A selective review of CFD transition models

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This paper aims to give an overview of the more widely used approaches to model transition in Computational Fluid Dynamics (CFD). Eight different methods are reviewed: the stability theory approach, the low Reynolds number turbulent closure approach, the intermittency transport method with integral correlations, the intermittency and the vorticity Reynolds number approach, the laminar fluctuation energy method, the $\bar{v^2} - f$ model, and the Large Eddy Simulations (LES) and Direct Numerical Simulations (DNS) for transition. The approaches are compared to one another, highlighting their respective advantages and drawbacks. From this analysis, a list of desirable features for CFD transition models is drawn up, against which the eight approaches are scored.

Key Words: *Transition, intermittency, boundary layer, CFD.*

I.   Introduction

Transition is a complex phenomenon, defined as the whole process of change from laminar to turbulent flow. The origin of turbulence and the accompanying transition from the laminar to turbulent regime, as often happens, for example, on aircraft wings or past turbine blades, is of fundamental importance to the whole fluid mechanics community. It affects strongly the evolution of losses and other factors of practical significance such as the distributions of wall shear stress and surface heat transfer. It is accompanied by many changes in flow characteristics, such as an abrupt change in the law of resistance, and both the skin friction and heat transfer may increase considerably. The most important feature of the phenomenon of transition is the increased diffusivity in the flow. The main reason of its complexity is that besides the simultaneous presence of laminar and turbulent flow, there is also the interaction between the two phases. In favour of the more widely investigated laminar and turbulent regimes, the study of this interaction was often overlooked.

This work aims to evaluate eight popular transition CFD models, showing the advantages and disadvantages of each one and making an assessment on them. The efforts by different research groups have resulted in a spectrum of models that can be used in different applications, while balancing the accuracy requirements and the computational resources available to the CFD user. However, the important effect of laminar-turbulent transition is not included in the majority of today's engineering CFD simulations. The reason for this is that transition modelling does not offer the same wide spectrum of CFD-compatible model formulations that is currently available for turbulent flows, even though a large body of publications is available on the subject. There are several reasons for this unsatisfactory situation. Firstly, the transition process involves a wide range of scales, with energy and momentum transfer predominately affected by non-linear (inertial) processes between eddies of different scales and it is very sensitive to physical flow features such as pressure gradients and the free stream turbulence level. Secondly, transition occurs through different mechanisms in different applications such as natural transition, bypass transition and separation-induced transition. The third complication arises from the fact that conventional Reynolds averaged Navier-Stokes (RANS) procedures do not lend themselves easily to the description of transitional flows, where both linear and non-linear effects

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are relevant. RANS averaging eliminates the effects of linear disturbance growth and it is therefore difficult to apply to the transition process.

II.   Current methods to predict transition in CFD

A.   Stability theory approach

The first approach is based on stability theory that avoids the aforementioned RANS limitation. Stability theory is based on the study of the behaviour of small flow disturbances to see whether they grow or not. The single oscillation of the disturbance is assumed to be

$$\psi\left(x,y,t\right) = \phi\left(y\right)e^{i(\alpha x - \beta t)} \tag{1}$$

and any arbitrary two-dimensional disturbance is assumed expanded in a Fourier series, each of its terms represents such a partial oscillation. If the disturbance amplitude grows, the flow is unstable and transition to turbulent flow is expected. The advantage of this approach is that the equations can be linearised, which makes this problem amenable to an analytical approach. Making use of the continuity and momentum equations for two-dimensional, incompressible, unsteady flow and neglecting quadratic terms in the disturbance velocity components results in the Orr-Sommerfield equation

$$(U - c)\left(\phi^{''} - \alpha^2\phi\right) - U^{''}\phi = -\frac{i}{\alpha R}\left(\phi^{''''} - 2\alpha^2\phi^{''} + \alpha^4\phi\right) \tag{2}$$

where the primes denote differentiation with respect to the dimensionless coordinate y/δ and R is the Reynolds number of the mean flow. The problem of stability thus reduces to an eigenvalue problem: for a given Reynolds number and wavenumber pair, there exist eigenvalues ω_k with corresponding eigenfunctions. The stability of each eigenmode is given by the imaginary part of ω_k. If this is positive, the amplitude of the corresponding disturbance grows exponentially and if it is negative, the amplitude decays. But it could be believed that it is enough to study the stability of each individual mode in order to determine whether the flow stays laminar or not. This is however not the case and the reason is that the eigenfunctions are non-orthogonal. Due to this fact, a sum of eigenmodes might show an initial growth even if linear stability predicts that all eigenmodes decay exponentially. This mechanism is known as the transient growth. Based on recent advances in hydrodynamic instability, it has been recognised that the dynamics of many wall-bounded shear flows is better described by a superimposition of normal modes rather than by a single (the least stable) mode. Even though the Laplace transform resides entirely in the stable half-plane, transient effects can cause energy amplification that may subsequently trigger non-linear saturation followed by secondary instabilities. The same viewpoint should hold for global modes, that is, the superimposition of mutually non-orthogonal global modes may result in a substantially different perturbation dynamics that is predicted by the global spectrum. In addition, by means of the stability equation, a theoretical critical Reynolds number is obtained that indicates the point on the wall at which amplification of some individual disturbances begins and proceeds downstream of it. The transformation of such amplified disturbances into turbulence takes up some time. It must, therefore, be expected that the observed position of the point of transition will be downstream of the calculated one, in other words, the experimental critical Reynolds number exceeds its theoretical value. Because the growth is so slow, transition to turbulence might not be complete until a streamwise distance that can be as large as 20 times farther downstream from the leading edge than the initial starting position of linear instability.[1] Moreover, since the transition model is based on the linear stability theory, it cannot predict the transition due to non-linear effects, such as high freestream turbulence or surface roughness. Where the exponential growth of two-dimensional waves results in a finite amplitude wave, the linear theory ceases to be valid. In fact, during the growth of the waves, spanwise distortion and three-dimensional non-linear interactions become relevant.

The more widely used method based on the stability theory is the so-called e^n method of Smith & Gamberoni[2] and Van Ingen.[3] They proposed to correlate the onset of transition with the amplification rate of the most unstable wave at each position to determine an amplitude disturbance ratio

$$\frac{A}{A_0} = exp\left[\int_{x_0}^{x} -\alpha\,dx\right] = exp\left[n\right] \tag{3}$$

with A_0 the initial amplitude disturbance at the first neutral-stability point. e^n methods are not compatible with general-purpose CFD methods as typically applied to complex geometries. The reason is that these

methods require a priori knowledge of the geometry and of the grid topology. In addition, they involve numerous non-local operations, such as tracking the disturbance growth along each streamline, that are difficult to implement into today's CFD methods,[4] and the typical industrial Navier Stokes solutions are not accurate enough to evaluate the stability equation. However, even the e^n method is not free from empiricism. This is because the transition n-factor is not universal and depends on the wind tunnel and free-stream environment. This means that it works well for flows that are not too different from the ones used for its calibration. The main obstacle to the use of the e^n model is that the required infrastructure needed to apply the model is complex. The stability analysis is typically based on velocity profiles obtained from highly resolved boundary layer codes that must be coupled to the pressure distribution of a RANS CFD code. The output of the boundary layer method is then transferred to a stability method, which then provides information back to the turbulence model in the RANS solver. The complexity of this set-up is mainly justified for special applications where the flow is designed to remain close to the stability limit for drag reduction, such as in laminar wing design. The problem is considerably more complex in 3D flows in which streamwise and crossflow disturbances can coexist. Moreover, when bypass transition occurs, this method does not work at all.

B.   Low Reynolds number turbulent closure approach

The second way to predict transition is to use low Reynolds number turbulence models. However, the ability of these turbulent models to predict transition is questionable. They typically suffer from a close interaction between the transition capability and the viscous sublayer modelling and this can prevent an independent calibration of both phenomena.[5] At best, low Reynolds number models can only be expected to simulate bypass transition, which is dominated by diffusion effects from the freestream. This is because standard low Reynolds number models rely exclusively on the ability of the wall damping terms to capture the effects of transition. Realistically, it would be very surprising if these models that were calibrated for viscous sublayer damping could faithfully reproduce the physics of transitional flows. It should be noted that there are several low Reynolds number models where transition prediction was considered specifically during the model calibration (Wilcox,[6] Langtry & Sjolander,[7] Walters & Leylek[8]). However, these model formulations still exhibit a close connection between the sublayer behaviour and the transition calibration. Re-calibration of one functionality also changes the performance of the other. It is therefore not possible to introduce additional experimental information without a substantial re-formulation of the entire model. Models like the Launder-Sharma model,[9] where the near-wall behaviour is described by the turbulence Reynolds number ($Re_t = k^2 / \nu \varepsilon$) perform better than those that use the local wall distance. However, no model gives a reliable result for any arbitrary combination of Reynolds number, Free-Stream Turbulence (FST) level and pressure gradient. Moreover, results are sensitive to the initial conditions, boundary conditions and numerical aspects such as the grid resolution and the computational domain extension. Westin and Hankes[10] and Craft et al.[11] improved the low Reynolds number closure approach for bypass and separation-induced transition by means of a non-linear eddy viscosity model that in general produces better results than a linear eddy viscosity model, but is still sensitive to boundary conditions and numerical aspects. Some improvements have been obtained with the low Reynolds number second-moment-closure (SMC) as used by Hanjalic et al.[12] The benefits are located in the provision to account for the anisotropy of the free-stream and of the near-wall Reynolds stress field, particularly in the ability to reproduce the normal-to-the-wall velocity fluctuations. Another merit is the exact treatment of the turbulent production and of the effects of streamline curvature. These characteristics help also in handling other forms of non-equilibrium phenomena, such as separation and reattachment, that are frequently encountered with different forms of transition. With this model, Hanjalic was able to predict the onset of transition for several known test cases without having to use any empirical triggering as recommended by the Special Interest Group on transition (SIG10) of ERCOFTAC. This technique has been successfully applied at high levels of FST intensity but not to flows with a FST level less than 3% . In addition, it is more complex to implement and also more computationally expensive than more empirical models (6,7,8).

C.   The intermittency transport method with empirical correlations

The third approach to predict transition, which is favoured by the turbomachinery community, is to use the concept of intermittency, as introduced by Dhawan and Narasimha,[13] to blend together laminar and turbulent flow regimes as done by Abu-Ghannam,[14] Mayle[15] and Suzen & Huang[16] based on empirical

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correlations. Many detailed investigations of the process of transition reveal that, over a certain range of Reynolds numbers around the critical value, the flow becomes intermittent, which means that it alternates in time between being laminar and turbulent. The physical nature of this flow can be properly described with the aid of the intermittency factor γ, which is defined as the fraction of time during which the flow at a given position remains turbulent, or in other words, it is the fraction of time that the flow is turbulent during the transition phase. By letting the intermittency grow from zero to unity, the start and the evolution of transition can be imposed. Mostly, this is done by multiplying the eddy viscosity in a two-equation turbulence model by the intermittency factor. In other words, once γ is determined, it is multiplied by the eddy viscosity in the mean-flow equations. In the pre-transitional regime, γ is set to zero and γ assumes a positive value only where the model is required to initiate transition. Suzen & Huang[16] developed an intermittency transport model that can produce both the experimentally observed streamwise variation of intermittency and a realistic profile in the cross-stream direction. The model combines the transport equation models of Steelant & Dick[17] and Cho & Chung.[18] Specifically, the transport of intermittency, γ, is given by

$$
\frac{\partial \rho \gamma}{\partial t} + \frac{\partial \rho u_j \gamma}{\partial x_j} = (1 - \gamma) \left[(1 - F) C_o \rho \sqrt{u_k u_k} \beta (s) + F \left(\frac{C_1 \gamma}{\kappa} \tau_{ij} \frac{\partial u_i}{\partial x_j} - C_2 \gamma \ \varrho \frac{\kappa^{\frac{3}{2}}}{\varepsilon} \frac{u_i}{(u_\kappa u_\kappa)^{\frac{1}{2}}} \frac{\partial u_i}{\partial x_j} \frac{\partial \gamma}{\partial x_j} \right) \right]
$$
$$
+ C_3 \rho \frac{\kappa^2}{\varepsilon} \frac{\partial \gamma}{\partial x_j} \frac{\partial \gamma}{\partial x_j} + \frac{\partial}{\partial x_j} \left(((1 - \gamma) \gamma \sigma_{\gamma t} \mu + (1 - \gamma) \sigma_{\gamma t} \mu_t) \frac{\partial \gamma}{\partial x_j} \right) \tag{4}
$$

where the modelling constant are $\sigma_{\gamma t} = 1.0$, $C_0 = 1.0$, $C_1 = 1.6$, $C_2 = 0.16$, $C_3 = 0.15$.

This approach neglects the interaction between the turbulent and non-turbulent parts of the flow during transition. In order to capture this interaction, a conditional averaging technique leading to a set of turbulent and a set of non-turbulent equations for mass, momentum and energy is necessary, as used by Steelant & Dick.[17] The conditional averaging is usually seen as too computationally expensive for engineering applications, as the number of equations doubles. Therefore, the intermittency concept is typically used in combination with globally averaged Navier-Stokes equations and the loss of some physical information is accepted. Despite its inability to capture the essence of the actual transition mechanism, single-point RANS turbulence closures offer more flexibility and better prospects for predicting a real flow with transition than the classical linear stability theory. Although much more limited in capturing the real physics than DNS or LES, statistical modelling is still the only viable method to compute complex flows with transition phenomena. It is recalled that natural transition is much rarer in industrial flows than bypass and separation induced transition. The RANS intermittency statistical models typically correlate the transition momentum thickness Reynolds number to local freestream conditions, such as the turbulence intensity and the pressure gradient. These models are relatively easy to calibrate and are often sufficiently accurate to capture the major effects of transition. In addition, correlations can be developed for the different transition mechanisms, ranging from bypass to natural transition as well as cross-flow instability or surface roughness. The main shortcoming of these models lies in their inherently non-local formulation. They typically require information on the integral thickness of the boundary layer and the state of the flow outside the boundary layer. While these models have been used successfully in special-purpose turbomachinery codes, the non-local operations involved with evaluating the boundary layer momentum thickness and determining the freestream conditions have precluded their implementation into general purpose CFD codes. Still, statistical RANS models can adequately capture the effects of transition in situations where most of the natural transition development stages are bypassed by some strong external disturbance.

D.   The laminar fluctuation energy method

A new and interesting class of transition models is based on the description of the laminar fluctuation energy in the pre-transitional region of a boundary layer. The pre-transitional region of boundary layers subject to free-stream turbulence resembles a laminar boundary layer in terms of the mean velocity profile. As the FST level is increased, the profile becomes noticeably distorted from the typical Blasius profile, with an increase in momentum in the inner region and a decrease in the outer region, even for a FST level as low as about 1%. This shift in mean velocity profile is accompanied by the development of relatively high-amplitude streamwise fluctuations, which can reach intensities several times that of the free-stream turbulence. This process results in an increase in skin friction and heat transfer in the pre-transitional region, and eventually leads to bypass transition through the eventual breakdown of the streamwise fluctuations.[19] It is important to note that

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these streamwise fluctuations are not turbulence in the usual sense of that word. This distinction was made for modelling purposes by Mayle and Schulz,[20] who proposed a "laminar-kinetic-energy"k_l equation to describe the development of such fluctuations upstream of transition. Structurally, these fluctuations are very different from turbulent fluctuations, since the energy is almost entirely contained in the streamwise component. Their dynamics are also considerably different. The familiar cascade of energy from larger to smaller scales is not present. Instead, fluctuations are amplified at certain scales determined by the boundary layer itself, and remain at a relatively low wavenumber. Dissipation is therefore also expected to be relatively low, except very near the wall due to the no-slip condition. All of these considerations have led to adopt a second kinetic energy equation by Mayle and Schultz [20] to describe these fluctuations. The growth of k_l has been shown experimentally[21] and analytically[22] to correlate with low-frequency normal velocity fluctuations (v') in the free-stream. The scale selectivity of the boundary layer was clearly demonstrated by Johnson and Ercan,[23] who plotted the amplification of six frequency bands in a pre-transitional boundary layer. The reasons for this selectivity and amplification are not yet completely understood. Volino[24] considered the possibility that the growth of k_l is due to a "splat mechanism", similar to that discussed by Bradshaw.[25] It is thought that the wall redirects the normal velocity fluctuation into a streamwise component, at the same time as creating local pressure gradients in the boundary layer, leading to disturbance amplification. This mechanism is decidedly different from typical turbulence production, and is adopted herein as a reasonable explanation of the development and amplification of k_l. Since "splats"are likely to occur only for eddies with large length-scales relative to the wall distance, the turbulent energy spectrum can be divided into wall-limited (large scales) and non-wall-limited (small scales) sections in the near-wall region (see Fig. 1), where the cut-off eddy size is designated by λ_{eff}, since it is the effective turbulent length scale for the small
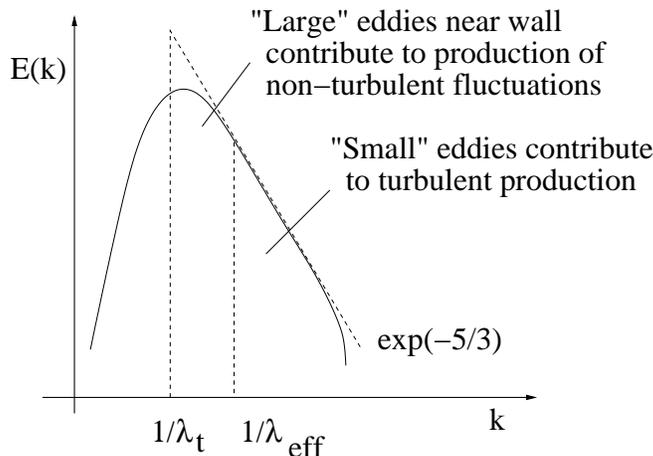

Figure 1. Illustration of wall-limiting concept leading to "splat mechanism" for production of k_l.

scale turbulence. Scales smaller than λ_{eff} interact with the mean flow as typical turbulence and larger scales contribute to the production mechanism for k_l. The laminar kinetic energy represents the magnitude of the "non-turbulent" streamwise fluctuations in the pre-transition boundary layer. Another region of interest is the transition zone itself. Jacobs and Durbin[19] showed that bypass transition is initiated by an instability of the upstream fluctuations, which leads to turbulent spot development and progression to full turbulence. It is not clear what initiates the instability. In the k_l model, a local transition parameter is implemented that depends on the turbulence energy, the effective length scale, and the fluid viscosity, based in part on measurements by Andersson et al.[26] Once this parameter reaches a certain value, transition is assumed to begin, which results in a transfer of energy from the streamwise fluctuations k_l to the turbulent fluctuations k_t . This is accompanied by a change in length scale of the turbulence, as would occur in an actual spot breakdown process. Downstream of transition, the model predicts a fully turbulent boundary layer. Almost all of the fluctuation energy is turbulent, but a small amount of k_l is still present within the viscous sub-layer. This agrees qualitatively with experimental observations indicating the presence of streamwise-oriented streaky structures in the viscous sublayer and buffer region that bear a resemblance to those in the pre-transitional region. Although bypass transition is recognised to occur for a FST level greater than 1%, the downstream location of transition is in fact shortened by turbulence intensities greater than

about 0.1%.[27] This suggests that there is a mixed transition regime involving elements of both natural and bypass transition. In order to include natural and mixed mode transition into the k_l model, modifications must be made to both the k_l production terms and the transition production term that governs the transfer of energy between streamwise fluctuations and the kinetic energy of turbulence. These modifications do not depend directly on turbulence quantities, but depend instead on the local mean flow and the laminar kinetic energy k_l . Recent examples of such models were formulated by Walters & Leylek[8] and Laurdeau et al.[28] A one-equation system is used to describe the "non-turbulent" fluctuations prior to transition,

$$\frac{\partial (\rho k_l)}{\partial t} + \frac{\partial (\rho u_j k_l)}{\partial x_j} = \rho P_{k_l} - \rho R - \rho D_L + \frac{\partial}{\partial x_j}\left[\mu \frac{\partial k_l}{\partial x_j}\right] \tag{5}$$

This equation lacks the usual shear-stress/strain related generation term, but it contains a source term that is argued to arise from the pressure-diffusion correlation. Thus, the equation returns, on calibration, the requisite rise in the fluctuation-energy level in the laminar regime, despite the absence of a shear stress, presumed to be zero. Information from this system is used to start and let grow the turbulent kinetic energy in a conventional two-equations $k - \omega$ RANS. The model is based on an eddy viscosity coefficient, determined by using three transport equations for the turbulent kinetic energy k, the laminar kinetic energy k_l, and the specific turbulent kinetic energy dissipation rate ω. The model automatically predicts the onset of transition without any intervention from the user and is based strictly on local variables, therefore it does not require the evaluation of any integral parameter. The principle is physically sound, but the technique is still too new to allow a judgement on its quality. It has not been extensively validated except for a few flat plate test cases and a turbine blade case. However, the initial results from this model are promising and indicate that the model appears to have the correct sensitivity to the freestream turbulence. It remains to be seen how accurately the model can predict the effects of pressure gradient and separation on transition, particularly at a low FST level, below $< 1\%$. Recent LES performed by Lardeau et al.[29] shed some light on the validity of the assumption underlying the RANS closure for the fluctuation level observed upstream of the transition onset. This simulation has shown that, from a statistical point of view, shear-stress/strain-induced production is mainly responsible for the elevation of the pre-transitional laminar fluctuation energy, a process that is akin to that observed in the turbulent state, although here is mainly in the upper part of the boundary layer. Indeed the ratio $<uv>/k$ is quite high over a significant portion of the pre-transitional boundary layer, that contradicts the base assumption of the model, that is the shear stress term in the production of k_l is zero. Further studies should be done on this model to improve its representation of pre-transitional flow.

E.   DNS for transition

Large Eddy Simulation (LES) and Direct Numerical Simulations (DNS) are suitable tools to predict transition (e.g. Durbin et al.[1]), although the proper specification of the external disturbance level and structure poses substantial challenges. In principle, laminar flow breakdown, the development of turbulent spots and transition to fully turbulent flow can be simulated very accurately using DNS.[30] A DNS computation is performed by solving the full time-dependent Navier-Stokes equations. Since there is no Reynolds averaging, then there is no requirement for turbulence closure by a turbulence model. In order to capture the small scales of turbulence, a DNS computation requires a very fine grid. Unfortunately, these methods are far too costly for typical engineering applications. For instance, a DNS simulation by Zheng[30] of a flat plate transitional boundary layer used approximately 50 million grid points and was performed in about four weeks on a supercomputer with 64 processors. Due to its large computational requirements, DNS is clearly not yet at the stage where it can serve as a practical tool for engineering design applications. This will be the case for a long time as additional computing resources in industry are usually used to simulate larger and more complex geometries. DNS simulations are currently used mainly as research tools and as a substitute for controlled experiments.

F.   LES for transition

Because of the significant computational costs associated with DNS, a number of researchers have applied the concept of Large Eddy Simulation to transitional flow. In LES computations, only the large scale eddies are resolved, the small scale eddies are modelled using an eddy viscosity approach such as that proposed by Smagorinsky.[31] One of the main problems with LES is that the predicted transition location is very sensitive to the choice of the Smagorinsky constant that is used to calibrate the subgrid eddy viscosity

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(Germano et al.[32]). Germano et al. have since proposed the dynamic subgrid-scale model to compute the Smagorinsky constant locally. The dynamic model has the advantage that in laminar boundary layers the subgrid eddy viscosity is automatically reduced to zero. Consequently, it is believed that this model should be more appropriate for predicting transitional flows. Nevertheless, the dynamic LES model is not a complete solution to the issues associated with applying LES to predict transitional flows. LES computations performed by Michelassi et al.[33] on a low-pressure turbine blade with periodically impinging wakes have indicated that, while the dynamic LES model was in good qualitative agreement with DNS results, noticeable differences were observed in the quantitative comparison.

G. The $\bar{v^2} - f$ model

Large Eddy Simulation (LES) for bypass transitional flow[34] suggested that v', the turbulence fluctuation in the wall-normal direction, plays an important role in the transition process. A wall-normal velocity disturbance slowly increases from close to the wall to regions of higher velocity. This motivated the use of the $\bar{v^2} - f$ model,[35] without the inclusion of γ, for transitional flows. The $\bar{v^2} - f$ model consists of three transport equations for the turbulent kinetic energy k, the dissipation of the turbulent kinetic energy ε, and the flow-normal component of the kinetic energy $\bar{v^2}$ along the streamlines. In addition, the model includes a Helmholtz type equation for a quantity f which models the pressure-strain term. The turbulent velocity and time scale are calculated from the standard $k - \varepsilon$ equations. The $\bar{v^2}$ transport equation is

$$\partial_t \bar{v^2} + U \cdot \bigtriangledown \bar{v^2} = kf - \bar{v^2}\frac{\varepsilon}{k} + \bigtriangledown \cdot \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \bigtriangledown \bar{v^2} \right] \tag{6}$$

where

$$kf = \phi_{22} - \varepsilon_2 + \frac{\bar{v^2}}{k}\varepsilon \tag{7}$$

represents redistributions of turbulent kinetic energy from the streamwise component. The Boussinesq approximation is used for the stress-strain relation. The $\bar{v^2} - f$ is solved separately from the mean flow and the k and ε transport equations. Results have shown that with this model the onset of transition at a low FST level is predicted early, moreover, the peak of u' is under-predicted. This could be due to the use of the Boussinesq stress-strain relation, in which the Reynolds stresses are assumed of the same shape as the viscous stresses. The results suggest that the dissipation rate transport equation for ε and, possibly, the 'elliptic relaxation' equation for f, which is responsible for energy distribution among different Reynolds-stress components, require further calibration, particularly in the transition region. The introduction of an intermittency factor γ into the model is likely to increase the model's sensitivity to a number of flow features, such as pressure gradients and the FST level, depending on how γ is coupled with the $k - \varepsilon - \bar{v^2}$ model.

H. The intermittency and vorticity Reynolds number approach

Based on the success of the intermittency concept in predicting transitional flows, a novel approach to avoid the need for non-local information in correlation-based models has been introduced by Menter et al.[36] In this formulation, only local information is used to activate the production term in the intermittency equation. The link between the correlation and the intermittency equation is achieved through the use of the vorticity Reynolds number Re_v.[37] Since the vorticity Reynolds number depends only on density, viscosity, wall distance, and vorticity, it is a local property and can be easily computed at each grid point. The model is based on two transport equations. One is an intermittency equation for γ. The intermittency function γ is coupled with the SST (Shear Stress Transport) $k - \omega$ turbulence model,[38] which is used to turn on the production term of the turbulent kinetic energy downstream of the transition point in the boundary layer. The second one is a transport equation formulated to avoid additional non-local operations introduced by the quantities used in experimental correlations. These correlations are typically based on freestream values, like the turbulence intensity or the pressure gradient outside the boundary layer. The additional transport equation is formulated in terms of the transition onset Reynolds number $Re_{\theta t}$, which is function of the boundary layer momentum thickness θ. Outside the boundary layer, the transported variable is forced to follow the value of $Re_{\theta t}$ provided by the experimental correlation. This information is then diffused into the boundary layer by a standard diffusion term. By this mechanism, the strong variations of the turbulence intensity and the pressure gradient in the freestream that are typically observed in industrial flows can be taken into account. This transport equation essentially takes a non-local empirical correlation

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and transforms it into a local quantity, which can then be compared to the local vorticity Reynolds number in order to determine where in the flow the transition criteria are satisfied. At every location in the flow where the vorticity Reynolds number exceeds the local transition Reynolds number, a source term in the intermittency equation is activated and turbulence is produced. This is the central mechanism by which the transition model operates. To predict transition in separated flows, a separated flow modification is applied to the model.[39]

III. Review of the current intermittency models for CFD

Closer inspection shows that many of the current transition models are not fully CFD compatible. Specifically, most formulations suffer from non-local operations that cannot be carried out (with reasonable effort) in general-purpose CFD codes. This is because modern CFD codes use mixed elements and this does not provide the platform for computing integral boundary layer parameters or allow the integration of quantities along the direction of external streamlines. Even where structured boundary layer grids are used, like hexahedra, the code often retains the data structure of an unstructured mesh. The information in the body-normal grid direction is therefore not easily available. In addition, most industrial CFD simulations are carried out on parallel computers using a domain decomposition approach. This means in the most general case that boundary layers can be split and distributed across different processors, prohibiting any search or integration algorithm. Consequently, the main requirements for a fully CFD-compatible transition model are:

1. Allow the calibrated prediction of the onset and the length of transition

2. Allow the inclusion of different transition mechanisms

3. Be formulated locally (no search or line-integration operations)

4. Avoid multiple solutions (same solution for initially laminar or turbulent boundary layer)

5. Not affect the underlying turbulence model in the fully turbulent regime

6. Allow a robust integration down to the wall with similar convergence as the underlying turbulence model

7. Be formulated independent from the coordinate system

8. Applicable to three-dimensional boundary layers

In order for a transition model to be useful for industrial predictions of transitional flows it must be accurate and robust, and must be based on a local formulation that is applicable to unstructured and massively parallelized CFD codes.

IV. Conclusion

This paper reviewed eight transition modelling approaches. The review highlighted the difficulty in combining classical CFD to transition models. This arises from the locality of CFD algorithms that conflicts with the modelling of transition as a process that takes place along streamlines. The more widely used transition models have been outlined, highlighting their respective benefits and shortcomings. The main requirements for a fully CFD compatible transition model have been identified. Transition models that have historically been compatible with modern CFD methods are the low Reynolds number models, but their limitations prompt further research towards more generally applicable formulations. The transition model of Menter satisfies most of the requirements for a fully CFD compatible transition model, but it is not coordinate system independent. This is a consequence of the fact that transition correlations are based on non-Galilean invariant parameters such as the turbulence intensity, which is defined based on the local freestream velocity. This is because the empirical correlations require the freestream velocity in order to determine the turbulence intensity. For stationary or rotating reference frames, this is not a problem as long as the relative velocity is used to compute the freestream turbulence intensity. However, when moving walls are present, like sliding walls, deforming walls or walls rotating at a different speed compared to the

rotating reference frame, as in the casing a turbine rotor half stage, then the freestream velocity relative to the wall will be in error. Galilean invariance is an important criterion for general turbulence models and future work should therefore focus on improving this aspect of the model. The main limitation of the Menter transition model right now is thought to be the accuracy of the empirical correlations, in which the physics of transition is entirely contained. As more experimental data become available, which can be used to calibrate the empirical correlations, the model accuracy should also improve. With this in mind, DNS results are becoming more and more important as numerical test cases for transition because they eliminate a lot of the uncertainty that is present in transition experimental data. The proposed transport equations do not attempt to model the physics of the transition process unlike turbulence models, but form a framework for the implementation of correlation-based models into general-purpose CFD methods. The other promising development route in modelling transition is the one proposed by Mayle & Schultz,[20] the laminar kinetic energy approach. This correlation-free technique predicts the onset of the transition process and is based on local variables, but so far it has not been extensively validated. However, the preliminary results are good and indicate that the model appears to have the correct sensitivity to the freestream turbulence level. Still, it is an open question as to whether this model can be extended to predict additional effects such as roughness or freestream length scales. Anyway, it is expected that the concept of laminar-kinetic-energy will be a very active area of transition research in the near future. To date, none of the transition models have been shown to satisfy all the requirements stated in section III and there is clearly a need in industry for an accurate and robust transition model, based on local state variables. Despite its complexity, transition should not be viewed as outside the range of RANS methods. In many applications, transition is constrained to a narrow area of the flow due to geometric features, pressure gradients and/or flow separation. Even relatively simple models can capture these effects with sufficient engineering accuracy. The challenge to a proper engineering transition model is therefore mainly in the formulation of a model that can be implemented into a general RANS environment.

Acknowledgements

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