamma < 25$ and at a high Taylor number, well beyond the published values for transition to turbulent flow. This regime is characterized by enhanced waviness near the end-walls as compared with the wavy flow in the central region. Once established, this inviscid, incompressible instability maintains its principal defining features, with changes in Taylor number, aspect ratio and radius ratio.

**CONCLUSIONS**

A review of existing literature concerning boundary-layer transition over rotating cones presents clear evidence of an alternative instability mode. Observations report that this mode leads to counter-rotating vortex pairs, consistent with occurrences of centrifugal instabilities, and is in contrast to co-rotating vortices present over rotating disks that arise from crossflow effects. It is suggested that this mode competes with the crossflow mode but is dominant only over slender cones, where $\Psi < 40^\circ$. Analytical progress has been made that confirms theoretically the existence of the centrifugal mode and preliminary predictions are aligned with experimental measurements over slender cones.

Low-speed and high-speed tests were performed on 152 mm and 37.3 mm diameter normal cylinders respectively. Experimental work confirmed the suitability of the zero-sweep Kestin & Wood theory as a basis for predicting streamwise streaks and vortical structures on normal cylinders. Although the Kestin & Wood work is related to unswept circular cylinders, it also provides an excellent starting point from which to obtain a predictive model that includes sweep effects.

This work has shown that organized and systemic fine-scale streamwise vorticity may occur more frequently on convex surfaces than hitherto appreciated. Streaks observed by surface flow visualization do have aerodynamic significance; they are not mere artefacts of the visualization medium. The conventional view of purely two-dimensional laminar boundary layers following blunt leading edges is not realistic. Such boundary layers need to be treated three-dimensionally, particularly when sweep is present. The vortical structures are counter-rotating for normal cylinders and co-rotating under high sweep conditions. Crossflow instabilities may have a major role to play in the transition process but the streamline curvature mode is still present, and seemingly unchanged, when the boundary layer becomes turbulent. On the suction surface of turbine blades the lateral spacing between vortical structures remains virtually constant and does not scale with boundary layer parameters.

The rotation of an inner cylinder laid co-axial with a stationary outer cylinder develops an organized secondary flow in the meridional plane of the annular gap. Counter-rotating vortex pairs fill the gap between the two cylinders.

Observations of cylinders at wide gap indicate that the pattern is consistent with that from the inviscid, incompressible Taylor vortex instability. Taylor number
increments have a de-stabilizing effect on the counter-rotating vortex cells, which are still identifiable, albeit with some difficulty, at \( Ta = 10.93 \times 10^6 \), in the turbulent flow.

Three rather different physical cases are being studied. A common theme of competition between modes and vortex types, whether counter-rotating or co-rotating, emerges. The objective of ongoing work is to obtain physical confirmation, enhanced understanding and predictive capability for the vortex structures encountered in rotating machines.

Prediction of these modes requires a sufficiently fine spanwise spacing for the streamwise structures to be resolved. Application of computational methods to these problems is likely to be expensive. A combined approach of analysis, computation and experiment is indicated.

**NOMENCLATURE**

- \( D \) Cylinder diameter
- \( Me \) Discharge Mach number
- \( Re \) Reynolds number
- \( Ta \) Taylor number
- \( Tu \) Free-stream turbulence level, \%
- \( d \) Annular gap width
- \( m \) Azimuthal wave number
- \( u \) Stream velocity
- \( u_m \) Velocity at moving probe
- \( u_r \) Velocity at reference probe
- \( \Omega \) Shaft speed
- \( A \) Sweep angle, with respect to normal
- \( \Gamma \) Aspect ratio
- \( \lambda \) Lateral spacing between vortex pairs
- \( \lambda_o \) Spacing between vortex pairs; unswept cylinder
- \( \mu \) Cylinder radius ratio
- \( \Psi \) Half angle, with respect to axis

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**REFERENCES**


