Experimental observations and theoretical predictions of streamwise vorticity on circular cylinders in crossflow and on turbine blades are considered. The observations on cylinders confirm earlier predictions and this forms a firm basis for referencing other measurements and predictions of vortical behavior. It also results directly in a correlation for predicting the spanwise wavelength of streamwise vortices on the convex surfaces of turbomachine blades. Highly resolved Large Eddy Simulation has shown that fine scale organized streamwise vorticity may exist on the convex surfaces of turbine and compressor blading and is predictable. For a turbine blade with a blunt leading edge the streamwise vorticity may persist on a time-average basis to influence the entire suction surface at suitably low Reynolds numbers typical of aircraft cruise conditions. The results emphasize the enormous computing resource required to resolve the flow on a routine basis for design purposes. It is demonstrated computationally that streamwise vorticity interacts with spanwise vorticity in leading edge bubbles to promote early transition and bubble closure. Time resolution is required to capture the flow complexity that is fundamental for an understanding of the physical behavior of the laminar boundary layer and its separation and transition. A narrow spanwise strip does not allow the streamwise vorticity to settle into the organized pattern. For streamwise vorticity to become organized, an adequate spanwise domain and run duration for time averaging are also essential. Any accurate treatment of laminar boundary layers at low Reynolds numbers needs to be performed three dimensionally and with a sufficiently fine spanwise spacing and duration of run to resolve streamwise vortical structures. Sweep of the body, wing or blade poses special problems. Not least is a serious lack of information on even the most basic cases. An attempt is made to relate the streamwise vorticity studied by Kestin and Wood to the more aggressive crossflow instability studied by Poll. More research is needed if designers are to be confident about the flow regimes they might expect to prevail for a given sweep angle.

Nomenclature

\begin{itemize}
  \item \textit{D} \quad \text{Cylinder diameter}
  \item \textit{n} \quad \text{Amplification factor in inception prediction}
  \item \textit{Re} \quad \text{Reynolds number}
  \item \textit{Tu} \quad \text{Free-stream turbulence level}
  \item \textit{u} \quad \text{Velocity component in x direction}
  \item \textit{x, y, z} \quad \text{Coordinate directions}
  \item \textit{A} \quad \text{Angle between normal to the inflow and axis of the body}
  \item \textit{\lambda} \quad \text{Spanwise spacing between vortex pairs}
\end{itemize}
I. Introduction

Previous investigations have revealed streamwise vortices and “streaky structures” on flat plates\(^1\) and on the suction surface of compressor blades\(^2,3\). Turbine blade designers are quite familiar with the phenomenon of Görtler vorticity; this is thought to occur predominantly on the concave pressure surfaces of turbine blades. This organized vortex system tends to increase heat transfer to the blade surface and also makes the flow and heat transfer difficult to predict.

Designers generally assume that streamwise vorticity of this kind is confined to these concave pressure surfaces. The authors have observed streamwise vorticity on suction surfaces, both experimentally\(^4\) and computationally\(^5,6\). In this paper examples of this streamwise vorticity on convex surfaces will be considered. This behavior had been predicted and observed previously in low speed flows, with attendant theories for wavelength. For a predominantly convex surface the behavior is consistent with the later predictions of Görtler\(^7\), who postulated instability on a convex surface from the concave streamlines ahead of the leading edge stagnation region.

The circular cylinder in cross flow will always be an important canonical case and it is difficult to imagine a solution for more complex geometries without the guidance of a thorough understanding of flow over a circular cylinder. The spanwise wavelength of the array of streamwise vortices was predicted theoretically and experimentally by Kestin and Wood\(^8\) and is confirmed experimentally in the present paper.

At high positive and negative incidence angles blades are prone to flow separation in the leading edge region. Any streamwise vorticity present might be expected to affect the separation behavior. This would then have an effect on the transition from laminar to turbulent flow. The implications of this behavior for laminar separation and boundary layer transition and their prediction will be discussed and have informed a new approach to modeling laminar separation\(^9\).

The introduction of sweep brings consideration of a wider range of instabilities. Prominent is cross-flow instability resulting from the inflectional behavior of a three-dimensional boundary layer. The Kestin and Wood theory may be regarded as the limiting case for zero sweep angle and it is of interest to work from that to consider stability over a wide range of sweep angles. It would be particularly reassuring to have experimental measurements to support the theoretical work and such a program of work is planned.

Figure 1. Suction Surface Flow Visualization on NRC Turbine Blade at a Discharge Mach Number of 1.16.
II. Streamwise Vorticity

Flow Visualization and Stability Theory

Highly loaded turbine blades were tested in a cascade tunnel at the National Research Council of Canada in Ottawa. Time-averaged surface flow visualization on the convex suction surface revealed the existence of organized streamwise vortical structures. The resulting streaks occupied the whole of the suction surface. The blades were tested for discharge Mach Numbers up to 1.16 and the behavior did not appear to be strongly dependent on Mach Number. A sample is presented in Figure 1. In addition to the predominantly streamwise structures, part of the rapidly growing end-wall boundary layer intrudes from the left hand side. These streamwise streaks have been shown to be quite common on the suction surface of turbine blades resulting in an examination of the fundamental flow behavior causing the phenomenon.

The crossflow over a circular cylinder is a canonical problem and the existence of regular streaks on circular cylinders was originally investigated by Kestin and Wood who, in 1970, published a stability analysis. They predicted a theoretical value of spanwise wavelength between vortex pairs, \( \lambda \), for a cylinder of diameter, \( D \), given by:

\[
\lambda = 1.79\pi D Re^{-0.5}
\]

This result (Eq. 1) is represented by the \( Tu = 0\% \) line in Figure 2. Kestin and Wood also undertook experimental work on circular cylinders which provided the results for non-zero turbulence levels.

In order to confirm the suitability of the Kestin and Wood theory as a basis for examining more complex surface geometries, and variables such as sweep, further experimental work was undertaken on circular cylinders. In 2002 Ackerman had performed surface flow visualization on a 37.26 mm diameter cylinder at a free stream Mach Number of 0.5. Streamwise streaks were observed before and after a separation bubble. Application of the visualization medium upstream of separation had been non-uniform. Because the liquid medium collected in a pool in the bubble a more uniform dispersion of the medium was observed downstream of the bubble. In this region spanwise periodicity showed up clearly (Figure 3) and formed a viable basis for measuring the spanwise wavelength. This experiment was performed at the relatively high Reynolds number of 675,000 and provided a point for comparison with the Kestin and Wood prediction.

![Figure 2. Measurements of Spanwise Wavelength on Circular Cylinders and Comparison with Kestin and Wood Predictions.](image)
More recently testing was undertaken on a 152 mm diameter aluminum cylinder in the Leicester University low speed research tunnel at three Reynolds numbers. Surface flow visualization has provided a further three points on the Kestin and Wood plot in Figure 2. All four results are in reasonable agreement with the Kestin and Wood theory and therefore provide further confirmation of the theory for the case of the circular cylinder in crossflow.

Although these results were obtained for a circular cylinder it is useful to ask whether the approach might form a basis for predicting streamwise vorticity on turbine blade suction surfaces. These typically have a rounded leading edge and subsequently the suction surface retains a strong convex curvature over the forward portion and is quite flat further downstream. The rapid changes in curvature of the convex surface raise the question of what effective diameter should be applied if comparing with the Kestin and Wood model. It has been found that if the diameter of the suction surface osculating circle is determined at the 10% true chord location as the value of $D$ for comparison with the Kestin and Wood theory, that theory provides a useful correlation for the spanwise wavelength of the suction surface streamwise vorticity on turbine and compressor blades.

The measurements of the spanwise wavelength of the array of vortices are compared with the predictions of Kestin and Wood in Figure 4. Surface flow visualization photographs have been analyzed from the work of a number of published experiments on blading. When examined in the same way as the surface flow visualization from the NRC cascade, two results from compressor blading and four from turbine blading were accessible. Figure 4 shows that these also gave reasonable agreement with the Kestin and Wood theory and experiments. Free-stream turbulence levels for the above cases were all in the range $0.2\% \leq Tu \leq 4.0\%$.

One outcome of these investigations is to establish that organized streamwise vorticity may occur more frequently on convex surfaces, such as turbine blade suction surfaces, than was previously appreciated. Investigations and predictions of flow behavior should be extended to encompass this possibility. These applications often also have an appreciable degree of sweep and it is appropriate to enquire how sweep affects the instabilities. The question of sweep is addressed in Section IV.

Figure 3. Surface Flow Visualization downstream of Separation Bubble on Circular Cylinder in NRC Tunnel at a Mach Number of 0.5.
Large Eddy Simulation of Compressor Blade Flows

An alternative approach to consideration of the role of streamwise vorticity that gives instantaneous predictions and unsteady behavior is obtained using Large Eddy Simulation (LES). A LES study of the flow around a controlled diffusion compressor blade (CD) has been undertaken. In this work a CD compressor cascade stator blade was simulated at a Reynolds number of 700,000. The blade was the Sanger design of CD blade as tested in cascade at NPS, Monterey. A wide range of inlet angles was computed, including high negative and positive incidence conditions. At all inlet angles the surface pressure distributions were well-predicted by the LES. Predictions of measured loss near the design inlet angle were good whilst at both high positive and negative incidences losses were less well predicted. At incidences above the design angle a laminar separation bubble was formed near the leading edge of the suction surface, culminating in transition to turbulence. Similar behavior was noted on the pressure surface at extreme negative incidence. At high negative incidence contra-rotating vortex pairs were found to form around the leading edge in response to an unsteady stagnation line across the span of the blade.

In Figure 5 the computed global axial velocity distribution is shown with the suction surface uppermost. As the flow is at high negative incidence, a laminar separation bubble exists near to the leading edge of the pressure surface. The instantaneous vortex structure can be observed in the region near the leading edge. In both Figures 5 and 6 the parameter represented by the coloring is the global $u$ velocity. It does appear that there is no particular reverse flow on the suction surface, and the streaks appear to convect away from the leading edge. An unusual and interesting feature is the propagation of toroidal structures forward of the leading edge. These appear to originate on the blade surface, are shed from the blade and grow as they move forward of the leading edge along the stagnation streamline. They then appear to collapse resulting in strong streamwise vorticity as the resulting contra-rotating vortex pair is stretched onto the suction surface. Some of the toroidal structures persist a little longer and are convected along the suction surface.

Figure 6 shows the flow in the separation region on the pressure surface resulting from this extreme negative incidence. These time-accurate results give an indication of the complexity of the flows in the leading edge region at off-design conditions.
It has also been demonstrated\(^5\) that the degree of organization of the predicted streamwise vorticity is a function of the spanwise extent of the flow-field. A narrow spanwise strip does not allow the streamwise vorticity to settle into the organized pattern. The extent of the calculation domain should ideally extend all the way to the end walls; this is also desirable if secondary and end-wall flows are to be predicted accurately. Although these LES results demonstrate that at low Reynolds numbers the complex leading edge flows can be predicted, they also emphasize the enormous computing resource that would be required to achieve this on a routine basis for design purposes. The grid needs to be very highly resolved to minimize the dependence on sub-grid modeling. The computation needs the time accuracy to capture the vortical events that are essential to an understanding of the physical behavior of the laminar boundary layer, laminar separation and transition. It also needs to be run for a sufficient duration that adequate time averaging can take place for the streamwise vortices to become organized. For this to happen a further requirement is for an adequate spanwise extent of the computational domain to allow the vorticity to become organized. This may require the computation to extend all the way to the end walls.

### III. Boundary Layer Transition

The onset of transition is generally the outcome of competition between different instability modes. Laminar stability theory describes "natural" transition under low free-stream turbulence. The basis of this was originally confirmed by Schubauer and Klebanoff\(^16\) and led to the \(e^n\) prediction method for transition onset\(^7,8\). Integral boundary layer calculations have traditionally been used to predict transition onset with calculation of the laminar layer momentum thickness Reynolds number proceeding from the origin until some previously determined criterion for transition commencement is met. Abu-Ghannam and Shaw\(^19\) produced a correlation for transition inception on a flat plate over a range of pressure gradients and free-stream turbulence levels.

"Bypass" transition under high free-stream turbulence, a concept originally introduced by Morkovin\(^20\), was erroneously presumed, by some workers, to imply instantaneous turbulent breakdown with zero length.
of transitional flow. However this was not the intention. Bypass transition does not necessarily exclude instability processes, which are essential for transition: only the long region of two-dimensional wave amplification preceding the appearance of three-dimensional disturbances (spanwise periodicity) in low turbulence flow is bypassed.

The method and location of free-stream turbulence measurements is an important issue. Due to the strong accelerations and decelerations experienced in turbomachine blading, the local value of free-stream turbulence at the transition onset location may differ greatly from that on the stagnation streamline, which may also differ from an average value over the flow field. Currently used transition onset correlations involve data from several workers, who may have adopted different bases for defining free-stream turbulence values: Abu-Ghannam and Shaw, for example, used neither a local value at breakdown, nor some mean value over the region of unstable flow, but rather an average value of free-stream turbulence taken midway between the leading edge of their plate and the location in question.

Transition has been ignored by some blade designers, as the boundary layer flow was supposed completely turbulent due to the high disturbance levels in a machine. This led to inappropriate distortion of experimental studies by artificially promoting transition. The viscous instability of a laminar boundary layer was originally investigated by Tollmien. Under low free-stream turbulence conditions instability is initiated when two-dimensional unstable Tollmien-Schlichting (TS) waves are formed; these propagate in the streamwise direction at less than 40% stream velocity. They develop three-dimensionality and spanwise variations and a concentration into peaks, valleys and hairpin vortices occurs. Turbulent spots are formed in the peak regions of vorticity. Klebanoff first observed the growth of turbulent spots upon which Emmons\textsuperscript{21} based his spot theory. The spots grow and coalesce to form continuously turbulent flow.

All transition models are empirical to some extent. Only direct numerical simulation (DNS) can predict the whole transition process without recourse to empirical data. DNS has shed great light on the detailed physics of transition in boundary layers and separated shear layers under both low and high free-stream turbulence conditions. Because it is so demanding in computer resources, however, it remains impractical for engineering calculations at engine-representative Reynolds numbers. LES computations can successfully predict quite detailed features of transitional flow, especially in separation bubbles where the Kelvin-Helmholtz (KH) instability predominates. However they are not yet capable of predicting the whole transition process and modeling is still needed at the sub-grid scales.
The early approach to transition modeling using Reynolds Averaged Navier Stokes procedures was to backward extrapolate from the turbulent flow region, and assume that conventional turbulence modeling could also describe transitional flow. This produces transition-like behavior, which is initiated by diffusion of turbulent energy from the free-stream to the boundary layer; unfortunately, however, it does not model the actual flow physics.

Recent studies have highlighted the significant influence of streamwise vorticity on separated flow transition phenomena under the high free-stream turbulence conditions experienced by embedded blade rows in a multi-stage turbomachine. Filtering of disturbances from the free-stream causes the appearance of streaky structures in the separated shear layer, similar to Klebanoff modes in attached boundary layers under a turbulent stream. Transition onset is then characterized by the appearance of vortex loops or hairpin eddies at a frequency related to the dominant KH frequency of inviscid instability in the separated shear layer. This suggests that turbulent breakdown in a laminar separation bubble under these conditions occurs through interactions of TS waves with streamwise structures having a much smaller spanwise length scale. Under high free-stream turbulence conditions DNS studies show turbulent breakdown originating from KH instability in the outer region of the boundary layer.

There are a number of different mechanisms by which periodic spanwise disturbances might impinge on separated flow regions on a turbomachine airfoil. The filtering of disturbances from a turbulent stream will be important regardless of the location of the separated flow region on the airfoil. The Kestin-Wood stagnation flow instability is relevant for turbine blades, which tend to have a blunt leading edge. Streamwise vorticity has also been observed in the suction surface laminar layers of compressor blades and is also relevant to the leading edge laminar separation bubbles commonly observed at off-design conditions. Spanwise periodicity on a turbine airfoil suction surface can also be introduced by the periodic impingement of wake jets arising from the relative motion of an adjacent upstream blade row: this produces a moving stagnation line that convects with the passing wake. The region of concave flow curvature at a separation point that is associated with deflection of the approaching boundary layer away from the surface is an additional candidate for invoking Görtler instability on compressor blades.

Regardless of the importance of these instability mechanisms in generating spanwise periodicity, it has recently been pointed out by Diwan and Ramesh that three-dimensionality of flow in a separating two-dimensional boundary layer is a topological necessity in any case, arguing that a two-dimensional separation line is topologically impossible. Similar topological arguments would require the existence of three-dimensionality, with associated spanwise periodicity and generation of streamwise vorticity, at an attachment line in the stagnation region of a two-dimensional stream.

The LES results of McMullan, typified by Figures 5 and 6, lend support to the above interpretations of transition and especially its role in closing a laminar separation bubble. These computational results would appear to be in accordance with the findings of Durbin et al. on the importance of mode interaction.

### IV. Sweep

Sweep is employed or encountered widely in the natural and physical worlds. It is used very deliberately in aeronautics, in turbomachinery for power generation and aircraft propulsion and in the open rotors encountered on wind and water turbines. The sweep angle, \( \alpha \), is defined as the angle between the normal to the inflow and the axis of the body. A circular cylinder is a basic generic body that has been studied to gain an understanding of sweep effects. The effect of sweep on circular cylinders has not been fully documented and there is much work to be done experimentally and computationally before a thorough understanding may be claimed. Wings and turbomachine blades are more complex bodies but would benefit strongly from a more complete understanding of circular cylinder flows.

The previously-described work of Kestin and Wood is a good place to begin a discussion of these effects. Although the Kestin and Wood work is related to unswept circular cylinders it does provide an excellent benchmark for sweep effects. The experimental results of Figure 2 demonstrate that the Kestin and Wood formula of Eq. (1) for the lateral spacing of streamwise vortex pairs, as a function of Reynolds number, gives a solid foundation from which other stability investigations may be viewed. The principal published collection of results for medium to high sweep angles is that of Poll covering the sweep range \( 55^\circ < \alpha < 71^\circ \). There are no published systematic flow visualization data covering a wide range of sweep angles. The authors are presently undertaking such an experimental investigation.

It is possible to use the predictions of Kestin and Wood directly as a benchmark for swept results. As an example the results of Poll have been referenced to the Kestin and Wood results and are shown in...
Figure 7. It is immediately obvious that the lateral spacings of vortices observed by Poll are of the same order as those of Kestin and Wood but are significantly larger. In addition as the sweep angle is increased the observed vortices become more widely spaced, and are likely to be more vigorous. Data in the useful range of sweep up to 50 degrees are virtually non-existent although the experiments of Dagenhart\textsuperscript{26} and Kohama\textsuperscript{27} are of interest. The latter publication gives an intriguing photograph showing two instability modes simultaneously. It seems possible that one mode is the remnant of the type of streamwise vorticity observed by Kestin and Wood at zero sweep, the other coarser mode being the vigorous cross-flow instability.

Although Figure 7 needs to be populated with data it could provide a useful road-map of sweep intensity. A designer, contemplating the choice of a sweep angle, would surely benefit from the knowledge of when the crossflow instability mode is likely to be encountered. Another paper by Kohama\textsuperscript{28} does point the way here. His liquid crystal work suggests that the crossflow instability is first observed at around 40 degrees sweep and becomes most strongly amplified at around 57 degrees.

**Generalization of Kestin and Wood to non-zero sweep**

A preliminary attempt to generalize Kestin and Wood’s prediction of vortex spacing\textsuperscript{8} (Eq. 1) to non-zero sweep angle has been made. The approach considers the thickening of the boundary layer due to the introduction of non-zero flow parallel to the leading edge of the cylinder with sweep. Rescaling Kestin and Wood’s formulation in terms of the modified boundary-layer thickness and regrouping parameters in terms of unswept quantities leads to the simple modification:

\[
\lambda = 1.79 \pi D / Re^{0.5} \cos(\Lambda).
\] (2)

Implicit in this is the assumption that the effect of non-zero sweep angle is merely to increase the boundary-layer thickness; it is crucial that the evaluation of the wall shear rate is also applicable for swept flows. This warrants further investigation, but, under the assumption that it is, we are able to use Eq. (2) to plot the lateral vortex spacing, normalized on the unswept case.

![Figure 7. Normalized Lateral Spacing of Vortices as a Function of Sweep Angle: Comparison of Experiments for Five Reynolds Numbers (Poll\textsuperscript{25}) and Theory of Eq. (2).](image-url)
This is shown in Figure 7, and we see good agreement with Poll’s measurements for large sweep. Clearly this approach provides no information on the underlying stability modes within the boundary layer and our studies continue.

Connection to rotating cones
The change in dominant instability mode with sweep angle, as observed by Kohama\textsuperscript{27,28}, has interesting parallels to observations of the transitional region in the boundary-layer flows over rotating cones. In particular, the experimental studies of cones with slender half-angles (rotating in either still fluid or uniform axial flows) by Kobayashi and co-workers\textsuperscript{29,30} show the existence of pairs of counter-rotating vortices. These are qualitatively similar to Poll and Kohama’s observations of unswept cylinders. The vortices are known to arise from a dynamic instability induced by the centrifugal force of the flow field. However, as the half-angle is increased, visualizations clearly show that these vortices change from pairs of counter-rotating vortices to co-rotating crossflow vortices, as observed on rotating disks\textsuperscript{31-36}. These are qualitatively similar to the vortices observed over cylinders with sufficient sweep.

The change in dominant mechanism is seen to occur over cones with half-angles in the range of 40 to 50 degrees. This is comparable to the apparent critical sweep angle for the change in dominant mode over swept cylinders, as reported by Kohama\textsuperscript{27,28}.

Although the rotating system clearly represents a fundamentally different system to the stationary, swept cylinder, the parallels drawn between high-level experimental observations of the two systems mean that the ongoing analysis of the rotating-cone system by Garrett, Hussain and Stephen\textsuperscript{37-39} could provide useful insights into the swept-cylinder problem. To date, Garrett \textit{et al.} have been able to correctly predict observable quantities for vortices in the crossflow-dominated transitional flow over broad cones, using a combination of numerical and asymptotic approaches. Furthermore, they have recently demonstrated the existence of an alternative centrifugal mode, predicted to be dominant over slender cones. The interplay between these two modes close to the critical half-angle is of particular relevance to this swept cylinder study.

V. Conclusions

The existence of fine scale organized streamwise vorticity in the canonical case of cross-flow over a circular cylinder was predicted by Kestin and Wood and has been confirmed experimentally. Their theory predicted the spanwise wavelength and the present experiments have demonstrated good agreement with the theory. Similarly streamwise vorticity has been observed on the suction surfaces of compressor and turbine blading. The Kestin and Wood theory provides the basis of a correlation predicting the spanwise wavelength of these disturbances on compressor and turbine blading.

It has been demonstrated computationally that the streamwise vorticity interacts with spanwise vorticity in leading edge bubbles to promote early transition and bubble closure. LES results demonstrate that instantaneous flow patterns in the leading edge region at off-design conditions can be extremely complex. At low Reynolds numbers these complex leading edge flows can be predicted but they also emphasize the enormous computing resource that would be required to achieve this on a routine basis for design purposes. The grid needs to be very highly resolved to minimize the dependence on sub-grid modeling. Time accuracy is required to capture the vortical events that are required for an understanding of the physical behavior fundamental for laminar boundary layers, laminar separation and transition. The computation also needs to be run for a sufficient duration that time averaging can take place for streamwise vortices to become organized. For this to happen a further requirement is that the spanwise extent of the computational domain should be adequate to allow the vorticity to become organized. This may require the computation to extend all the way to the end walls, which is also desirable if secondary and end wall flows are to be predicted.

The boundary layers of swept flows need particular attention. The governing instabilities are not well understood over the complete range of sweep angles. The streamwise vorticity observed at zero sweep is eventually overcome by the more vigorous crossflow instability. At present there is no published guidance for designers on the conditions under which this happens. An approach to obtaining such information is suggested. A preliminary attempt to generalise the Kestin and Wood theory to non-zero sweep angles has been made.
This relates the cross-flow instability of Poll with sweep angles of 55° to 71° to the attachment line instability of Kestin and Wood. Experimental work at zero sweep has produced a solid foundation giving a reference for subsequent work over a wide range of sweep angles. Work is in progress to cover such a range and to further refine the theory.

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