Validating Controllers for Internal Stability Utilizing Closed-Loop Data

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***Abstract*—We introduce novel tests utilizing a limited amount of experimental and possibly noisy data obtained with an existing known stabilizing controller connected to an unknown plant for verifying that the introduction of a proposed new controller will stabilize the plant. The tests depend on the assumption that the unknown plant is stabilized by a known controller and that some knowledge of the closed-loop system, such as noisy frequency response data, is available and on the basis of that knowledge, the use of a new controller appears attractive. The desirability of doing this arises in iterative identification and control algorithms, multiple-model adaptive control, and multi-controller adaptive switching. The proposed tests can be used for SISO and/or MIMO linear time-invariant systems.**

***Index Terms*—Iterative identification and control, multicontroller adaptive switching, multiple model adaptive control, robust control.**

I. Introduction

Novel tests for verifying that the introduction of a new controller will stabilize an unknown plant using a limited amount of noisy input-output experimental data obtained from the plant connected to an existing known stabilizing controller are introduced in this manuscript. Suppose a feedback control interconnection $[P,\, C_0]$, comprised of the unknown plant P and the stabilizing controller C_0, is internally stable, and further suppose that the use of a new controller C_1 appears attractive. We develop validation tests based on the available knowledge of C_0 and C_1, and the limited data obtained from experiments on the current closed-loop system $[P,\, C_0]$, but not directly on P, to conclude whether a proposed controller C_1 (instead of C_0) stabilizes the feedback loop. These tests exploit phase information of the current closed-loop data, analogously to the Nyquist stability criterion, to ascertain closed-loop stability with the new controller. The proposed tests offer robustness to noise.

Manuscript received June 29, 2009; revised July 19, 2009. First published October 16, 2009; current version published November 04, 2009. This work was supported by the ARC Discovery-Projects Grants DP0342683 and DP0664427 and by National ICT Australia. National ICT Australia is funded by the Australian Government's Department of Communications, Information Technology, and the Arts and the Australian Research Council through Backing Australia's Ability and the ICT Research Centre of Excellence programs, and The financial support of the Engineering and Physical Sciences Research Council under EPSRC Grant EP/F06022X/1, and the Royal Society (via a research grant). Recommended by Associate Editor B. Ninness.

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Digital Object Identifier 10.1109/TAC.2009.2031587

There exist iterative control design methods which utilize the closed-loop data collected from an existing feedback interconnection in order to replace the current controller with a better performing controller, see e.g., [1]–[6] and the references therein. However, the existing stability tests to ascertain internal stability with the new controller before its implementation in the closed-loop are based either on the identification of a parametric 'full order model' of the current closed-loop transfer functions, or on an identification of a parametric 'full order model of the plant' from the current closed-loop transfer functions and the current controller or on the 'full estimation' of frequency bounds on the magnitude of the current closed-loop transfer functions [7]–[13]. One may argue that a mismatch exists between the nature of the aforementioned tests and their usual application. Iterative methods [3]–[5] are based on limited closed loop experiments which are utilized to obtain information for the design of small controller changes; see also [14]–[17]. The existing controller validation tests use the identification of the full dynamics of the current closed-loop system, and hence the amount of experimental effort required for validation purposes can be much larger than that required for the design of the controller update. In contrast, we shall show in Section IV that our validation tests require gathering of information only on an easily-estimated frequency region whose size depends on the size of the controller change. This links the experimental effort to the size of the controller update. In particular, if the controller change has limited size then it is sufficient to obtain an estimate of the phase of the current closed-loop system up to a finite frequency which can be inferred from the closed-loop bandwidth.

In spite of copious research attempts, many state-of-the-art adaptive control design methodologies do not explicitly rule out the possibility of placing a destabilizing controller in the closed-loop [18], [19], and hence the notion of 'safe adaptive control' is at stake. Our data-based stability tests of Section IV ensure against insertion of a destabilizing controller in the closed-loop, in the absence of any form of identified model of the plant. This promises to address, in retrospect, the so-called transient instability problem [15] in the context of multiple model adaptive control (MMAC) [18]–[22]. Here there is the possibility that the controller connected to the unknown plant at any particular time and frozen thereafter in combination with the plant provides an unstable closed loop. This happens partly because it is not always straightforward to accurately predict the new closed-loop transfer function that will result from changing a controller from one known controller to another known controller, when the first closed-loop transfer function is approximately known [18], [19], [23], [24]. Furthermore, it is shown in [25], [26] that the data-driven Unfalsified Adaptive Control approach of [27] and the references therein can engender the worst problem of inserting a destabilizing controller in the closed-loop; moreover, such a destabilizing controller can remain in the loop for a long period of time resulting in very large closed-loop signals. It is shown in [25] that for a simple academic example, a maximum value of 1.228×10^6 was recorded for the plant input signal $u(t)$ when the reference signal $r(t)$ was a sinusoid of magnitude 1. Indeed, one cannot even put a global upper bound on the time during which the destabilizing controller is attached.

The structure of the technical note is as follows. Section II collects the required and necessary definitions and notations, and elucidates the problem of concern by citing the internal stability results from the relevant literature. In Section III, we will present a framework for implementing controllers in a specialized way, which enables presentation of the proposed experimental setting. This builds on our earlier results [28], [29] and leads to the development of the proposed novel validation tests of Section IV for SISO/MIMO systems. Simulation examples of Section V show the versatility and applicability of the proposed tests. Section VI contains concluding remarks and future research directions.

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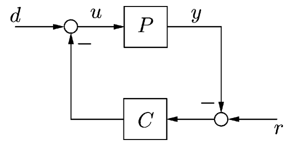


Fig. 1. Standard feedback configuration.

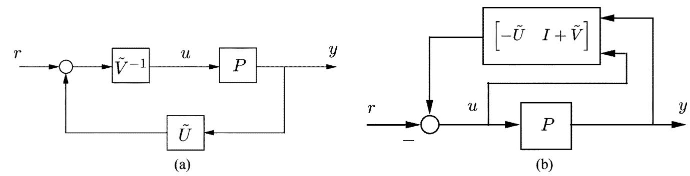


Fig. 2. Observer-form controller implementation; $C = \tilde{V}^{-1}\tilde{U}$. (a) An alternative implementation of the controller C. (b) Feedback interconnection equivalent to (a).

Notations and Background

We shall denote by $\mathcal{H}_\infty$ the space of functions bounded and analytic in the open right-half complex plane, and the same function spaces with prefix $\mathcal{R}$ their real-rational proper subspaces. The plant is assumed to be a MIMO linear time-invariant system (although at times we will restrict attention to scalar systems). The transfer function of the plant belongs to $\mathcal{R}^{n\times n}$, the set of real rational transfer functions, and is denoted by P. The transfer function of the controller is denoted by C. The eigenvalues of $A \in \mathbb{C}^{m\times m}$ are denoted by $\lambda_1, \ldots, \lambda_m$ and its spectral radius $\rho(A) = \max_{1\le i\le m} |\lambda_i|$. The deteminant of a matrix is denoted by $\det$ and its singular values by $\sigma_i(\cdot)$ with the largest singular value $\overline{\sigma}(\cdot)$ and the smallest singular value $\underline{\sigma}(\cdot)$. The number $\mathrm{wno}(\cdot)$ denotes the winding number of the Nyquist diagram of a scalar transfer function, evaluated on a contour along the imaginary axis and indented to the right around any pure imaginary pole. The nearest integer function $\mathrm{nint}[\cdot]$ returns the integer closest to $[\cdot]$ with the additional rule that half-integers are always rounded to even numbers. We will extensively utilize the coprime stable factor representations of P and C, and assume that the plant and all controller transfer functions are always proper. We denote $G(j\omega)^*$ as the complex conjugate transpose of frequency-response function $G(j\omega)$ at each ω, i.e., $G(j\omega)^* = G(-j\omega)^T$.

Definition 1: The *unwrapped* phase of a transfer function is denoted by unwarg and refers to the phase of the frequency response when it is in the form of a continuous function of the frequency.

The unwrapped phase is computed from the phase frequency response by changing absolute jumps greater than π to their 2π complements, and ensures that all appropriate multiples of 2π are included in the phase-frequency response; see [30] for a phase unwrapping technique.

What follows is a collection of well-known internal stability results for the interconnection $[P,\ C]$ in Fig. 1 and definitions linked to coprime factor representation; see [31].

Definition 2: The interconnection $[P,\ C]$ is "well-posed" if the transfer function matrix mapping $\begin{bmatrix} r \\ d \end{bmatrix}$ to $\begin{bmatrix} y \\ u \end{bmatrix}$ exists. Put another way, $[P,\ C]$ is well-posed if $(I - CP)^{-1} \in \mathcal{R}$.

Given such well-posedness, the four transfer functions of Fig. 1 can be written as

$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} P \\ I \end{bmatrix} (I - CP)^{-1} [-C \quad I] \begin{bmatrix} r \\ d \end{bmatrix} = H(P,C) \begin{bmatrix} r \\ d \end{bmatrix}.$$

Definition 3: The interconnection $[P,\ C]$ is said to be "internally stable" if it is well-posed and $H(P,C) \in \mathcal{RH}_\infty$; i.e., each of the four transfer functions in $\begin{bmatrix} r \\ d \end{bmatrix} \mapsto \begin{bmatrix} y \\ u \end{bmatrix}$ belongs to $\mathcal{RH}_\infty$.

Definition 4: The ordered pair $\{N, M\}$, with $M, N \in \mathcal{RH}_\infty$, is a right-coprime factorization (*rcf*) of $P \in \mathcal{R}$ if M is invertible in $\mathcal{R}$, $P = NM^{-1}$, and N and M are right-coprime over $\mathcal{RH}_\infty$. Furthermore, the ordered pair $\{N, M\}$ is a normalized *rcf* of P if $\{N, M\}$ is a *rcf* of P and $M^*M + N^*N = I$.

Definition 5: The ordered pair $\{\tilde{U}, \tilde{V}\}$, with $\tilde{U}, \tilde{V} \in \mathcal{RH}_\infty$, is a left-coprime factorization (*lcf*) of $C \in \mathcal{R}$ if $\tilde{V}$ is invertible in $\mathcal{R}$, $C = \tilde{V}^{-1}\tilde{U}$, and $\tilde{U}$ and $\tilde{V}$ are left-coprime over $\mathcal{RH}_\infty$. Furthermore, the ordered pair $\{\tilde{U}, \tilde{V}\}$ is a normalized *lcf* of C if $\{\tilde{U}, \tilde{V}\}$ is a *lcf* and $\tilde{V}\tilde{V}^* + \tilde{U}\tilde{U}^* = I$.

Before presenting a characterization of closed-loop stability in terms of coprime factorizations, we define

$$G := \begin{bmatrix} N \\ M \end{bmatrix} \quad , \quad \tilde{K} := [-\tilde{U} \quad \tilde{V}] \tag{1}$$

where G will be referred to as the graph symbol of P, and $\tilde{K}$ will be referred to as the inverse graph symbol of C. Then the following results hold.

Theorem 6: [31, Prop. 1.9] Let G and $\tilde{K}$ be defined as in (1). Then the following are equivalent:

i) $[P,\ C]$ is internally stable;
ii) $(\tilde{K}G)^{-1} \in \mathcal{RH}_\infty$;
iii) $\det(\tilde{K}G)(j\omega) \neq 0\ \forall\omega$ and $\mathrm{wno}\det(\tilde{K}G) = 0$.

In this work we will refer to the so-called "observer-form implementation" of the controller, where the factor $\tilde{V}^{-1}$ of C, see Definition 5, is implemented in the feed-forward path and the factor $\tilde{U}$ of C is implemented in the feedback path as depicted in Fig. 2(a), see [32] and [31, ch. 5]. This specialized implementation helps in avoiding restrictions on the response from r to y posed by the poles and zeros of the controller as further analysed in Section III-B. In Fig. 2 and the rest of this manuscript, we shall not consider the exogenous signal d (as in Fig. 1) since it is not relevant to the discussion.

Simple manipulations show that the controller equation can also be rewritten as

$$u = \begin{bmatrix} -\tilde{U} & I + \tilde{V} \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix} - r \tag{2}$$

which is depicted in Fig. 2(b) and clearly justifies why this configuration is referred to as the observer-form. A similar controller implementation is also utilized in [25], [33]; see [34], and [35] for further discussion on observer-based controllers and the link to the controller implementation in Fig. 2(b).

We formulate the problem of interest next and present the preliminary results, based on phase information, which define the experimental setting for stability tests.

II. Problem Setup and Proposed Experimental Setting

The considered problem is that given an unknown plant, which is stabilized by a known controller C_0, and a limited amount of experimental data obtained with C_0, how can one verify—without actual insertion in the closed-loop—if the introduction of the new controller C_1 will stabilize the plant? The following theorem defines the *experimental setting* for the proposed stability tests.

Theorem 7: Let $[P, C_0]$ be internally stable. Let $C_0 = \tilde{V}_0^{-1}\tilde{U}_0$ and $C_1 = \tilde{V}_1^{-1}\tilde{U}_1$ be left coprime factorizations over $\mathcal{RH}_\infty$. Consider Fig. 3 and define $T : r \mapsto z$ to be

$$T = [-\tilde{U}_1 \quad \tilde{V}_1] \begin{bmatrix} P(I - C_0P)^{-1} \\ (I - C_0P)^{-1} \end{bmatrix} \tilde{V}_0^{-1} \tag{3}$$

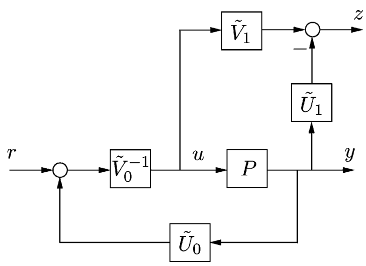


Fig. 3. Experimental setting: $C_0 = \tilde{V}_0^{-1}\tilde{U}_0, C_1 = \tilde{V}_1^{-1}\tilde{U}_1$.

and let unwarg denote the unwrapped phase as in Definition 1. Then the following are equivalent:

a) $[P, C_1]$ is internally stable;

b) $T^{-1} \in \mathcal{RH}_\infty$;

c) $\det T(j\omega) \neq 0 \ \forall \omega$ AND $\operatorname{wno} \det T = 0$;

d) $\det T(j\omega) \neq 0 \ \forall \omega$ AND $\operatorname{unwarg} \det T(j\infty) = \operatorname{unwarg} \det T(j0)$.

Proof: Note that $T = (\tilde{K}_1 G)(\tilde{K}_0 G)^{-1}$ since

$$\begin{aligned} T &= \tilde{V}_1 (I - C_1 P)(I - C_0 P)^{-1} \tilde{V}_0^{-1} \\ &= \begin{bmatrix} -\tilde{U}_1 & \tilde{V}_1 \end{bmatrix} \begin{bmatrix} P(I - C_0 P)^{-1} \\ (I - C_0 P)^{-1} \end{bmatrix} \tilde{V}_0^{-1} \\ &= \tilde{K}_1 \underbrace{\left[G(\tilde{K}_0 G)^{-1} \right]}_{: r \to \begin{bmatrix} y \\ u \end{bmatrix}} = (\tilde{K}_1 G)(\tilde{K}_0 G)^{-1} . \end{aligned}$$

The proof is completed by noticing that:

- (b) and Theorem 6ii) are equivalent since $(\tilde{K}_0 G), (\tilde{K}_0 G)^{-1} \in \mathcal{RH}_\infty$;
- (c) and Theorem 6iii) are equivalent since $[P, C_0]$ internally stable $\Leftrightarrow \det(\tilde{K}_0 G)(j\omega) \neq 0 \ \forall \omega$ AND $\operatorname{wno} \det(\tilde{K}_0 G) = 0$ as $\operatorname{wno} \det(T) = \operatorname{wno} \det(\tilde{K}_1 G) - \operatorname{wno} \det(\tilde{K}_0 G)$;
- (d) and (c) are equivalent because $T \in \mathcal{RH}_\infty$ and is bi-proper and therefore

$$\operatorname{wno} \det(T) = \mathcal{Z}(T) = \frac{1}{\pi}[\operatorname{unwarg} \det T(j\infty) - \operatorname{unwarg} \det T(j0)]$$

where $\mathcal{Z}(T)$ denotes the number of open RHP zeros of T. ■

Note that the assumption here is that the plant P is unknown and hence one cannot explicitly construct the transfer function T in closed-form. However, the stable mapping from r to z (resulting from $T : r \to z$) can be studied in a safe experiment, i.e., one in which no instability can occur, as shown in Fig. 3. Even though we do not have an explicit characterization of T since P is unknown, the reference signal r and the constructed output signal z (computed as a filtered version of the measured signals $\begin{bmatrix} y \\ u \end{bmatrix}$ via $\tilde{K}_1$) can be used experimentally to infer the required properties of T.

Next, we propose our stability tests in Section IV, but before that, an alternative controller implementation is proposed in the following subsection to address any concern regarding the specialized implementation of the controller depicted in Figs. 2(a) and 3.

A. Alternative Implementation of Controllers

The controller implementation in Fig. 3 is a form of implementing controller in which the coprime factors of the controller are implemented in the forward-path and the feedback-path of the closed loop. Note, however, that the results of Theorem 7 can be extended to include experimental settings involving alternative controller implementations. If C_0 is implemented only in the forward path, provided C_0 and C_1 are *bi-proper and minimum-phase*, one can study the stable mapping $T_1 : r \to \tilde{r}$ with $\tilde{r} = C_1^{-1} u + y$, as a replacement for T, in a safe experiment. Similarly, for the feedback-path implementation of C_0, if C_0 and C_1 have *no right half-plane poles*, the stable mapping $T_2 : r \to \tilde{w}$ with $\tilde{w} = u + C_1 y$ can be examined by utilizing the aforementioned results. We adopt the observer-form implementation of the controller depicted in Fig. 2(a), and propose the following alternative implementation [29] to circumvent concerns one may have in splitting up the physical controller in two coprime factors before injecting the reference signal r.

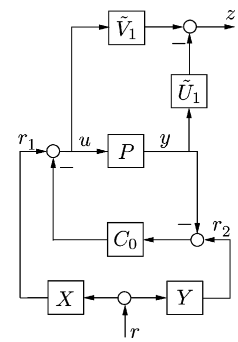


Fig. 4. Equivalent experimental setting.

Let $\begin{bmatrix} X \\ Y \end{bmatrix}$ be a right inverse of $[\,-\tilde{U}_0 \quad \tilde{V}_0\,]$. In other words, let $P_0 = XY^{-1}$ be some plant that stabilizes $C_0 = \tilde{V}_0^{-1}\tilde{U}_0$ and satisfies the corresponding Bezout identity. Note that P_0 does not have to be an estimate of P. Then, it is easy to see in Fig. 4 that

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix} r \quad \text{and} \quad \begin{bmatrix} y \\ u \end{bmatrix} = H(P, C_0) \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}.$$

Because $H(P, C_0) = G(\tilde{K}_0 G)^{-1} \tilde{K}_0$, it follows that

$$\begin{bmatrix} y \\ u \end{bmatrix} = G(\tilde{K}_0 G)^{-1} r$$

which is the mapping from r to $\begin{bmatrix} y \\ u \end{bmatrix}$ in Fig. 2(a) and (b). Note that the requirement $[\,-\tilde{U}_0 \quad \tilde{V}_0\,] \begin{bmatrix} X \\ Y \end{bmatrix} = I$ can be relaxed to $[\,-\tilde{U}_0 \quad \tilde{V}_0\,] \begin{bmatrix} X \\ Y \end{bmatrix} = Z$ where Z is a unit in $\mathcal{RH}_\infty$ since the transfer function from r to $\begin{bmatrix} y \\ u \end{bmatrix}$ then becomes

$$\begin{bmatrix} y \\ u \end{bmatrix} = G(\tilde{K}_0 G)^{-1} Z r = G(\hat{\tilde{K}}_0 G)^{-1} r$$

with $\hat{\tilde{K}} = Z^{-1} \tilde{K}_0$, i.e., only changing the particular coprime factor representation of the controller.

An interesting observation is that there are several plants P_0 that stabilize C_0 and furthermore there are several coprime factorizations of $P_0 = Y^{-1} X$. This choice can be used in the synthesis of X and Y to determine the frequency and bandwidth characteristics of the physical reference signals r_1 and r_2.

III. Proposed Data-Based Stability Tests

In this section, we put forward data-based stability tests on the basis of the experimental setting defined in Section III with the aim of verifying condition (d) in Theorem 7. For the development of these data-based stability tests, following assumptions are introduced.

Assumption 8: The factors $\tilde{V}_0$ and $\tilde{V}_1$ are such that $\tilde{V}_0(j\infty) = \tilde{V}_1(j\infty) = I$.

Assumption 9: The transfer functions PC_0 and PC_1 are strictly proper.

It is evident that Assumption 8 is without loss of generality and Assumption 9 captures a typical situation. Notice that the transfer function T can be written as

$$T = \tilde{V}_1(I - C_1P)(I - C_0P)^{-1}\tilde{V}_0^{-1} \tag{4}$$

for which under Assumptions 8 and 9 we have

$$\det T(j\infty) = \frac{\det \tilde{V}_1(j\infty)}{\det \tilde{V}_0(j\infty)} \frac{\det(I - C_1P)(j\infty)}{\det(I - C_0P)(j\infty)} = 1\,. \tag{5}$$

Thus, $\det T(j\infty)$ is strictly positive and known and will be used as a datum for the verification of condition (d) in Theorem 7. To start, we have the following *falsification* test for step responses.

Theorem 10: Let the suppositions of Theorem 7 and Assumptions 8 and 9 hold. Let e_i denote a reference signal where a step is applied at the $i-$th input while the other inputs are kept at 0. Perform n experiments with reference signal $r(t) = e_i(t)$, $i = 1, \ldots, n$ and let $\bar{z}_i$ be the steady state output of T recorded in each experiment. Define $\bar{Z} = [\bar{z}_1, \ldots, \bar{z}_n]$. Then

$$[P, C_1] \text{ is internally stable } \Rightarrow \det \bar{Z} > 0\,.$$

Thus, if $\det \bar{Z} \leq 0$, stability of $[P, C_1]$ is falsified.

Proof: A necessary condition for condition (d) in Theorem 7 to hold true is that $\det T(j0)$ and $\det T(j\infty)$ have the same sign. By the final value theorem, we have

$$\bar{Z} = [\bar{z}_1\ \bar{z}_2,\ \ldots\ \bar{z}_n] = \lim_{s \to 0} s \left[T(s)\frac{1}{s}\right] = T(j0)\,.$$

Hence, $\det T(j0) = \det \bar{Z}$. The proof is complete by noticing that if condition (d) in Theorem 7 holds, $\det T(j0)$ must have the same sign as $\det T(j\infty)$, which was set to 1 without loss of generality. ■

It readily follows that our result on steady-state step response measurements in Theorem 10 is robust and in fact unaffected by all disturbances with finite energy. Suppose that the plant input signal u and plant output signal y in the experimental setting of Fig. 4 are corrupted by disturbances w_1 and w_2 respectively. Due to linearity, it holds that $z(t) = z_1(t) + z_2(t)$, where $z_1(t)$ is due to the reference input $r(t)$ and $z_2(t)$ is due to $\begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$. Given the stable mapping, we then have that $\lim_{t\to\infty} z_2(t) = 0$ (and consequently $z(\infty) = z_1(\infty)$) whenever $\begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$ are energy bounded disturbances.

The experimental test devised above is quite simple to carry out; it simply consists in recording the steady state values of m step responses. However such an experiment can only be used to check a necessary stability condition. Condition (d) in Theorem 7 can be verified in both its necessary and sufficient parts by using more sophisticated identification techniques. In principle, one could inject a white noise signal r or a full sine sweep, measure the corresponding output z and compute the *full* frequency response for T. However, this is not desirable on the grounds of complexity and hence one needs to determine an alternative and smarter experiment. There are two key points that have to be noticed in designing the experiment. First, there is no need to estimate the full frequency response of T, but what is instead needed is to measure its frequency response up to a certain finite frequency ω_0. Second, the measurement can tolerate error, as its purpose is simply to facilitate computation of a certain phase change. A mechanism to estimate ω_0 is advanced next by exploiting the structure of T.

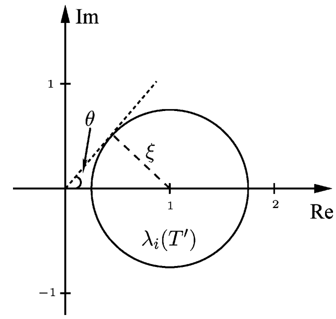


Fig. 5. Graphical representation of the condition on $\lambda_i(T')$. The angle θ is of the form π/n.

Lemma 11: Let the suppositions of Theorem 7 hold. Then T can be expressed as

$$T = I + T' \tag{6}$$

$$T' = \begin{bmatrix} -(\tilde{U}_1 - \tilde{U}_0) & (\tilde{V}_1 - \tilde{V}_0) \end{bmatrix} \begin{bmatrix} P(I - C_0P)^{-1} \\ (I - C_0P)^{-1} \end{bmatrix} \tilde{V}_0^{-1}. \tag{7}$$

Proof: The expression for T' is derived as follows:

$$\begin{aligned} T' &= \tilde{K}_1 G(\tilde{K}_0 G)^{-1} - I = (\tilde{K}_1 - \tilde{K}_0) G (\tilde{K}_0 G)^{-1} \\ &= \begin{bmatrix} -(\tilde{U}_1 - \tilde{U}_0) & (\tilde{V}_1 - \tilde{V}_0) \end{bmatrix} \begin{bmatrix} P \\ I \end{bmatrix} (I - C_0P)^{-1}\tilde{V}_0^{-1}. \end{aligned}$$

The last expression coincides with (7). ■

The expression for the transfer function T presented in Lemma 11 shows that T is the sum of a known term (i.e., I) and a term which, under Assumptions 8 and 9, is strictly proper. Hence, it can be expected that measuring the frequency response of T up to a frequency, say ω_0, where the response of T' has nearly vanished is enough to characterize the full frequency response of T. This fact is utilized in Theorem 12.

Theorem 12: Suppose the hypothesis of Theorem 7 and Assumption 8 and 9 hold. Define $T' \in \mathcal{RH}_\infty{}^{n\times n}$ by $T' = T - I$ as in Lemma 11. Let $\omega_0 \in [0, \infty)$ be a frequency such that

$$\begin{cases} \rho(T'(j\omega)) < 1, & n = 1 \\ \rho(T'(j\omega)) < \sin\left(\frac{\pi}{n}\right), & n \geq 2 \end{cases} \quad \forall\omega \geq \omega_0. \tag{8}$$

Then the condition

$$\begin{cases} \det T(j\omega) \neq 0\ \forall\omega \in [0, \omega_0) \qquad \text{AND} \\ 2\pi \times \text{nint}\left[\frac{\text{unwarg}\det T(j\omega_0)}{2\pi}\right] = \text{unwarg}\det T(j0) \end{cases} \tag{9}$$

is equivalent to condition (d) in Theorem 7.

Proof: Lemma 11 shows that $T(j\infty) = I$ under Assumption 8 and 9. For the case $n = 1$, (8) becomes $|T'(j\omega)| < 1 \quad \forall\omega \geq \omega_0$, which certainly implies

$$2\pi \times \text{nint}\left[\frac{\text{unwarg}T(j\omega_0)}{2\pi}\right] = \text{unwarg}T(j\infty)\,.$$

For $n \geq 2$ in (8), observe that $|\lambda_i(T')| \leq \rho(T')$ and

$$\det T = \prod_{i=1}^{n} \lambda_i(I + T') = \prod_{i=1}^{n} \left[1 + \lambda_i(T')\right]\,.$$

Since inequality (8) holds, $[1 + \lambda_i(T')]\,(j\omega)$ lies in the interior of a circle of center 1 and radius $\xi = \sin(\pi/n)\ \forall\omega \geq \omega_0$ and $\det T(j\omega) \neq 0\ \forall\omega \in [\omega_0, \infty)$. Hence the angle θ depicted in Fig. 5 is precisely π/n. Consequently, the angle of the complex number $1 + \lambda_i(T')$ for each i and each $\omega \in [\omega_0, \infty)$ lies in $(-\pi/n, \pi/n)$. Thus, the angle of the complex number $\det T(j\omega)$ is in $(-\pi, \pi)$ for each $\omega \in [\omega_0, \infty)$ and $\det T$ can never complete a contour around the origin $\forall \omega \in [\omega_0, \infty)$, because the contour can never cross the negative real axis. Since $\det T(j\omega)$ is a continuous function of frequency and is equal to unity at infinite frequency

$$2\pi \times \text{nint}\left[\frac{\text{unwarg}\det T(j\omega_0)}{2\pi}\right] = \text{unwarg}\det T(j\infty).$$

■

Theorem 12 states a necessary and sufficient condition for the stability of $[P, C_1]$ and implies the estimation of the frequency response of the current closed-loop system up to a certain frequency ω_0. For the application of Theorem 12 it is important to recall that $\rho(T'(j\omega)) \leq \overline{\sigma}(T'(j\omega))$ and that under Assumptions 8 and 9, $\overline{\sigma}(T'(j\omega))$ tends to zero as ω tends to infinity. In practice, it is reasonable to assume that one has a rough estimate of the bandwidth of the current closed-loop $[P, C_0]$ which can be used to obtain an estimate of ω_0 by assuming that $\overline{\sigma}(T'(j\omega))$ remains below the right-hand side of inequality (8) over some known high-frequency region. In practice a conservatively larger value makes the choice of ω_0 robust. Notice also that the structure of T' in (7) is as such that $\rho(T'(j\omega))$ depends on the size of the controller change. In order to better illustrate this, consider for example the SISO case where C_0 and C_1 are stable. In this case, one can choose $\tilde{V}_1 = \tilde{V}_0 = 1, \tilde{U}_0 = C_0$ and $\tilde{U}_1 = C_1$. Therefore, T' can be written as

$$T' = \frac{(C_0 - C_1)P}{1 - C_0 P}.$$

Hence, a small controller change certainly reduces the frequency ω_0 and, as a consequence, reduces the experimental effort.

It is important to notice that since the value $\text{nint}[\text{unwarg}\det T(j\omega_0)/2\pi]$ is only used in condition (9), a rough estimate of $\text{unwarg}\det T(j\omega_0)/2\pi$ is enough and hence the test can tolerate considerable estimation errors. Moreover, the structure of T' in (7) is such that $\overline{\sigma}(T'(j\omega))$ depends on the size of the controller change. A small controller change certainly implies a smaller frequency ω_0 and hence reduced experimental effort. The estimate of the frequency response of $T(j\omega)$ up to ω_0 can be obtained by using either parametric [36] or non parametric [37] estimation methods. In practice, at each frequency one can use

$$\overline{\sigma}(T) \leq \|T\|_F = \sqrt{\sum_{i,j} |T_{ij}|^2}$$

along with $\rho(T(j\omega)) \leq \overline{\sigma}[T(j\omega)]$ to find an upper bound on the eigenvalues of $T(j\omega)$ in order to check the condition in (8). Alternatively the inequality

$$\overline{\sigma}[T(j\omega)] \leq \sqrt{n}\|T(j\omega)\|_1$$

can be utilized at each frequency; see [38, ch. 2] for useful matrix norm inequalities. The unwrapped phase can be obtained by phase unwrapping techniques of [30]. Next, two simulation examples illustrate the features and applicability of the stability tests proposed in Theorem 10 and Theorem 12.

IV. Simulation Examples

We shall consider a MIMO system and a SISO system to exemplify the advantages and effectiveness of the stability tests proposed in Theorems 7 and 12. Although the theorems do not assume that the plant is known, for the sake of simulation the underlying unknown plants are given.

A. Example 1: A MIMO System

Let the unknown plant, $P \in \mathcal{R}^{2\times 2}$, be given by

$$P = \frac{1}{s^2+2s+4}\begin{bmatrix} -(s-2) & 2(s+0.5) \\ -3 & -(s-2)\end{bmatrix}$$

and let C_0 be a stabilizing controller, $[P, C_0] \in \mathcal{RH}_\infty$, given by

$$C_0 = \frac{2(s+2)(s^2+2s+4)}{s(s+1)(s^2+2s+7)}\begin{bmatrix} (s-2) & 2(s+0.5) \\ -3 & (s-2)\end{bmatrix}$$

with a left coprime factorization, $C_0 = \tilde{V}_0^{-1}\tilde{U}_0$

$$\tilde{V}_0 = \frac{(s+1)}{(s^2+3.89s+3.8)(s^2+1.94s+2.58)(s^2+2.03s+4.07)} \cdot \begin{bmatrix} \tilde{V}_0^{11} & \tilde{V}_0^{12} \\ \tilde{V}_0^{21} & \tilde{V}_0^{22}\end{bmatrix}$$

$$\tilde{U}_0 = \frac{(s+2)(s^2+2s+4)}{(s^2+3.89s+3.8)(s^2+1.94s+2.58)(s^2+2.03s+4.07)} \cdot \begin{bmatrix} \tilde{U}_0^{11} & \tilde{U}_0^{12} \\ \tilde{U}_0^{21} & \tilde{U}_0^{22}\end{bmatrix}$$

$$\begin{aligned}
\tilde{V}_0^{11} &= -0.22s(s^2+4.72s+6.01)(s^2+2.24s+4.51)\\
\tilde{V}_0^{12} &= 0.71s(s+2.03)(s^2+1.98s+3.8)\\
\tilde{V}_0^{21} &= 0.27s(s-3.12)(s+2.04)(s^2+2s+3.9)\\
\tilde{V}_0^{22} &= -0.71s(s+1.93)(s+0.2)(s^2+2.02s+4.14)\\
\tilde{U}_0^{11} &= -0.437(s+1.65)(s^2+1.31s+1.81)\\
\tilde{U}_0^{12} &= -0.872(s+1.88)(s^2+1.96s+2.94)\\
\tilde{U}_0^{21} &= 0.545(s+2.36)(s^2+2.44s+3.74)\\
\tilde{U}_0^{22} &= -0.341(s+1.62)(s^2+0.78s+2.35).
\end{aligned}$$

Theorem 7 puts forward a solution to the problem of checking in advance using collected closed-loop data if the controller C_1 given here by

$$C_1 = \frac{2(s^2+2s+4)(s-2)}{(s^2+s+1)(s^2+2s+7)}\begin{bmatrix} (s-2) & 2(s+0.5) \\ -3(s-0.33) & (s-2)\end{bmatrix}$$

with a left coprime factorization, $C_1 = \tilde{V}_1^{-1}\tilde{U}_1$

$$\tilde{V}_1 = \frac{(s^2+s+1)(s^2+2s+7)}{(s+3.15)(s+2.04)(s+1.85)(s+0.34)(s^2+2.04s+4.09)} \cdot \begin{bmatrix} \tilde{V}_1^{11} & \tilde{V}_1^{12} \\ \tilde{V}_1^{21} & \tilde{V}_1^{22}\end{bmatrix}$$

$$\tilde{U}_1 = \frac{(s-2)(s^2+2s+4)}{(s+3.15)(s+2.04)(s+1.85)(s+0.34)(s^2+2.04s+4.09)} \cdot \begin{bmatrix} \tilde{U}_1^{11} & \tilde{U}_1^{12} \\ \tilde{U}_1^{21} & \tilde{U}_1^{22}\end{bmatrix}$$

$$\begin{aligned}
\tilde{V}_1^{11} &= -0.22(s+2.95)(s+2.06) \quad \tilde{V}_1^{12} = -0.13(s+1.93)\\
\tilde{V}_1^{21} &= -0.03(s+7.43)(s+1.97)\\
\tilde{V}_1^{22} &= -0.16(s+1.83)(s+0.64)\\
\tilde{U}_1^{11} &= -.43(s-1.01)(s+1.8)(s+.42)\\
\tilde{U}_1^{12} &= -.87(s+2.01)(s+3.54)(s+0.27)\\
\tilde{U}_1^{21} &= .88(s-3.6)(s+1.85)(s+0.33)\\
\tilde{U}_1^{22} &= -0.43(s+.13)(s^2+2.92s+2.17).
\end{aligned}$$

We set up the experimental configuration of Fig. 3 and perform two experiments with reference signals $r(t) = step(t)\cdot e_1$ and $r(t) = step(t)\cdot e_2$. The steady state values of the step responses of $T: r \to z$ are

$$\bar{Z} = \begin{bmatrix} -0.75 & 0.476 \\ -0.391 & 1.27\end{bmatrix}$$

with $\det(\bar{Z}) = -0.7664 < 0$ and hence the stability of $[P, C_1]$ is falsified. Indeed, computing $H(P, C_1)$ shows that it has three RHP poles which conforms with the results.

B. Example 2: A SISO System

This example demonstrates the effectiveness of the stability tests proposed in Theorem 12. Let the unknown SISO plant be given by

$$P = \frac{-186.66(s-5)(s+4.5)}{(s+10)^2(s+7)(s+6)}$$

and let C_0 be a stabilizing controller, $[P, C_0] \in \mathcal{RH}_\infty$, given by

$$C_0 = \frac{0.021(s+10.92)(s+8.87)(s+7.31)(s+5.93)}{(s^2+8.6s+19.84)(s^2-0.603s+5.34)}$$

with a left coprime factorization, $C_0 = \tilde{V}_0^{-1}\tilde{U}_0$

$$\tilde{V}_0 = \frac{(s^2+8.603s+19.84)(s^2-0.602s+5.34)}{(s^2+8.64s+19.97)(s^2+1.83s+6.96)}$$

satisfying Assumption 8, $\tilde{V}_0(j\infty) = 1$, and

$$\tilde{U}_0 = \frac{0.021(s+10.92)(s+8.87)(s+7.31)(s+5.93)}{(s^2+8.64s+19.97)(s^2+1.83s+6.96)}.$$

Suppose that the data collected from the closed-loop suggests the use of a new controller C_1 given by

$$C_1 = \frac{0.33(s+0.586)(s+2.99)(s+3.416)}{(s+2)(s^2+2.26s+3.52)}$$

with a left coprime factorization, $C_1 = \tilde{V}_1^{-1}\tilde{U}_1$,

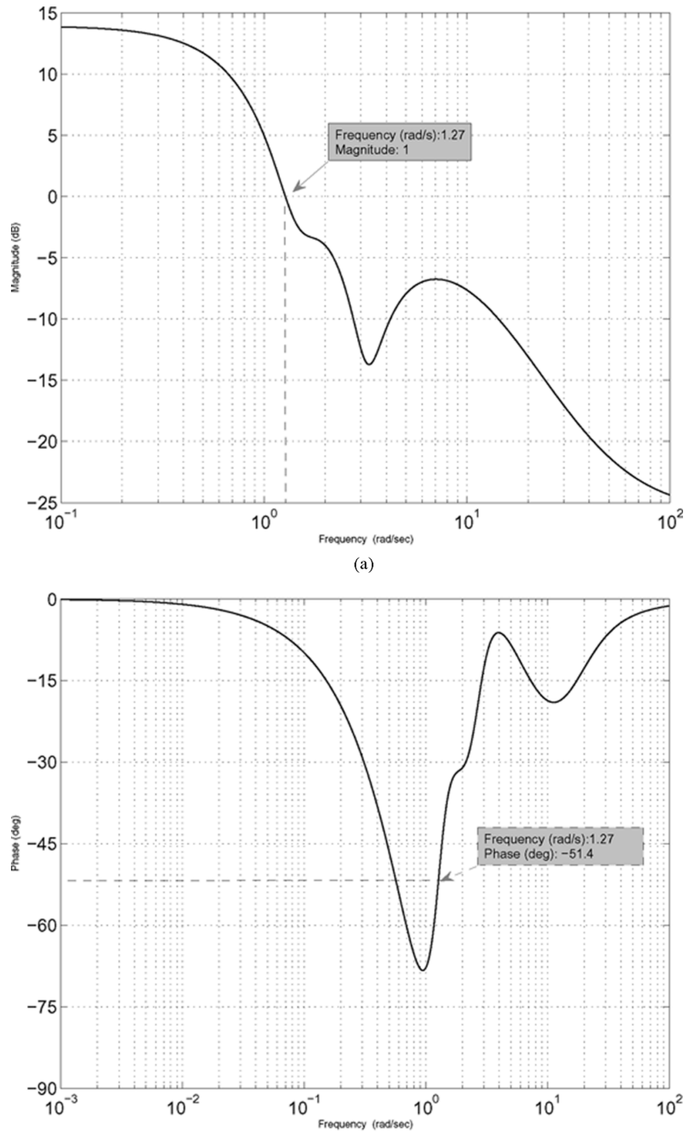
$$\tilde{V}_1 = \frac{(s+2)(s^2+2.26s+3.52)}{(s+1.87)(s^2+2.81s+3.712)}$$

satisfying Assumption 8, $\tilde{V}_1(j\infty) = 1$, and

$$\tilde{U}_1 = \frac{0.33(s+0.586)(s+2.99)(s+3.416)}{(s+1.87)(s^2+2.81s+3.712)}$$

is attractive. Setting up the experimental configuration of Fig. 3 for simulation and utilizing Theorem 10 to check if C_1 is stabilizing, we perform experiments with reference signal $r(t) = step(t)$ and the step response is measured at the output z. The steady state of $T: r \to z$ is $\bar{z} = 4.74 > 0$ which does not falsify the stability of $[P, C_1]$. Thus, we shall use the results of Theorem 12 to check if C_1 is stabilizing.

As shown in Fig. 6(a), the simulation reveals that $|T-1| \leq 1\ \forall\omega \geq 1.27$ rad/s. Given that $\mathrm{unwarg}T(j0) = 0$ and $\mathrm{unwarg}T(j\omega_0) = -0.285\pi$, shown in Fig. 6(b), the condition in Theorem 12 holds and hence C_1 is stabilizing. Indeed, computing $H(P, C_1)$ shows that C_1 is stabilizing. Obviously, but importantly, even with uncertainty of around 3 *dB* error in magnitude and 30 degrees error in phase, one could draw the same conclusion with confidence, and hence the measurement can tolerate error. The versatility of our proposed tests for ensuring safe adaptive control in practice are further illustrated in [39] via presenting two benchmark examples.



(a)

(b)

Fig. 6. Example 2: Magnitude and phase responses. (a) Magnitude response of $T' = T - 1$. (b) Phase response of $T(j\omega)$.

V. Conclusions

We have proposed new validation tests for MIMO/SISO linear time-invariant systems which aim to protect internal stability with the introduction of a new controller C_1 by utilizing a limited amount of experimental data obtained from the stable closed-loop interconnection $[P, C_0]$. One of the proposed tests, Theorem 10, uses step response properties of a closed-loop mapping T to falsify the controller C_1. The second test, Theorem 12, proposes a type of phase test analogous to the Nyquist criterion and utilized the noisy frequency response information of the closed-loop mapping T up to a finite frequency ω_0 to check if C_1 will stabilize the unknown plant P. This was all achieved despite the restrictive assumption that no *a priori* information about the plant was available.

The simulations of Section V clearly showed the effectiveness and applicability of the proposed tests. This promises to address the so-called transient instability problem in the context of multiple model adaptive control (MMAC) and iterative identification and control ideas discussed in the introduction; see also [15], [18], and [19] and the references therein.

We have extended the proposed stability tests to enable one to infer some performance specifications of the closed-loop with C_1 before its actual insertion into the closed-loop. The new results builds on the properties of T to predict some aspects of performance like gain margin, phase margin, overshoot, rise time, bandwidth, etc.

We have also extended our results to the nonlinear case and proposed novel nonlinear analysis and tests [40], [41], utilizing the kernel representation of nonlinear systems, for ensuring that the introduction of a new nonlinear controller will stabilize the unknown plant.

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