The Effect of Neutron Irradiation and Thermal Cycling on Permanent Deformations in Uranium under Load


The Cottrell "creep" effect in polycrystalline uranium irradiated in a neutron flux at low temperatures is explained, using a model in which each grain continues to deform anisotropically whilst stressed to a yield point by the mutual restraints of the aggregate. An estimate of the analogous (time-independent) deformation occurring during a large thermal cycle is given. The model is developed to cover the case in which the internal stresses do not cause yielding, but are sufficient to lead to an accelerated creep mechanism under small applied stresses. The cases of creep accelerated by irradiation at temperatures in the range 350-450 °C and by small thermal cycles in the temperature range above 300 °C are worked out in detail, and results indicate that an error of two orders of magnitude can be made by neglecting such effects and relying on isothermal creep data. The integrated effect of internal stresses during relaxation in producing a component of primary creep is analysed, and shown to give a permanent deformation of several elastic deflections occurring over a period subsequent to a large thermal cycle. Such deformations will be additive to the time-independent deformation occurring during the large thermal cycle. A brief review of experimental work is included and suggests that experimental agreement with the theories is good.

Konobeevsky et al. produced the first experimental suggestion of a considerable increase in plasticity of polycrystalline uranium under neutron irradiation at low reactor temperatures. Subsequently, Cottrell proposed in considerable detail a mechanism of repeated yielding in the grains due to build-up of internal stresses by the known anisotropic growth of α-uranium grains and predicted the deformation behaviour of polycrystalline uranium under external loads. Roberts and Cottrell carried out creep experiments on uranium springs during neutron irradiation and found that the predictions agreed with the experimental results. Experiments reported by Zaimovsky et al. further substantiated the existence of the effect.

In this paper the theoretical development of Cottrell's work carried out by the present authors since 1956 is reported. Cottrell's repeated-yielding model is described, and a continuously deforming model (cf. Anderson and Bishop*) for polycrystalline uranium under neutron irradiation is analysed and found to give the same result. The model is then used (cf. Bishop*) to examine a conjecture by Cottrell that large thermal cycles could produce time-independent permanent deformations in uranium under the smallest applied stresses.

The model is shown to be applicable more generally to uranium under internal-stress systems that are greater than the applied stress but less than the yield stress. The deformation mode is then one of conventional creep rather than time-independent yield flow. The model is used to examine the nature of the Cottrell creep phenomenon in reactor temperatures between 350 and 450 °C under conditions for which the growth rate of uranium single crystals is insufficient to generate a yield-stress system in the aggregate. It is found that, even without yielding, a range of temperatures exists over which the conventional creep rate under a given stress will be accelerated. A second example of an accelerated creep rate in uranium is shown to occur during small-amplitude thermal cycling (cf. Anderson*), owing to the anisotropy of the crystal thermal-expansion coefficients. An estimate of the internal stress in such a case is made and the corresponding acceleration factors estimated. The experimental investigations are outlined briefly. The model is finally used to derive an estimate for the component of primary creep occurring during the relaxation of internal stresses when the creep and relaxation mechanisms are identical.

Cottrell's "Creep" Theory

A brief summary of Cottrell's theory is given here. Consider a random α-uranium polycrystal. The constraint between grains will result in the production of internal stresses under neutron irradiation at temperatures of ∼200 °C. Since the strain-rate tensor (ε) for a single crystal under irradiation can be taken as shown in Table I, where a, b,

<table>
<thead>
<tr>
<th>Table I</th>
<th>Growth Tensor for α-Uranium</th>
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<tbody>
<tr>
<td>a</td>
<td>-ε_x</td>
</tr>
<tr>
<td>b</td>
<td>0</td>
</tr>
<tr>
<td>c</td>
<td>0</td>
</tr>
</tbody>
</table>

and ε refer to the 100, 010, and 001 crystal axes, respectively, there will be no net dimensional changes in a random (i.e. isotropic) aggregate. Hence if Υ is a measure of the yield strength of a crystal, ε_x the growth rate, and E an appropriate elastic constant, the yield point will be reached in each grain in a time t_m given by:

\[ t_m \approx \frac{Y}{E} \epsilon_x \]

For a typical set of values in the BEPO reactor, this is about a week. After this period, plastic flow will occur in each grain to nullify the growth strains. Hence, the macroscopic strain will be zero, although local surface deformations will accompany plastic distortion of the surface grains. All these effects are observed. If now a tensile stress ε is applied to the specimen, an elastic strain of the order of ε/E will occur, together with a plastic strain of the same order due to local plastic relaxations under the combined influences of the internal and applied stresses. Since it will take a time of the order of t_m to regenerate the internal stress, this strain will occur in time t_m. Hence the growth strain rate \( \dot{\epsilon} \) will be given by

\[ \dot{\epsilon} = \frac{Y}{E} \epsilon \]

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Anderson and Bishop: Effects of Irradiation &c. on

\[ \dot{\varepsilon}_m = \frac{\sigma}{Y} \dot{\varepsilon}_x \]

Using this formula and the elementary elastic theory for helical springs, Roberts and Cottrell\(^\text{a}\) predicted the growth rate for a uranium wire wound into a helix. With the relevant constants they predicted a growth rate of \(1.16\, (\pm 0.2)\) elastic deflections/week.

There is a close correspondence between the predicted values and the experimental results. Since the experimental work was undertaken as a result of the prediction, the results are a striking endorsement of the physical intuition leading to the theory. In the theory, the process is analysed as though it occurred in a series of steps with internal stress generation alternating with relaxation under load. It is not, of course, implied that this is what occurs. In the following section the present authors attempt an order-of-magnitude calculation with the process treated as continuous.

The Continuous Deformation Model of Cottrell Creep

Consider the irradiation of an isotropic aggregate of \(\alpha\)-uranium crystals. Each crystal, if unconstrained, would expand as shown in Table I. Clearly, this free growth in an aggregate would lead to the production of internal voids. Although in practice a few voids are observed, these are not consistent with the free growth of the grains. Moreover, in many specimens no voids occur. It will be assumed that there is displacement continuity at grain boundaries, since this is in keeping with experimental observation. Consider irradiation from an initially stress-free state. At first stresses of elastic order are generated, until at some stage depending on local geometry the first grain will be stressed to the yield point. Thereafter there will be a continuous change in the local stresses, with both elastic strains and plastic strains of elastic order occurring. The experimental evidence leads to the conclusion that preferred elongation does not occur in a random aggregate. Even if this is not exactly true, it is permissible to assume that virtually all the grains in the aggregate are eventually stressed to the yield point and flow plastically. In an isotropic aggregate it will be statistically true that the plastic strains will nullify the growth strains. It is quite impracticable to calculate the internal stresses and strains in an aggregate. In the treatment given here, the assumption is made that each grain undergoes the same overall deformation as the aggregate.

Bishop and Hill\(^\text{b}\) have established that the plastic-strain increment corresponding to a uniform stress in a single crystal which deforms by glide can be determined from a knowledge of the yield criterion. This fact is based on a maximum-work theorem which they prove. The extension of the theorem to crystals that deform by glide, and by any plastic deformation mechanism obeying a critical stress law, is immediate. The experimental evidence on deformation in \(\alpha\)-uranium is not sufficient at the moment to define the yield criterion. It is not even clear that the plastic deformation modes out of pile are relevant to in-pile behaviour. This difficulty extends to the stress/strain relationships. A form will be assumed which is consistent with existing knowledge of crystal and macroscopic plasticity. Hence the following work may be regarded in one of two ways: either as relevant to a material which has certain defined properties, the material having some properties similar to those of \(\alpha\)-uranium, or alternatively as relevant to \(\alpha\)-uranium with a quantitative error introduced by assuming the form of the yield criterion and the stress/strain rate relations.

The following problem is considered here. We require to define the steady-state behaviour under a small macroscopic tensile stress \(\sigma_x\) of an isotropic aggregate of elements each yielding according to the criterion

\[ \sigma_{x1}^2 + \sigma_{y1}^2 + \sigma_{z1}^2 + 2\tau_{x1}^2 + 2\tau_{y1}^2 + 2\tau_{z1}^2 = \frac{Y^2}{2} \]

where \(\sigma_{yx}\) are the stress components relative to a set of Cartesian axes \((x,y,z)\) in the aggregate, with stress/strain increment relations of the Lévy-von Mises type, namely

\[ \frac{\dot{\varepsilon}_{xx}}{\varepsilon_{xx}} = \frac{\dot{\varepsilon}_{yy}}{\varepsilon_{yy}} = \frac{\dot{\varepsilon}_{zz}}{\varepsilon_{zz}} = \frac{\dot{\gamma}_{xy}}{\gamma_{xy}} = \frac{\dot{\gamma}_{xz}}{\gamma_{xz}} = \frac{\dot{\gamma}_{yz}}{\gamma_{yz}} = \lambda \]

and with the growth rates \((\varepsilon_{yx})_{hce}\) in three orthogonal directions \((a,b,c)\) fixed in the grains. The growth rates of uranium are taken as in Table I.

The \(\sigma_{yx}\) are the “reduced” stress components, i.e. the ordinary stress components minus the hydrostatic stress. When the applied stress is a simple tensile stress \(\sigma_x\), the components of macroscopic strain \((\varepsilon_{yx})_{x}\) in the \((x,y,z)\) axes are as given in Table II.

| Table II: Macroscopic Strain-Rate Tensor for a Tensile-Creep Specimen |
|------------------|------------------|------------------|
| \( x \)          | \( y \)          | \( z \)          |
| \( \dot{\varepsilon}_x \) = -\( \dot{\varepsilon}_y \) = -\( \dot{\varepsilon}_z \) = \( \dot{\varepsilon}_t \) = 0 |
| \( \dot{\varepsilon}_y \) = -\( \dot{\varepsilon}_x \) = 0 |
| \( \dot{\varepsilon}_z \) = -\( \dot{\varepsilon}_x \) = 0 |

Assume now that each grain undergoes the same strain rate as the aggregate. The permanent deformation rates in each grain \(\dot{\varepsilon}_{xx}, \ldots\) are then given by:

\[ \dot{\varepsilon}_{xx} = \dot{\varepsilon}_x - (\varepsilon_{yx})_{xx}, \quad \dot{\varepsilon}_{yy} = \dot{\varepsilon}_y - (\varepsilon_{yx})_{yy}, \quad \dot{\varepsilon}_{zz} = \dot{\varepsilon}_z - (\varepsilon_{yx})_{zz} \]

\[ \dot{\gamma}_{xy} = -\dot{\epsilon}_{xy}, \quad \dot{\gamma}_{xz} = -\dot{\epsilon}_{xz}, \quad \dot{\gamma}_{yz} = -\dot{\epsilon}_{yz} \]

The Lévy-von Mises equations (2) may be used to show that:

\[ \frac{\dot{\gamma}^2}{2} = \frac{\dot{\varepsilon}_x^2}{2} + \frac{\dot{\varepsilon}_y^2}{2} + \frac{\dot{\varepsilon}_z^2}{2} + \frac{\dot{\gamma}_{xy}^2}{2} + \frac{\dot{\gamma}_{xz}^2}{2} + \frac{\dot{\gamma}_{yz}^2}{2} = \frac{Y^2}{2} - 3\dot{\varepsilon}_t^2 + \dot{\varepsilon}_x^2 X \]

Since \(E(x, y, z)\) is an invariant,

\[ E(x, y, z) = E(a, b, c) \]

and from Table I

\[ E(a, b, c) = 2e_0^2 \]

whence

\[ \frac{\dot{\gamma}^2}{2} = \frac{\dot{\varepsilon}_x^2}{2} = \frac{Y^2}{2} - \frac{3e_0^2}{2} + 2e_0^2 \]

Again, using the Lévy-von Mises equations

\[ \sigma'_{xx} = \frac{\dot{\varepsilon}_x - (\varepsilon_{yx})_{xx}}{\lambda} \]

and when \(\dot{\varepsilon}_x \ll \dot{\varepsilon}_t\) for small applied stresses

\[ \sigma'_{xx} \cong \frac{Y}{\sqrt{3}e_0} \left[ - (\varepsilon_{yx})_{xx} + \dot{\varepsilon}_t \left( 1 - \frac{1}{2} \frac{\varepsilon_{yx}^2}{e_0^2} \right) \right] \]

for each grain. It is further assumed that the stress averaged over all grains in any direction is equal to the applied stress in that direction. Denoting such averages by a bar

\[ \overline{\sigma}_{xx} = \frac{\sigma_{xx}}{3} \cong \frac{Y}{\sqrt{3}e_0} \left[ - (\varepsilon_{yx})_{xx} + \dot{\varepsilon}_t \left( 1 - \frac{1}{2} \frac{\varepsilon_{yx}^2}{e_0^2} \right) \right] \]
where $\sigma_t$ is the applied tensile stress.

Straightforward algebra shows that $\left(\frac{\sigma_{22}}{\sigma_t}\right)^2 = \frac{4}{15}$

and hence that

$$\sigma_t \approx \frac{Y}{\sqrt{3}} \varepsilon_t$$

or, more usually, expressing the tensile strain rate as a function of stress

$$\varepsilon_t \approx \frac{5}{2 \sqrt{3} Y} \sigma_t$$

(4)

This equation shows a close correspondence with Cottrell’s original result

$$\varepsilon_t \approx \frac{5}{2} \frac{Y}{Y} \varepsilon_t$$

which is, in turn, in good agreement with experimental results. The method of derivation is an alternative to that given previously by Anderson and Bishop.8

The Application to Large Thermal Cycles

The $\alpha$ phase of uranium exhibits marked anisotropy as regards thermal expansion. The orthorhombic crystal of the present model will be taken to have thermal-expansion coefficients in the directions of the crystal axes varying with temperature as shown in Fig. 1. As an example, the mean expansion coefficients in the principal directions are roughly:

<table>
<thead>
<tr>
<th>Direction</th>
<th>Coefficient of Expansion, $\times 10^{-3}$</th>
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<tbody>
<tr>
<td>$a$</td>
<td>60</td>
</tr>
<tr>
<td>$b$</td>
<td>-5</td>
</tr>
<tr>
<td>$c$</td>
<td>-30</td>
</tr>
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</table>

The linear expansion coefficient of an isotropic aggregate will be taken as the mean of these three values. When an aggregate of randomly oriented crystals is heated, the grains will expand directionally, and either internal stresses will build up or intergranular tearing will take place. There will normally be a range in which the former process occurs. If

the temperature rise is large and rapid enough, the initial elastic straining of the grains will be followed by plastic straining to maintain coherence. Thus, for large temperature fluctuations at rates greater than those at which creep will relax internal stresses, all the grains will be brought to the yield point. This, it should be emphasized, is entirely irrespective of external loading. The aggregated plastic strains in an isotropic aggregate will give a uniform expansion in the absence of external loading.

Consider a specific grain. If it is assumed that each grain deforms similarly to the aggregate, the plastic strain added to the (free) expansion will equal the aggregate expansion. Hence the plastic strain in each grain is known. If the stress/strain relationships are known, the existing internal stresses can be deduced.

Suppose now that the aggregate is loaded. The stress state in each grain will be modified and the plastic strain components will be varied. It is found that when the resulting strain is averaged for an aggregate a macroscopic permanent strain is associated with an applied load, no matter how small it may be. The theory is derived below.

Theory

Let the coefficients of expansion in the $a$, $b$, and $c$ directions be $a_a$, $a_b$, $a_c$ per °C with the mean expansion $a = \frac{1}{3}(a_a + a_b + a_c)$. Initially, each grain will be strained elastically for small temperature rises. The elastic strains per °C which result from each crystal remaining the same shape are: $-\varepsilon_a$, $-\varepsilon_b$, $-\varepsilon_c$, $-\varepsilon_a + \varepsilon_b + \varepsilon_c$, $\frac{1}{3}(\varepsilon_a - \varepsilon_b - 2\varepsilon_c)$, $\frac{1}{3}(\varepsilon_b + \varepsilon_c - 2\varepsilon_a)$, $\frac{1}{3}(\varepsilon_a + \varepsilon_b - 2\varepsilon_c)$, $\frac{1}{3}(\varepsilon_a - \varepsilon_b + \varepsilon_c)$, $\frac{1}{3}(\varepsilon_a - 2\varepsilon_b + \varepsilon_c)$, $\frac{1}{3}(\varepsilon_a - 2\varepsilon_c + \varepsilon_b)$. Taking $a_a = a_c$ these are $\frac{1}{3}(a_a - a_b)$, $\frac{1}{3}(a_a - a_b)$, $\frac{1}{3}(a_a - a_b)$. If we take Poisson’s ratio to be $\frac{1}{3}$, this corresponds to a uniform tensile strain in the $b$ direction of $\frac{1}{3}(a_a - a_b)$. If the tensile strain to yield is $\varepsilon_y$, then the temperature rise required to bring each grain to the yield point from an initially stress-free condition is

$$\Delta T_p = \frac{\varepsilon_y}{3}(a_a - a_b)$$

and taking as an example $\varepsilon_y \approx 0.001$; $\Delta T_p = 43^\circ$.

Hence a temperature rise of 43 °C will bring each grain of an initially stress-free aggregate to the yield point, or a range of ±43 °C will just cycle the grains to the yield point in each direction. Therefore a plastic strain with components in the $a$, $b$, $c$ direction of $\frac{1}{3}(a_a - a_b)$, $\frac{1}{3}(a_a - a_b)$, $\frac{1}{3}(a_a - a_b)$ per °C will occur. Suppose the temperature rise in this plastic range is $(\Delta T - \Delta T_p)$. Then the tensile strain in the $b$ direction is

$$\varepsilon = \frac{1}{3}(a_a - a_b)(\Delta T - \Delta T_p)$$

Let the $b$ direction be inclined at an angle $\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, to a specimen direction $z$ in which a strain $\varepsilon_z$ due to an applied stress $\sigma_z$ occurs. Taking orthogonal specimen axes $x$, $y$, $z$ where the $b$-axis lies in the $y$, $z$ plane, it can be shown that the plastic strain in each grain relative to the specimen axes is as in Table III.

<table>
<thead>
<tr>
<th>Table III</th>
<th>The Plastic Strain in a Grain Relative to Specimen Axes</th>
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<tbody>
<tr>
<td>$x$</td>
<td>$\varepsilon_z$</td>
</tr>
<tr>
<td>$y$</td>
<td>$\varepsilon_z\sin\theta \cos\phi$</td>
</tr>
<tr>
<td>$z$</td>
<td>$\varepsilon_z\cos\theta$</td>
</tr>
</tbody>
</table>

The yield criterion and the Lévy-von Mises relations will be taken as in equations (1) and (2) and the mean deviatoric stress is given by
\[ \bar{\sigma}_{z} = \int_{0}^{\pi} \sigma_{z} \sin \theta \, d\theta. \]

The averaging technique used above implies that the applied stress

\[ \bar{\sigma}_{z} = \frac{\sigma_{z}}{Y}. \]

whence it may be shown that

\[ \frac{\sigma_{z}}{Y} = \frac{\epsilon_{z}}{4 \epsilon_{z}} + \frac{3 \epsilon_{z}^{2} - \epsilon_{z} - \epsilon_{z}^{2}}{4 \epsilon_{z}^{2} - \epsilon_{z} - \epsilon_{z}^{2}} \quad (5) \]

It is observed that the particular anisotropic mode of expansion considered here permits integration in a closed form, whereas the growth deformation mode applied earlier permits only a series expansion in the applied stresses. The result of the present analysis to first order in the applied stresses brings equation (5) to the form

\[ \frac{\sigma_{z}}{Y} = \frac{4 \epsilon_{z}}{5 \epsilon_{z}} \quad \ldots \quad (6) \]

In a complete cycle during which the temperature twice undergoes the amplitude \( \Delta T \) °C, the plastic strain due to a small applied load \( \bar{\sigma}_{z} \) is given by

\[ \epsilon_{z} = \frac{5}{4} \frac{\bar{\sigma}_{z}}{Y} \times \frac{2}{3} (a_{a} - a_{0})(2\Delta T - \Delta T_{a} - \Delta T_{b}) \quad \ldots \quad (7) \]

where \( \Delta T_{a} \) and \( \Delta T_{b} \) are defined as the thermal cycle to attain yielding at the upper and lower temperatures of the cycle.

**Brief Review of Experimental Work**

Roberts\(^{11}\) reported the results of large thermal cycles applied to loaded uranium springs in the range up to 400° C. The experiments indicated that for the first few cycles up to 300° C, the uranium cannot support even small loads without undergoing plastic strain. After a few cycles applied in fairly quick succession, the material becomes able to support small loads elastically. It was suggested in this paper that this change of behaviour is due partly to a redistribution of internal stresses and partly to an increase in yield stress by work-hardening. This "shakedown" condition was disturbed both by tensile straining to yield and by increasing the upper temperature of the cycling range. After a tensile straining to yield, the initial plastic flow recurred, whilst the effect of increasing the upper cycling temperature from 300-400° C was to produce an increment of strain for each cycle, which gave no indication of diminishing with increasing number of cycles.

Extensive investigations of single thermal cycles have been reported in the range 300-600° C by Young, Gardiner, and Rotsey\(^{11}\). This work enabled a clear distinction to be drawn between the time-independent plastic flow component of strain of the type analysed in the preceding section and the strain occurring during relaxation of the internal stresses generated between grains at the end of a complete thermal cycle. Ancillary tensile tests carried out at the strain rates appropriate to the thermal-cycling conditions enabled a good estimate to be made of the internal stress at the end of a thermal cycle. The theory developed to account for relaxation strain was in good agreement with experiment. (A general theory of relaxation strain is given later in the present paper).

Young, Gardiner, and Rotsey\(^{11}\) have also noted a reduction in the plastic strain that occurred per cycle when a number of cycles were carried out in quick succession. Thus, 12 rapid cycles were required to double the plastic strain obtained on a single cycle.

McIntosh and Heal\(^{11}\) have reported early results on large thermal cycles applied to uranium creep specimens which were unloaded during the application of the cycle but reloaded when the cycle was complete. The strain obtained subsequently is related to relaxation strain rather than the time-independent component. The results are in reasonable agreement with the theories developed by Young, Gardiner, and Rotsey\(^{11}\) and by the present authors in this paper.

**Accelerated Creep**

In earlier sections the behaviour of a uranium aggregate which continues to deform whilst stressed to the yield point either by irradiation growth or by thermal cycling, has been considered. In the present section the yield mechanism for permanent deformation is replaced by a creep-deformation mechanism and the effects of irradiation growth, thermal cycling, and stress relaxation are considered. Such a mechanism becomes important when the irradiation growth rate and the thermal cycling amplitude is insufficient to raise the aggregate to the yield stress internally, but is large enough to produce internal stresses greater than the applied stress. Under these circumstances creep rates greatly in excess of the nominal creep rate obtained by neglecting internal stresses can be obtained.

**The Acceleration of Creep by Irradiation Growth**

It is a matter of fundamental importance that the derivation of the Cottrell "creep" equation given above makes no mention of the conventional creep behaviour of uranium. Physically, the condition for such an approximation is that the yield-stress system acting in each crystal is insufficient to produce conventional creep deformations of the order of the crystal growth rate. Since the conventional creep strength of uranium roughly halves for each 20° C rise in temperature, and the single-crystal growth rate appears also to decrease with increasing temperature, it is clear that a temperature exists above which internal stresses generated by growth are less than the yield stress. Nevertheless they can be sufficient to deform the crystals significantly by creep. The analysis of the Cottrell creep mechanism given above will then no longer apply, since the yield criterion represented by equation (1) is not satisfied. An estimate based on the most recent data suggests that this temperature lies in the range 350-400° C. An analysis of this problem has been made by Blackburn,\(^{11}\) whose estimate of this temperature would lie in roughly the same range.

To analyse the creep mode of deformation, a modified form of the Lévy–von Mises equations can be introduced,

\[ \frac{\epsilon_{xx}}{\sigma_{xx}} = \frac{\epsilon_{yy}}{\sigma_{yy}} = \frac{\epsilon_{zz}}{\sigma_{zz}} = \frac{\gamma_{xy}}{\tau_{xy}} = \frac{\gamma_{yz}}{\tau_{yz}} = \frac{\gamma_{zx}}{\tau_{zx}} = A \Delta n^{-1} \quad \ldots \quad (8) \]

where \( \Delta n^{2} = \sigma_{xx}^{2} + \sigma_{yy}^{2} + \sigma_{zz}^{2} + 2\tau_{xy}^{2} + 2\tau_{yz}^{2} + 2\tau_{zx}^{2} \quad (9) \)

\[ A = \exp(P-Q/RT), \quad Q = \text{activation energy for creep}, \quad R = 2 \text{cal/g-mole °C}, \quad T = \text{temperature (°K)}, \quad \text{and the (x,y,z) axes are the specimen axes}. \]

Such equations have been used successfully by several authors in the analysis of creep rates during the application of combined stresses to polycrystalline specimens. (See Johnson\(^{11}\) for a comprehensive review of such
applications.) The application to uranium single crystals will thus be subject to the same limitations as the yield criterion used and discussed earlier.

Consider the steady-state behaviour under a small macroscopic tensile stress \( \sigma_t \) of an isotropic aggregate of elements each deforming in creep according to equation (8) and with the growth rates \( \varepsilon_k \) shown in Table 1.

First, take the case of uniform deformation* in each grain. Then if \( \varepsilon_t \) is the macroscopic strain rate, equation (8) becomes

\[
\varepsilon_t = \frac{-\varepsilon_{xx} \varepsilon_{yy}}{\sigma_{xy}} = \frac{\varepsilon_{xx} - \varepsilon_{yy}}{\sigma_{xy}} = \frac{\varepsilon_{xx} - \varepsilon_{yy}}{\sigma_{xy}} = \frac{\varepsilon_{xx} \varepsilon_{xy}}{\sigma_{xy}} = \frac{\varepsilon_{xx} \varepsilon_{xy}}{\sigma_{xy}}
\]

or eliminating \( I \) and using

\[
(\varepsilon_{xy})^2 + (\varepsilon_{yy})^2 + (\varepsilon_{xx})^2 + 2(\varepsilon_{xy})^2 = 2(\varepsilon_{xx})^2 + 2(\varepsilon_{xy})^2 + 2(\varepsilon_{yy})^2 = 2(\varepsilon_{xx})^2
\]

\[
\sigma_{xx} = \left( \frac{\varepsilon_{xx} - \varepsilon_{yy}}{\varepsilon_{xx} + \varepsilon_{yy}} \right) \left( \frac{3\varepsilon_{xx}^2}{2(\varepsilon_{xx}^2 + \varepsilon_{yy}^2)} \right) \left( \frac{3\varepsilon_{xx}^2 + \varepsilon_{yy}^2}{4\varepsilon_{xx}^2 + 4\varepsilon_{yy}^2} \right)
\]

Expanding to first order,

\[
\sigma_{xx} = \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2\varepsilon_{xx}^2} + \frac{\varepsilon_{xx} - \varepsilon_{yy}}{4\varepsilon_{xx}^2}
\]

or, averaging over all grains,

\[
\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2\varepsilon_{xx}^2} + \frac{\varepsilon_{xx} - \varepsilon_{yy}}{4\varepsilon_{xx}^2}
\]

and so

\[
\varepsilon_t = \frac{\varepsilon_{xx} \varepsilon_{xy}}{\sigma_{xy}} = \frac{\varepsilon_{xx} \varepsilon_{xy}}{\sigma_{xy}} = \frac{\varepsilon_{xx} \varepsilon_{xy}}{\sigma_{xy}} = \frac{\varepsilon_{xx} \varepsilon_{xy}}{\sigma_{xy}}
\]

Defining \( I_0 \) as the value of \( I \) (see equation (9)), when the applied stress is zero

\[
I_0 = (\sqrt{3} \varepsilon_{xx})^4 + \frac{4n + 1}{\sqrt{3} \varepsilon_{xx}} \frac{n}{4n + 1} \frac{I_0}{I_0}
\]

and

\[
\varepsilon_t = \frac{\varepsilon_{xx} \varepsilon_{xy}}{\sigma_{xy}} = \frac{\varepsilon_{xx} \varepsilon_{xy}}{\sigma_{xy}} = \frac{\varepsilon_{xx} \varepsilon_{xy}}{\sigma_{xy}} = \frac{\varepsilon_{xx} \varepsilon_{xy}}{\sigma_{xy}}
\]

Equation (10) gives the creep rate \( \varepsilon_t \), when \( \varepsilon_{xx} \leq I_0 \).

Equation (12) is of interest since, as \( I_0 \rightarrow \infty \) (the yield stress) and \( n \rightarrow \infty \), the solution tends towards that already obtained for Cottrell creep during yield flow.

One further equation may be derived which has implications extending beyond problems associated with uranium. Let \( \varepsilon_t \) denote the creep rate during a tensile creep test under an applied stress \( I_0 \) with no internal stresses present. Then using

\[
\frac{\varepsilon_t}{A} = \varepsilon_t (I_0)
\]

* The case of uniform deformation was first solved analytically by Hodgkins. The present solution is substantially his method.

to eliminate \( A \) from (11), and the resulting equation to eliminate \( \varepsilon_t \) from (12), it is found that

\[
\varepsilon_t = \frac{5n}{4n + 1} \frac{\varepsilon_{xx}}{\sigma_{xy}} \frac{\varepsilon_t (I_0)}{I_0}
\]

or

\[
\varepsilon_t = \frac{5n}{4n + 1} \frac{\varepsilon_{xx}}{\sigma_{xy}} \frac{\varepsilon_t (I_0)}{I_0}
\]

such a law may be shown to hold when \( \varepsilon_{xx} < I_0 \) for any method of producing the internal stress \( I_0 \). In the example discussed above the internal stress is constant.

It is clear from the variables considered in equation (14) that the presence of constant internal stresses in a metal which are greater than the applied stress will cause an acceleration of the natural creep rate in the absence of such stresses. Blackburn, Harnby, and Stobo have put forward a model which allows account to be taken of internal stresses which vary as a function of time and position in the metal. Such a general theory often requires a digital computer to obtain numerical results. The present method giving order-of-magnitude answers is applicable so long as equivalent constant internal stresses \( I_0 \) can be defined for any internal stress system which is a function of space and time. A considerable amount of work has been published on internal stresses in metals other than uranium. The knowledge of internal stress systems has not, however, been applied to predictions of accelerated creep behaviour. As an example of fundamental contributions to the topic, the work of Lassolo and Boas and Honeycombe is cited.

Let us now consider the particular problem of internal stresses due to thermal cycles of small amplitude.

The Acceleration of Creep by Small-Amplitude Thermal Cycles

The anisotropy of thermal expansion of uranium in the (100), (001), and the (010) directions is shown in Fig. 1, and an analysis of the effects of large thermal cycles has been made above. In the present section we are concerned with thermal cycles which are insufficient to bring the aggregate to the yield point, but which are nevertheless sufficient to induce large internal stresses in the uranium. We consider an idealized creep test in which a tensile specimen of uranium is subjected to small thermal cycles of amplitude \( \Delta T \) about a mean temperature \( T \). The thermal cycle is assumed to give an instantaneous temperature change from \( T - \Delta T \) to \( T + \Delta T \), followed by a holding time at the upper temperature that is short compared with the stress-relaxation time of the uranium. The temperature is then assumed to be reduced again to \( T - \Delta T \) instantaneously, and held at that temperature for a period short compared with the stress-relaxation time, and the cycle is then repeated. Thus, the type of thermal cycle is a rectangular wave form.

If the uranium is stress-free at temperature \( T \), the internal stresses at \( T + \Delta T \) and \( T - \Delta T \) will differ only in sign. The absolute magnitude of the internal stress is thus the same throughout the thermal-cycling scheme considered.

The problem is to estimate the value of \( I_0 \), the internal stress in the absence of applied stress, and to insert this value in equation (14). Now.

\[
\varepsilon_t = \varepsilon_{xx}^2 + \varepsilon_{yy}^2 + \varepsilon_{xx}^2
\]

and the elastic equations of equilibrium are

\[
\varepsilon_{xx} = \varepsilon_r \Delta T + (\varepsilon_{xx} - \varepsilon_{yy}) \varepsilon_{xy} / E
\]

\[
\varepsilon_{yy} = \varepsilon_r \Delta T + (\varepsilon_{xx} - \varepsilon_{yy}) \varepsilon_{xy} / E
\]

\[
\varepsilon_{xy} = \varepsilon_r \Delta T + (\varepsilon_{xx} - \varepsilon_{yy}) \varepsilon_{xy} / E
\]

where

\[
\varepsilon_r = \varepsilon_r \Delta T + (\varepsilon_{xx} - \varepsilon_{yy}) \varepsilon_{xy} / E
\]
where \( \varepsilon_{aa}, \varepsilon_{bb}, \varepsilon_{cc} \) are the coefficients of expansion along the \( a, b, c \) directions. The stains \( \varepsilon_{ma}, \varepsilon_{mc} \) are made up of two components—one due to the thermal expansion and one due to the internal stresses induced. The resultant deformation will be supposed to be homogeneous in the absence of applied stress. Thus

\[
\varepsilon_{aa} = \varepsilon_{bb} = \varepsilon_{cc} = (\varepsilon_a + \varepsilon_b + \varepsilon_c) \Delta T/3 + (1 - 2\nu)\varepsilon/E
\]  
(19)

and

\[
\sigma_{aa} = \frac{\sigma_b + \sigma_c - 2\sigma_a}{3} \cdot \frac{E}{1 + \nu} \cdot \Delta T
\]  
(20)

\[
\sigma_{bb} = \frac{\sigma_c + \sigma_a - 2\sigma_b}{3} \cdot \frac{E}{1 + \nu} \cdot \Delta T
\]  
(21)

\[
\sigma_{cc} = \frac{\sigma_a + \sigma_b - 2\sigma_c}{3} \cdot \frac{E}{1 + \nu} \cdot \Delta T
\]  
(22)

giving finally

\[
I_0 = \frac{E \cdot \Delta T}{1 + \nu} \sqrt{\sigma_{aa}^2 + \sigma_{bb}^2 + \sigma_{cc}^2 - \sigma_{bb} \sigma_{cc} - \sigma_{cc} \sigma_{aa} - \sigma_{aa} \sigma_{bb}}
\]  
(23)

or when, for uranium, \( \sigma_a \approx \sigma_c \)

\[
I_0 \approx \frac{E \cdot \Delta T}{1 + \nu} |\sigma_a - \sigma_b|
\]  
(24)

The problem of obtaining an equivalent internal stress does not arise for this idealized type of thermal cycling, since the absolute value remains numerically constant throughout. In practice, thermal cycles will range from a saw-tooth waveform to approximately rectangular waveforms and some stress relaxation will occur.

Consider a sinusoidal thermal cycle of short period, i.e. with no stress relaxation. The average internal stress generated will be less than the internal stress corresponding to the peak amplitude. A fairly good approximation to the average internal stress during such a cycle will be given by the root mean square value of internal stress during the cycle. Thus, for a maximum internal stress of \( I_0 \), the mean internal stress during a sinusoidal cycle would be \( I_0/\sqrt{2} \). An alternative method of averaging is to take the simple mean of the internal stress. The average value is then given by \( 2I_0/\pi \). The latter method of averaging has been justified in some specific cases by Blackburn, Harnby, and Stobo.19

The effect of relaxation will be of importance when the period of cycling is large. It should be noted, however, that the effect of stress relaxation on mean internal stress is not necessarily great. Consider a specimen under rectangular thermal-cycling conditions. If the relaxation behaviour is linear (i.e. stress relaxes linearly with time) and if the internal stress in each specimen is repeated exactly on each cycle, it is simple to show that the internal stress will have an average value given by equation (24). Thus, there is no effect of stress relaxation on such a model and effects of stress relaxation will be associated with departures from linear relaxation rather than with relaxation from a constant stress.

The effect caused by small thermal cycles may be estimated by defining an acceleration factor \( R \) as the ratio of the cycled creep rate to the isothermal creep rate. Thus

\[
R = \frac{\sigma_i}{I_0 \cdot \varepsilon/\Delta T}
\]

\[
= \left( \frac{I_0}{\varepsilon_0} \right) \frac{5n}{4n + 1}
\]  
(25)

\[
= \left( \frac{I_0}{\varepsilon_0} \right)^{n-1} \frac{5n}{4n + 1}
\]  
(26)

The following example illustrates the type of effect predicted. Consider creep at 500° C with a cycling amplitude of ±1° C. Taking

\[
E = 9.4 \times 10^4 \text{ lb/ln}^2
\]

\[
\nu = 0.23
\]

\[
I_0 = 447 \text{ lb/ln}^2 \text{ for a rectangular wave}
\]

\[
= 316 \text{ lb/ln}^2 \text{ for a sine wave (root mean square)}
\]

\[
= 285 \text{ lb/ln}^2 \text{ for a sine wave (simple average)}
\]

or, using the average result for a sine curve,

\[
I_0 = 300 \text{ lb/ln}^2
\]

and for creep tests with applied stresses of 100 and 10 lb/ln² the results are shown in Table IV.

<table>
<thead>
<tr>
<th>Applied Stress, lb/ln²</th>
<th>Rectangular</th>
<th>Sinusoidal</th>
<th>Rectangular</th>
<th>Sinusoidal</th>
<th>Rectangular</th>
<th>Sinusoidal</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>4.69</td>
<td>3.33</td>
<td>10.72</td>
<td>5.89</td>
<td>22.9</td>
<td>10.3</td>
</tr>
<tr>
<td>10</td>
<td>49.6</td>
<td>33.3</td>
<td>338</td>
<td>186</td>
<td>2290</td>
<td>1030</td>
</tr>
</tbody>
</table>

The principal conclusions to be drawn are that: (1) The error made in assuming isothermal behaviour (i.e., the value of \( R \)) increases considerably as the applied stress level is lowered. (2) The effect of the creep parameter \( n \) has a critical effect on \( R \)—the smaller values of \( n \) leading to smaller values of \( R \). (3) The shape of the cycle is not likely to introduce a new order-of-magnitude effect. Allowance for the shape effect can be made with sufficient accuracy using the approximation suggested in the present section.

Brief Review of Experimental Work on Small Thermal Cycles

The first experimental work undertaken explicitly to check the predictions of the theory of creep acceleration by small thermal cycles was reported by McIntosh and Heal.19 The work was carried out in the Creep Section of the Culcheth Laboratories of the U.K.A.E.A. and reported by Anderson. The cycling technique used was developed by W. T. Burke. The results obtained were in good agreement with the predictions based on isothermal creep data (i.e., the values of \( n, R, \) and \( Q \) used in equation (9)) which had been obtained up to 1958.

Work reported subsequently by Young, Gardner, and Rotseyn22 and Blackburn, Harnby, and Stobo21 has indicated that the isothermal creep data equations require some modification. In particular they derive a value of \( n \approx 2.5 \). Analysis of further data by Gardner and Miller26 has led to substantial confirmation of this value. Blackburn, Harnby, and Stobo21 report the results of tests on thermal cycling carried out at 500° C, whilst Gardner and Miller26 have analysed all the relevant data over the range 450–650° C.
Both sets of authors conclude that the theoretical estimates of internal stresses caused by a small thermal cycle (i.e., equation (24), with a suitable allowance for shape effect) are in good agreement with experimental results.

The Primary Creep Strain Occurring during Relaxation of Internal Stresses

When a uranium creep specimen is heated up to temperature, the internal stresses built up are considerable. A similar development of large residual stresses has been demonstrated by Young, Gardiner, and Rosely\(^{18}\) in a specimen after a large thermal cycle. At higher temperatures the relaxation rates and the creep rates are both increased.

The results on the acceleration of creep by irradiation growth will now be applied to derive an estimate for the strain occurring during relaxation. A similar estimate has been given without proof by Young, Gardiner, and Rosely\(^{18}\).

Using equation (14) in its differential form

\[
de_{r} = \frac{\sigma_{r}}{E} \frac{d \sigma_{r}}{d \sigma_{0}}
\]

the problem is to integrate the values of \(d_{r}\) over the internal-stress range from the initial value of \(I_{r}^{0}\) to the final value of \(I_{r}^{1}\) which may be taken as \(\approx 2\sigma_{r}^{*}\).

The second important equation is the relaxation equation. If creep and relaxation are identical processes in uranium, then the elastic strain due to stress relaxation is balanced by the creep of the grain under the internal stress. Thus

\[
d \frac{d \sigma_{r}}{d \sigma_{0}} + d \epsilon_{r} = 0
\]

and so, integrating equation (27)

\[
\epsilon_{r} \approx \frac{\sigma_{r}}{E} \log \left( \frac{I_{r}}{I_{r}^{0}} \right)
\]

or using \(I_{r}^{0} \approx 2\sigma_{r}^{*}\)

\[
\epsilon_{r} \approx \frac{\sigma_{r}}{E} \log \left( \frac{I_{r}}{2\sigma_{r}^{*}} \right)
\]

It is to be noted that a detailed knowledge of the actual creep and relaxation law is not required to obtain this result. Expressing the result in quantitative terms, it may be expected that during relaxation the strain occurring in a loaded specimen will amount to several elastic deflections. Such a deformation is not very temperature-sensitive. With conditions of the thermal cycle and temperature kept constant, the number of elastic deflections would be slightly increased for small applied stresses. This result is of importance in accounting for a component of primary creep in terms of a secondary creep characteristic. The result is of general value in that such a phenomenon is to be expected in any metal in which large internal stresses have been generated at the beginning of a creep test.

Conclusions

(1) A continuously yielding mechanism may be used to describe the phenomenon of "Cottrell creep" under small applied stresses.

(2) A similar mechanism leads to a time-independent increment of strain during the application of a large thermal cycle to a loaded uranium specimen.

(3) When the grains do not deform at the yield stress, an analogous mechanism of plastic flow due to creep may be postulated. Such a mechanism leads to creep rates in excess of the predicted isothermal behaviour when internal stresses are greater than the applied stress are generated. The observed creep rate can be two orders of magnitude greater than the isothermal creep rate.

(4) Available experimental results have produced good agreement with the theoretical work carried out for uranium.

(5) The application of the present techniques to creep problems with other metals subjected to low applied stresses must be a fruitful field of research.

Acknowledgements

The present work comprises many of the logical consequences of the numerical prediction of irradiation creep, by Professor A. H. Cottrell. The debt owed by us to other colleagues should be clear from the text. The work was carried out at the National Engineering Laboratories, East Kilbride, in 1956 and subsequently in the Culcheth and Springfields laboratories of the United Kingdom Atomic Energy Authority. The authors would like to acknowledge the permission of the Managing Director of the Reactor Group, U.K.A.E.A., to publish the paper.

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