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Vortex-Speed Selection within the Rotating-Disk Boundary Layer

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ABSTRACT
A theoretical investigation into the vortex-speed selection within the rotating-disk boundary layer is conducted. Two methods are used which are different to previous investigations in the literature and enable analysis of a particular vortex speed with respect to the rotating surface. Using these methods (together with arguments based on critical Reynolds numbers, unstable parameter ranges and linear growth rates) it is hypothesised that crossflow modes travelling at around 75% of the disk surface are likely to be selected in applications where smooth, clean disks are used, for example in Chemical Vapour Deposition (CVD) reactors.

1. INTRODUCTION
Since the seminal paper of Gregory Stuart & Walker [1] was published in 1955, the stability of the rotating-disk boundary layer has been an important open problem in fluid mechanics. Over the years GSW’s observations have been clarified by a number of experimentalists (for example [2, 3, 4, 5, 6]), and the flow characteristics are nicely summarised in the flow visualisation in Figure 1. We see that as the disk is rotated, a region of laminar flow is observed within the boundary layer around the axis of rotation, at larger radial positions a region of co-rotating spiral vortices appears, followed by the sharp transition to turbulence at a repeatably consistent radial location. The onset of the vortices is known to occur at a critical Reynolds number (as defined by equation (1)) of around 285, and turbulence at around 510. Physically the three regions move closer towards the axis of rotation as the rotation rate is increased (although the critical Reynolds numbers are maintained).

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This particular boundary-layer flow has remained important for two main reasons: firstly, its similarity to that over a swept wing (indeed this was GSW’s motivation), and secondly, as a model for general three-dimensional rotating boundary layers. The flow therefore has both practical and theoretical relevance and this has motivated a huge body of literature describing experimental and theoretical investigations over the last five decades. A summary of the literature is contained within the excellent review papers of Reed & Saric [7] and Saric et al. [8] and no attempt to summarise this body of work is made here. However, with regards to this present study two theoretical papers stand out as being particularly important, namely those due to Malik [9] and Lingwood [10].

Malik and Lingwood have clarified the theoretical approaches that can be used to model the important features of the flow. In particular, the appearance of the spiral vortices can be modelled by the onset of local convective instability (Malik), and the appearance of turbulence is closely related to the onset of local absolute instability (Lingwood). Although many papers have been published which investigate and use these concepts, these two papers have been
highlighted due to their similarly with the numerical methods used by myself within this paper.

As will be discussed later, two spatial modes are now known to dictate the convective instability characteristics of the boundary-layer flow; these are typically called type I and type II modes. Type I modes arise from inviscid crossflow instabilities and are a consequence of the inflectional nature of the steady laminar flow profiles; type II modes arise from streamline-curvature effects within the flow. The neutral curve for convective instability is mapped out by the interplay of the two spatial branches arising from these modes as the Reynolds number is varied. The resulting neutral curve has two characteristic lobes: an upper lobe due to the type I mode and a lower lobe due to the type II mode (see Figure 7, for example).

The onset of absolute instability is also known to occur from the interplay of spatial branches. In particular, Lingwood demonstrates that absolute instability occurs when the type I branch ‘pinches’ with another (convectively stable) branch, typically labelled type III. Instability modes of type I, II and III also arise in the related boundary-layer flows over rotating spheres and cones, and a detailed description of these is given by myself and co-workers [11, 12, 13].

My previous work has been concerned with the extension of the main results from the rotating-disk literature to the related flows over rotating cones and spheres (both in and out of an imposed axial flow) [11]–[17], with particular emphasis on aerodynamic and industrial applications. However, in this paper I return to the rotating-disk problem in an attempt to address the unsolved aspect of vortex-speed selection within the convectively unstable region.

Early visualisation experiments of the rotating-disk boundary layer showed stationary vortices that rotate with the surface of the disk, and subsequent theoretical analyses have therefore assumed this. However, more recently the experimental investigations of Corke and co-workers [18, 19] have shown that travelling modes can be important in the transition process over highly polished, clean disks. In applications where clean disks are used, for example in types of chemical vapour deposition (CVD) reactors used for depositing thin films of optical and electrical materials on substrates (as discussed by myself and co-workers in [20]), better predictions for the onset of instability can be gained from studying all possible vortex speeds.

With regards to previous theoretical investigations it appears that only a small number of studies have considered travelling mode disturbances [21, 22, 23]. These papers conclude that disturbances arising from the type II mode can move at different speeds relative to the disk surface and with considerably lower critical Reynolds numbers than for the type I mode. However, as will be
discussed in §3.1, these investigations model non-stationary modes using a method that is difficult to interpret in terms of actual vortex speeds with respect to the disk surface, and no predictions of preferred vortex speed have been possible. In this present paper an alternative formulation is presented that enables a straightforward interpretation of vortex speed. Details of the computational aspects of this investigation are presented along with a theoretical prediction for the preferred vortex speed over clean disks. Arguments based on critical Reynolds numbers, unstable parameter ranges and linear growth rates are used.

In §2 the problem is formulated and the governing equations derived. The numerical methods used to solve the equations are described in §3 and two distinct pseudo-algorithms are introduced which enable the study of travelling modes in §4. For completeness, the existing method used for studying travelling modes in the literature is summarised in §3. Conclusions are drawn in §5.

2. MATHEMATICAL FORMULATION

2.1. The Steady Flow

Consider a rigid circular disk that is rotating in otherwise still fluid about its axis of symmetry with angular velocity $\Omega^*$. Note that an asterisk denotes a dimensional quantity in all that follows. We choose the fixed orthogonal curvilinear coordinate system $(r^*, \theta^*, z^*)$ representing radial, azimuthal and surface-normal variation respectively, with the origin located at the centre of the disk. Using this frame of reference necessarily eliminates the appearance of Coriolis terms in the governing equations and is crucial to the methods presented here. The choice of a fixed frame is in contrast to most other related studies in the literature, including both Malik and Lingwood.

The numerical stability analysis is conducted at local points along the disk surface $r^*_L$. The non-dimensionalising length, velocity, pressure and time scales are $\delta^*, r^*_L\Omega^*, \rho^*r^*_L\Omega^*^2$ and $\delta^*/\Omega r^*_L$ respectively, which lead to the local Reynolds number

$$ R_L = \frac{r^*_L\Omega^*\delta^*}{\nu^*} = r^*_L, $$

which is interpreted as the non-dimensional radial position on the disk. Note that $\delta^* = (\nu^*/\Omega^*)^{1/2}$ is the boundary-layer thickness. The steady velocities are non-dimensionalised as

$$ U(\eta) = \frac{U^*}{r^*_L\Omega^*}, \quad V(\eta) = \frac{V^*}{r^*_L\Omega^*}, \quad W(\eta) = \frac{W^*}{(\nu^*/\Omega^*)^{1/2}}, $$

(2)
where \( U, V \) and \( W \) are the non-dimensional velocities in the \( r^* \)-, \( \theta^* \)- and \( z^* \)-directions respectively and \( \eta = z^*/\delta^* \) is the non-dimensional distance from the disk surface in the normal direction.

The equations that govern the steady mean flow in the boundary layer are non-dimensionalised using (2) as

\[
WU' + (U^2 - V^2) = U'',
\]

\[
WV' + 2UV = V'',
\]

\[
W' + 2U = 0,
\]

where a prime denotes differentiation with respect to \( \eta \). The non-dimensional boundary conditions are

\[
U = W = V - 1 = 0 \quad \text{on} \quad \eta = 0,
\]

\[
V = U = 0 \quad \text{as} \quad \eta \rightarrow \infty,
\]

which represent the no-slip condition on the disk surface and the quiescent fluid condition away from the disk, respectively. Equations (3)–(6) are the familiar von Kármán equations [24] in this frame of reference, and are solved using a fourth-order Runge–Kutta integration method, in conjunction with a two-dimensional Newton–Raphson searching routine to iterate on the outer boundary conditions.

### 2.2. Linear Disturbance Equations

In order to derive the disturbance equations we consider the instantaneous non-dimensional flow quantities to be given by

\[
\hat{U}(\eta, r, \theta, t; R_L) = \frac{r}{R_L} U(\eta) + \hat{u}(\eta, r, \theta, t; R_L),
\]

\[
\hat{V}(\eta, r, \theta, t; R_L) = \frac{r}{R_L} V(\eta) + \hat{v}(\eta, r, \theta, t; R_L),
\]

\[
\hat{W}(\eta, r, \theta, t; R_L) = \frac{1}{R_L} W(\eta) + \hat{w}(\eta, r, \theta, t; R_L),
\]

\[
\hat{P}(\eta, r, \theta, t; R_L) = \frac{1}{R_L^2} P(\eta) + \hat{p}(\eta, r, \theta, t; R_L),
\]

where the hatted quantities are small unsteady perturbations and the unhatted quantities are the steady basic flow as determined by equations (3)–(6). The
appropriate non-dimensional Navier–Stokes equations are then linearised with respect to these perturbation quantities. In order to make the resulting perturbation equations separable in $r$, $\theta$ and $\eta$, it is necessary to use an approximation of the sort usually called the parallel-flow approximation where we ignore variation in $R_L$ with local radius and assume that $\eta/r_L << 1$. The resulting stability equations are then strictly local, with location $R_L = r_L$ appearing as a parameter. The assumption that $R_L > 1$ (equivalent to $\delta^* << r_L^*$) necessarily prohibits analysis close to the centre of the disk. The perturbation quantities can then be expressed in normal-mode form

$$(\hat{u}, \hat{v}, \hat{w}, \hat{p}) = (u(\eta), v(\eta), w(\eta), p(\eta)) \exp(i(\alpha r + \beta R_L \theta - \gamma t)) + \text{c.c.}$$

The wavenumber in the $r$-direction, $\alpha = \alpha_r + i \alpha_i$, is complex as required by the spatial convective analysis presented here; $-\alpha_i$ is later referred to as the spatial growth rate. The frequency, $\gamma$, and circumferential wavenumber, $\beta$, are real. It is assumed that $\beta$ is $O(1)$. The integer number of complete cycles of the disturbance round the azimuth is $n = \beta R_L$, and we identify this with the number of spiral vortices. We also note that the disturbance phase velocity in the azimuthal direction is $c = \gamma/\beta$, we identify this as the speed at which the vortices rotate with respect to the disk surface. The formulation is such that $c = 1$ corresponds to vortices that rotate with the surface of the disk; $c > 1$ corresponds to vortices rotating faster than the disk surface, and $c < 1$ corresponds to slowly rotating vortices.

The perturbation quantities may be written as a set of six first-order ordinary-differential equations using the transformed variables:

$$\phi_1 = (\alpha - i/R_L)u + \beta v, \quad \phi_2 = (\alpha - i/R_L)u' + \beta v', \quad \phi_3 = w$$
$$\phi_4 = p, \quad \phi_5 = (\alpha - i/R_L)v - \beta u, \quad \phi_6 = (\alpha - i/R_L)v' - \beta u'.$$

These equations are

$$\frac{\phi_1'}{R_L} = \frac{1}{R_L} \left( [\alpha^2 + \beta^2] + i R_L (\alpha U + \beta V - \gamma) + [U]_s \right) \phi_1$$
$$+ \frac{W \phi_5}{R_L} + \left( \frac{i}{R_L} \right) \left( \alpha - \frac{i}{R_L} \right) \phi_3$$
$$+ \frac{1}{R_L^2} \left( \alpha^2 + \beta^2 \right) \phi_4 - \frac{2V \phi_5}{R_L}, \quad (7)$$

$$\frac{\phi_2'}{R_L} = \left( \frac{\alpha - i}{R_L} \right) \phi_2 - \frac{iR_L}{R_L} \left( \alpha - \frac{i}{R_L} \right) \phi_3$$
$$+ \frac{1}{R_L^2} \left( \alpha^2 + \beta^2 \right) \phi_4 - \frac{2V \phi_5}{R_L}, \quad (8)$$
where the subscripts $v$ and $s$ indicate which of the $O(1/R_L)$ terms arise from the viscous and streamline-curvature effects respectively.

It should be noted that these equations are equivalent to those used by Lingwood [10] who formulates the problem in the rotating frame of reference. However, her use of the rotating frame necessarily leads to Coriolis terms that do not appear in equations (7)–(12). It is clear that the different frame of reference also leads to a different interpretation of $\gamma$.

The streamline-curvature terms represent the effect of deflection of the inviscid-flow streamlines through the action of the pressure gradient. By neglecting these terms in equations (7)–(12), the system of equations is demonstrated to be consistent with the Orr–Sommerfeld equation for the rotating disk in the form

$$
\left(\frac{i}{R_L}\phi_3'''' - 2(\alpha^2 + \beta^2)\phi_3''' + (\alpha^2 + \beta^2)^2\phi_3\right) - (\alpha U + \beta V - \gamma)(\phi_3'' - (\alpha^2 + \beta^2)\phi_3) - (\alpha U'' + \beta V'')\phi_3 = 0.
$$

Further, neglecting both the streamline-curvature and viscous terms in the perturbation equations leads to Rayleigh’s equation in the form

$$
(\alpha U + \beta V - \gamma)(\phi_3'' - (\alpha^2 + \beta^2)\phi_3) - (\alpha U'' + \beta V'')\phi_3 = 0.
$$
The Orr–Sommerfeld and Rayleigh equations are stated here only to demonstrate the consistency of the formulation with the standard equations of stability theory; the focus of this study is the solution of the full equations (7)–(12).

3. TRAVELLING-MODE ANALYSIS
In all that follows the eigenvalue problem defined by equations (7)–(12) is solved with the homogeneous boundary conditions

\[ \phi_i = 0, \quad \eta = 0, \]
\[ \phi_i \to 0, \quad \eta \to \infty, \]

where \( i = 1, 2...6 \). This eigenvalue problem will be solved for certain combinations of values of \( \alpha, \beta \) and \( \gamma \) at each Reynolds number, \( R_L \). From these we form the dispersion relation, \( D(\alpha, \beta, \gamma; R_L) = 0 \), with the aim of studying the occurrence of convective instabilities. In each investigation presented below, the spatial branches are calculated using a double-precision fixed-step-size, fourth-order Runge–Kutta integrator with Gram–Schmidt orthonormalisation and a Newton–Raphson linear search procedure, based on a numerical code originally developed by Lingwood (personal communication, 1999) and discussed in [11].

Before we proceed to consider the methods used for the study of travelling modes in this paper, it is interesting to briefly consider the existing method used in the literature.

3.1. The Existing Method
Traditional rotating-disk studies are performed in the rotating frame of reference and non-stationary modes are modelled by setting the perturbation frequency to non-zero values. Effectively the following pseudo-algorithm is implemented when solving the relevant dispersion relation \( D(\alpha, \beta, \gamma; R_L) \):

1. Fix \( \gamma \neq 0 \).
2. Fix \( R_L \) (i.e. the location of the analysis).
3. For a particular value of \( \beta \), solve \( D(\alpha, \beta; R_L, \gamma) \) for \( \alpha \).
4. March through values of \( \beta \) to map the spatial branches in the complex \( \alpha \)-plane.
5. Record neutrally stable parameter values (i.e. where the spatial growth rate is zero).
6. Repeat from step 2 for a different value of \( R_L \) within the required range.
7. Plot the location of the neutral parameter values against \( R_L \) to form the neutral curve in either the \( (R_L, \beta) \)- or \( (R_L, \alpha_r) \)-plane.
Steps 3–5 can be interpreted as sampling the stability (as determined by the spatial growth rate) of a range of disturbance waves at a particular location.

By considering the phase speed of the waves in the azimuthal direction, $\gamma/\beta$, it is clear that non-stationary modes are being analysed in this frame of reference when $\gamma \neq 0$. However, given the resulting neutral curve for a particular $\gamma \neq 0$, the algorithm demonstrates that the value of $\beta$ is changing and therefore so is the phase speed. An analysis of instability modes travelling at particular phase speeds is therefore impossible using this method. However, it is possible to calculate the resulting phase speed at any point on the neutral curve if required.

### 3.2. New Methods

The stability of travelling modes in the fixed frame of reference is considered in §4 using the two distinct methods that are introduced below.

#### 3.2.1. Method 1

In this method the phase speed is not controlled by the analysis, rather it is an output. In this sense the method is similar to the existing method, however it goes further by considering the envelope of all possible neutral curves. The pseudo-algorithm is as follows:

1. Fix $n = \beta R_L$ (i.e. the number of vortices).
2. Fix $R_L$ (i.e. the location of the analysis).
3. For a particular value of $\gamma$, solve $D(\alpha, \gamma; R_L, n)$ for $\alpha$.
4. March through values of $\gamma$ to map the spatial branches in the complex $\alpha$-plane.
5. Record neutrally stable parameter values (i.e. where the spatial growth rate is zero).
6. Repeat from step 2 for a different value of $R_L$ within the required range.
7. Plot the location of the neutral parameters values against $R_L$ to form the neutral curve in the $(R_L, \alpha_r)$-plane.
8. Repeat from step 1 with a different value of $n$ within the required range.

From this a global neutral curve is formed from the envelope of the individual curves pertaining to each $n$. Using this method we are able to predict the vortex speed from parameter values at the onset of instability using the relationship $c = \gamma/\beta = \gamma R_L/n$ at the global minimum of $R_L$.

#### 3.2.2. Method 2

Full neutral curves for disturbances travelling at a particular speed $c$ relative to the rotating-disk surface can be calculated using the following pseudo-algorithm:

1. Fix $c = \gamma/\beta$ (i.e. the speed of disturbances).
2. Fix $R_L$ (i.e. location of the analysis).
3. For a particular value of \( \beta \), solve \( D(\alpha, \beta; R_L, c) \) for \( \alpha \).
4. March through values of \( \beta \) to map the spatial branches in the complex \( \alpha \)-plane.
5. Record neutrally stable parameter values (i.e. where the spatial growth rate is zero).
6. Repeat from step 2 for a different value of \( R_L \) within the required range.
7. Plot the location of the neutral parameters values against \( R_L \) to form the neutral curve in either the \( (R_L, \beta) \)- or \( (R_L, \alpha_r) \)-plane.

This routine results in a single neutral curve that applies to disturbances travelling at speed \( c \) with respect to the disk surface.

4. RESULTS

4.1. Method 1

Method 1 has been applied around the predicted onset of the type I mode from the stationary analysis of Malik [9]. We find that the global curve does indeed predict that \( c = 1.0 \) at the onset of this mode. As expected, critical parameter values at this global minimum coincide with those calculated by explicitly setting \( c = 1.0 \) (for example, \( R_{L,C} = 285.36 \)), this is further discussed in [11] where the global curve is presented. From this we conclude that the most dangerous type I mode, in terms of critical Reynolds number, is stationary. However, this does not tell us anything about the type II mode.

4.2. Method 2

4.2.1. Critical \( R_L \) and unstable parameter ranges

Disturbance speeds have been considered in the range \( c = 0.5-20 \) and neutral curves computed using Method 2. Figure 2 shows the neutral curves in terms of \( \alpha_r, n = \beta R_L, k_\delta = \sqrt{\alpha_r^2 + \beta^2} \) and \( \epsilon = \arctan (\beta/\alpha_r) \) for \( c = 0.8, 1, 5 \) and \( 20 \), where \( k_\delta \) and \( \epsilon \) are the effective wavenumber and waveangle of the vortices, respectively. Note that \( n \) and \( \epsilon \) are observable quantities in experiments which motivates their use here. Recall that \( c = 0.8 \) corresponds to disturbances travelling at 80% of the disk surface speed, and \( c = 5 & 20 \) correspond to disturbances travelling at speeds much greater than the disk surface.

In each case we find that the type I mode is the most dangerous for stationary disturbances (\( c = 1 \)), which is consistent with the results from Method 1. In addition, we find that the lobe arising from the type II mode is sensitive to the disturbance speed. In particular, the type II lobe is quickly eliminated for \( c < 1 \).
and exaggerated for $c > 1$. For increasingly large values of $c$ the type II lobe appears to be limiting to a particular shape in terms of $\alpha_r$ and $k_\delta$, whilst the type I lobe remains dependent on the value of $c$.

The result that quickly travelling type II modes are the most dangerous (in the sense of lowest critical $R_L$) is consistent with the previous theoretical results referred to above. However, it is important to note that the range of waveangles and vortex numbers predicted to be unstable to quickly travelling modes is extremely narrow. In a sense this is a stabilising effect because only a very narrow range of vortex parameters can be selected.

4.2.2. Linear growth rates
In our attempt to determine the speed of the most dangerous mode within the boundary-layer flow we appear to have come against two competing processes: the critical Reynolds numbers for the onset of the type II mode reduce (to some limit) with increased $c$, but the range of parameter values that the corresponding
vortices can exist at becomes increasingly narrow, thereby prohibiting selection. We therefore need to consider the growth rates of the unstable modes in order to better understand the selection of vortex speed.

This is done by modifying the pseudo-algorithm for Method 2. More specifically, rather than recording the neutrally stable ($\alpha_i = 0$) parameter values in step 5, we record the entire spatial branch; the maximum value of $-\alpha_i$ then gives the maximum spatial growth rate at the value of $c$ and $R_L$.

The results of such an investigation are given in Figure 3 where unstable spatial branches are plotted for stationary disturbances ($c = 1$) at various Reynolds number. In this case we see that the type I lobe is dominant with significantly larger growth rates than those contained in the very small type II lobe. Note that the neutral curve for $c = 1$ appears in the $(R_L, \alpha)$-plane of this figure.

Figure 4 plots the spatial branches at alternative values of $c$ in order to visualise the growth rates of both modes. We see that the growth rates within the type II lobe increase relative to the type I mode as $c$ increases. However,
more importantly, we note that the globally maximum growth rates are for the type I mode and these peak between $c = 0.7$ and $c = 0.8$. This is further investigated by introducing the quantity $R_D = R_L - R_{L,C}$ (the distance from the critical Reynolds number for the onset of convective instability) and plotting $\max (-\alpha_i)$ against $R_D$; as shown in Figure 5. We see that vortices with $c = 0.75$ have the maximum growth rate within the region of convective instability. It is therefore most likely that this is the preferred vortex speed to be selected in experiments where roughness elements are not present and travelling modes are known to exist. Note that the maximum growth rates are only plotted for a range equal to 200 in $R_L$. Although these plots could be extended in principle, the region of vortices is known to extend only through this range (recall that beyond $R_L \approx 510$ the flow is known to be turbulent).
4.2.3. Into the region of absolute instability

Note that in producing plots of the maximum spatial growth rates shown in Figures 3–5, it is only possible to consider the convective instability over a finite distance in $R_L$ before the region of absolute instability is entered. Lingwood [10, 25] demonstrated that the onset of absolute instability occurs at around $R_L = 507$; although we are able to consider the convective instability beyond this Reynolds number by avoiding parameters within the region of absolute instability, eventually the position of maximum growth rate coincides with the location of a pinch point. At this point the characteristic branch exchange between the type I and type III branches occurs and the maximum spatial growth rate in the convective sense is undefined. This is demonstrated in Figure 6 where the spatial branches for $c = 1$ are seen at various Reynolds numbers as the pinch point at $R_L = 1025$, $\alpha = 0.1548 - 0.1656i$, $\gamma = \beta = 0.05356$ is approached.

Absolute instability is a spatio-temporal instability and so the region of absolute instability contained within the region of convective instability is irrespective of $c$. How far the convective instability analysis for maximum growth rates can be extended for each $c$ is determined by the occurrence of a pinch point in the absolutely unstable region with $\gamma = c\beta$. 

![Figure 5. Maximum linear growth rates within the region of convective instability for $c = 1$ (-), $c = 0.8$ (···) and $c = 0.75$ ( - -).](image)
Figure 6. Spatial branches at various $R_L$ for $c = 1$, approaching a pinch point at $R_L = 1025$ (x) and after the branch exchange.

Figure 7. Neutral curves for stationary ($c = 1$) convective instabilities (−) and absolute instability (−·) in the $(R_L, \alpha_r)$- and $(R_L, \gamma_r)$-planes, together with the location of maximum convective growth rate (···).
5. CONCLUSION
In this paper I have conducted a theoretical investigation into the vortex speed most likely to be selected within the rotating-disk boundary layer. The two methods used are different to previous investigations into travelling mode disturbances and enable interpretation in terms of a particular vortex speed with respect to the rotating surface. Although the streamline-curvature (type II) mode for disturbances has significantly lower critical Reynolds numbers as the disturbance speed increases, the unstable parameter range in terms of physical quantities significantly reduces, thereby prohibiting selection. Furthermore, crossflow modes travelling at around 75% of the disk surface have the highest growth rates of all modes investigated. In practice it is known that roughness elements occurring on the rotating-disk surface act to select stationary vortices, however in applications where care has been taken to create smooth, clean disks (for example in CVD reactors) it is hypothesised that vortices rotating at around 75% of disk surface will be selected.

It is known that the transition characteristics of the rotating-sphere boundary layer are similar to that for the rotating disk (as discussed by myself in [11, 12]). It is therefore interesting to note the experimental observation by Kobayashi & Aria [26] of vortices that rotate at 76% of the sphere surface speed in certain conditions. Although I first discussed this in [12], it is now suggested that the occurrence of these vortices could be linked to the arguments presented here for the rotating disk. This is an interesting area for further study.

It is acknowledged that an approximation similar to the parallel-flow approximation was made in the derivation of the governing equations in §2. This approximation is found in many other boundary-layer investigations and means that the perturbation equations solved here are not rigorous at $O(1/R_L)$. Although it is clear that the approximation will lead to inaccuracies at the predicted critical Reynolds numbers, it is my opinion that these will be small. The excellent agreement obtained between the numerical and high-$R_L$ asymptotic investigations for the rotating disk due to Malik [9] and Hall [27] respectively (and the rotating cone of various half-angle due to myself and co-workers [17]) for stationary disturbances shows that the affects of the approximation are negligible at high Reynolds number.

Further experiments in the sense of Kobayashi & Aria are required to confirm the hypothesis of this investigation.

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