Redistributive policies with heterogeneous social preferences of voters*

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Abstract

There is growing evidence on the roles of *fairness* and *other-regarding preferences* as fundamental human motives. Call voters with fair preferences, as in Fehr-Schmidt (1999), *fair-voters*. By contrast, traditional political economy models are based on *selfish-voters* who derive utility solely from 'own' payoff. In a general equilibrium model with endogenous labor supply, a mixture of fair and selfish voters choose optimal policy through majority voting. First, we show that majority voting produces a unique winner in pairwise contests over feasible policies (the Condorcet winner). Second, we show that a preference for greater fairness leads to greater redistribution. An increase in the number of fair voters can also lead to greater redistribution. Third, we show that in economies where the majority are selfish-voters, the decisive policy could be chosen by fair-voters, and vice-versa. Fourth, while choosing labor supply, even fair voters behave exactly like selfish voters. We show how this apparently inconsistent behavior in different domains (voting and labor supply) can be rationalized within the model.

Keywords: Direct democracy; redistribution; other-regarding preferences.

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1. Introduction

Traditional economic theory relies on the twin assumptions of rationality and self-interested behavior. The latter is generally taken to imply that individuals are interested primarily in their own pecuniary payoffs (*selfish-preferences*). This view is not always in conformity with the evidence. The purely selfish-individual model is unable to explain a range of phenomena from many diverse areas such as collective action, contract theory, the structure of incentives, political economy and the results of several experimental games.

Individuals are also often motivated by the pecuniary and non-pecuniary payoffs of others. A substantial fraction of individuals exhibit *social-preferences* (or *other regarding preferences*), i.e., care about the consumption and well being of others, say, through feelings of altruism or envy. Evidence indicates that about 40 percent of individuals exhibit selfish preferences while the remaining exhibit social preferences[1]. Hence, in many cases, it is appropriate that one considers a *mixture* of selfish and fair individuals. The results from a range of experimental games, such as the ultimatum game, the gift exchange game and the public-good game with punishment can easily be reconciled if we assume individuals to have social preferences.[2]

It would also seem plausible that social-preferences also motivate a human desire to redistribute. The experimental results of Ackert et al. (2007), Tyran and Sausgruber (2006) and Bolton and Ockenfels (2006) are strongly supportive of the importance of social preferences in the domain of voting models.

The aim of our paper is to introduce a mixture of voters with selfish and fair preferences into a standard general equilibrium model with endogenous labor supply that, in the past, has incorporated only voters with selfish preferences. The model we choose is due to Romer (1975), Roberts (1977), and Meltzer and Richard (1981); also known as the 'RRMR' model. In this model, in a well established tradition in political economy, voters vote directly (direct democracy) on a linear progressive income tax to choose public redistribution, rather than delegate such decisions to their elected representatives (representative democracy).[3]

In actual practice, a range of representative democracy issues are important[4], however, the RRMR model abstracts away from these and chooses direct democracy as the political

[1] See for instance, Fehr and Fischbacher (2002) and Brown et al (2004).

[2] The modern literature on fairness is vast and expanding rapidly. For some recent surveys, see Fehr and Fischbacher (2002), Camerer (2003), Fehr and Schmidt (2005). For a flavour of some of the theories of fairness, see Rabin (1993), Fehr and Schmidt (1999), Bolton and Ockenfels (2000), Charness and Rabin (2002), Falk and Fischbacher (2006). For the neuroeconomic foundations of fairness, see Fehr at al. (2005).

[3] The pioneering work on the existence of a Condorcet winner in a unidimensional policy space is Black (1948a,b,c).

[4] See, for instance, Persson and Tabellini (2000) for a comprehensive exposition of such issues.

institution.[5] Recent research indicates that direct democracy is becoming increasingly important. Figures given in Matsusaka (2005a,b) (who terms the increasing trend in direct democracy as the "storm in ballot box lawmaking") are instructive. In the US, 70 percent of the population lives in a state or city where the apparatus of direct democracy is available. There have been at least 360 citizen initiated measures in the last 10 years in the US. In Europe, at least 29 referenda on monetary and market integration have been held.

We consider a two-stage game that is embedded within the general equilibrium RRMR model. In the first stage, voters vote on the redistributive tax rate, anticipating the outcome of the second stage. In the second stage, voters, as worker-consumers, play a one-shot Nash game. Each voter chooses own-labor supply, given the labor supply choices of the other voters and given the redistributive tax policy determined from the first stage.

We pose the following questions. Can one characterize an appropriate political economy equilibrium when deciding on the level of redistribution for a society? Does one expect societies with greater fairness to redistribute more? What sort of redistributive outcome does one expect, when, as the evidence suggest, there is a mixture of both selfish and fair voters? How does this outcome change when we change our mixture of fair and selfish voters? The aim of our paper is to address these questions.

Our main results are as follows. First, we demonstrate the existence of a Condorcet winner[6] for an economy where voters have the Fehr and Schmidt (1999) preferences for fairness. The nature of these preferences is discussed below, in detail. We allow the voters to differ in their social preferences (i.e., different degrees of envy and altruism). Insofar as one believes that issues of fairness and concern for others underpin the human tendency to redistribute, this result opens the way for future developments in behavioral political economy. Second, we find that greater fairness induces greater redistribution. This result is true in two senses. In the first sense, a stronger fairness preference of the existing voters increases redistribution. In the second sense an increase in the number of fair voters at the expense of selfish voters may also increase redistribution. Third, the introduction of selfish-poor (fair-rich) voters in an economy where the median voter is fair (selfish) can have a large impact on the redistributive outcome and may actually reduce (increase) redistribution. In other settings, even in the presence of a majority of fair (selfish) voters the redistributive outcome is identical to that of an economy comprising solely of selfish (fair) voters. Fourth, we show how apparently inconsistent behavior can be rationalized within the model. In particular, it is entirely consistent for fair voters to

[5] However, our model can be imbedded as a subgame in a larger indirect-democracy game. Thus, the results in the paper apply for any institutional setting in which the median voter theorem holds. We are grateful to Referee 2 for this observation.

[6] The Condorcet winner is defined more formally in section 4 below. It is a policy such that a majority of the voters prefer it to any other feasible policy in pairwise contests between policies.

use social preferences when deciding on redistribution but act *as if* they maximized selfish preferences when choosing their own labor supply.

The plan of the paper is as follows. Section 2 discusses issues of fairness and voting. Section 3 describes the theoretical model and derives some preliminary results. Section 4 establishes the existence of a Condorcet winner for voters with heterogeneous social preferences. Section 5 considers an economy where there is a mixture of fair and selfish voters. Finally, section 6 concludes. Proofs are relegated to Section 7. We do not discuss any applications as these are beyond the scope of the current paper. However, Dhami and al-Nowaihi (2008) give brief sketches of potential applications that could be fruitfully developed in the future.

2. A discussion of some relevant issues of fairness and voting

In this section, we discuss issues of the choice of an appropriate model of fairness in a voting context and the empirical evidence on models of voting and fairness. We also give a brief critique of the existing literature and the relation of our paper to the literature.

2.1. Which model of fairness?

There are several models of fairness. We choose to use the Fehr-Schmidt (1999) (henceforth, FS) approach to fairness. In this approach, voters care, not only about their own payoffs, but also their payoffs relative to those of others.[7] If their payoff is greater than that of other voters then they suffer from *advantageous-inequity* (arising from, say, altruism) and if their payoff is lower than that of other voters they suffer from *disadvantageous-inequity* (arising from, say, envy).

Several reasons motivate our choice of the FS model. The FS model is tractable and explains the experimental results arising from several games where the prediction of the standard game theory model with selfish agents yields results that are not consistent with the experimental evidence. These games include the ultimatum game, the gift-exchange game, the dictator game as well as the public-good game with punishment[8].

[7] Bolton and Ockenfels (2000) also provide another approach to inequity averse economic agents, but it cannot explain the outcome of the public good game with punishment, which is a fairly robust experimental finding. See also Charness and Rabin (2002) for another approach that recognizes inequity averse behavior.

[8] In the first three of these games, experimental subjects offer more to the other party relative to the predictions of the Nash outcome with selfish preferences. In the public good game with punishment, the possibility of ex-post punishment dramatically reduces the extent of free riding in voluntary giving towards a public good. In the standard theory with selfish agents, bygones are bygones, so there is no ex-post incentive for the contributors to punish the free-riders. Foreseeing this outcome, free riders are not deterred, which is in disagreement with the evidence. Such behavior can be easily explained within the FS framework.

Experimental results on voting lend support to the use of the FS model in such contexts. Tyran and Sausgruber (2006) explicitly test for the importance of the FS framework in the context of direct voting. They conclude that the FS model predicts much better than the standard selfish-voter model. In addition, the FS model provides, in their words, "strikingly accurate predictions for individual voting in all three income classes." The econometric results of Ackert et al. (2007), based on their experimental data, lend further support to the FS model in the context of redistributive taxation. The estimated coefficients of altruism and envy in the FS model are statistically significant and, as expected, negative in sign. Social preferences are found to influence participants' votes over alternative taxes. They find evidence that some participants are willing to reduce their own payoffs in order to support taxes that reduce advantageous-inequity or disadvantageous-inequity.

The FS model focusses on the role of inequity aversion. However, a possible objection is that it ignores the role played by intentions. These have been shown to be important in experimental results (Falk et al. (2002)) and are treated explicitly in theoretical work (Rabin (1993), Falk and Fischbacher (2006)). However, experimental results on the importance of intentions come largely from bilateral interactions. Economy-wide voting, on the other hand, is impersonal and anonymous, thereby making it unlikely that intentions play any important role in this phenomenon.

2.2. A critique of the literature on voting and fairness

There is a relatively small theoretical literature that considers voters with other regarding (or fair) preferences, which we now describe.

While we consider changes in a linear progressive income tax that affects all taxpayers, Tyran and Sausgruber (2006), in their important work, examine pure transfers of income from the rich to the poor that do not affect the middle-income voter[9]. Some rich voters, on account of their fairness, vote for transfers to the poor in circumstances where a rich, but selfish, voter would have voted otherwise. Hence, a majority of the fair voters might vote for redistribution when voting under selfish preferences would predict no redistribution. Tyran and Sausgruber (2006) do not address general equilibrium considerations or those regarding the existence of a Condorcet winner.

Bolton and Ockenfels (2006) examine the preference for equity versus efficiency in a voting game. Groups of three subjects are formed and are presented with two alternative policies: one that promotes equity while the other promotes efficiency. The final outcome is chosen by a majority vote. About twice as many experimental subjects preferred equity

[9] They do introduce a cost of such redistribution to the middle income voters, but it is not an integral part of the redistributive fiscal package considered.

as compared to efficiency. Furthermore, even those willing to change the status-quo for efficiency are willing to pay, on average, nearly half relative to those who wish to alter the status-quo for equity.

Galasso (2003) modifies the RRMR model to allow for fairness concerns. However, his notion of fairness takes a very specific form. It is not fully consistent with any of the accepted models of fairness. In particular, in Galasso (2003), fair voters care about their own payoffs but suffer disutility through a term that is linear in their payoffs relative to the worse off voter in society.[10] Since this concern for fairness arises from a linear term, preferences continue to be strictly concave and a median voter equilibrium exits. Within this framework there is greater redistribution when there is a mean preserving spread in inequality. However, this leaves open the question of whether a median voter equilibrium will exist in a standard model of fairness, that accords well with the evidence, such as the FS model, and what the properties of the resulting equilibrium will be? Galasso (2003) also does not address questions about a mixture of voters with heterogenous preferences, which is a central contribution of our paper.

Dhami and al-Nowaihi (2010) give plausible conditions under which a Condorcet winner will exist when voters have identical social preferences (e.g. all voters could be selfish or all could have an identical preference for fairness). This establishes the results for the benchmark case and opens the way for formal models in behavioral political economy. However, these results do not extend to the case where, as the evidence indicates, there is a mixture of voters with selfish and fair preferences.

3. Model

We consider a general equilibrium model as in Meltzer and Richard (1981). Let there be $n = 2m - 1$ voter-worker-consumers (henceforth, voters). Let the skill level of voter j be s_j, $j = 1, 2, ..., n$, where

$$0 < s_i < s_j < 1, \text{ for } i < j, \tag{3.1}$$

Hence, s_m is the median skill level. Denote the skill vector by $\mathbf{s} = (s_1, s_2, ..., s_n)$. Each voter has a fixed time endowment of one unit and supplies l_j units of labor and so enjoys $1 - l_j$ units of leisure, where

$$0 \leq l_j \leq 1. \tag{3.2}$$

[10] The latter term captures some notion of social justice. Others have included such a term to incorporate social justice e.g. Charness and Rabin (2002). However, they posit preferences, different from Galasso (2003), that are a convex combination of the total payoff of the group (this subsumes selfishness, in so far as one's own payoff is part of the total, and altruism) and a Rawlsian social welfare function. These sorts of models are able to explain positive levels of giving in dictator games, and reciprocity in trust and gift exchange games. However, they are not able to explain situations where an individual tries to punish others in the group at some personal cost, for instance, punishment in public good games.

Labor markets are competitive and each firm has access to a linear production technology such that production equals $s_j l_j$. Hence, the wage rate offered to each voter coincides with the marginal product, i.e., the skill level, s_j. Thus, the before-tax income of voter j is given by

$$y_j = s_j l_j. \tag{3.3}$$

Note that 'skill' here need not represent any intrinsic talent, just ability to translate labor effort into income[11]. Let the average before-tax income be

$$\overline{y} = \frac{1}{n} \sum_{j=1}^{n} y_j. \tag{3.4}$$

We make the empirically plausible assumption that the income of the median-skill voter, y_m, is less than the average income,[12]

$$y_m < \overline{y}. \tag{3.5}$$

The government operates a linear progressive income tax that is characterized by a constant marginal tax rate, t, $t \in [0, 1]$, and a uniform transfer, b, to each voter that equals the average tax proceeds,

$$b = t\overline{y}. \tag{3.6}$$

The budget constraint of voter j is given by

$$0 \leq c_j \leq (1 - t)\, y_j + b. \tag{3.7}$$

In view of (3.3), the budget constraint (3.7) can be written as

$$0 \leq c_j \leq (1 - t)\, s_j l_j + b. \tag{3.8}$$

3.1. Preferences of Voters

We define a voter's preferences in two stages. First, let voter j have a continuous *own-utility* function, $\widetilde{u}_j(c_j, 1 - l_j)$, defined over own-consumption, c_j, and own-leisure, $1 - l_j$.

Second, and for the reasons stated in the introduction, voters have *other-regarding preferences* as in Fehr-Schmidt (1999). Let $\mathbf{c}_{-j}$ and $\mathbf{l}_{-j}$ be the vectors of consumption and

[11] As Referee 2 pointed out, it may, therefore, have been more appropriate to use "productivity" instead of "skill". However, our usage is in conformity with the literature where, in many places, "skill" is used when "productivity" would have been more appropriate.

[12] The assumption that $y_m < \overline{y}$ is needed for Propositions 4 and 5 but not for Propositions 1, 2 or 3. The necessary and sufficient condition on exogenous parameters for $y_m < \overline{y}$ to be true is given by (3.26) below; we are grateful to Referee 2 for pointing this out.

labor supplies, respectively, of voters other than voter j. Under Fehr-Schmidt preferences the *FS-utility* of voter j is

$$\widetilde{U}_j\left(c_j, l_j; \mathbf{c}_{-j}, \mathbf{l}_{-j}, t, b, \alpha_j, \beta_j, \mathbf{s}\right) = \widetilde{u}_j\left(c_j, 1-l_j; t, b, s_j\right)$$
$$-\frac{\alpha_j}{n-1}\sum_{k\neq j}\max\left\{0, \widetilde{u}_k\left(c_k, 1-l_k; t, b, s_k\right) - \widetilde{u}_j\left(c_j, 1-l_j; t, b, s_j\right)\right\}$$
$$-\frac{\beta_j}{n-1}\sum_{i\neq j}\max\left\{0, \widetilde{u}_j\left(c_j, 1-l_j; t, b, s_j\right) - \widetilde{u}_i\left(c_i, 1-l_i; t, b, s_i\right)\right\}, \tag{3.9}$$

where

$$\text{for } \textit{selfish} \text{ voters } \alpha_j = \beta_j = 0, \text{ so } \widetilde{U}_j = \widetilde{u}_j, \tag{3.10}$$

$$\text{for } \textit{fair} \text{ voters } 0 < \beta_j < 1,\ \beta_j < \alpha_j, \text{ so } \widetilde{U}_j \neq \widetilde{u}_j. \tag{3.11}$$

Thus, $\widetilde{u}_j$ is also the utility function of a selfish voter, as in the standard textbook model. From (3.9), the fair voter cares about own payoff (first term), payoff relative to those where inequality is disadvantageous (second term) and payoff relative to those where inequality is advantageous (third term). The second and third terms which capture respectively, *envy* and *altruism*, are normalized by the term $n-1$. Notice that in FS preferences, inequality is *self-centered*, i.e., the individual uses her own payoff as a reference point with which everyone else is compared to. Also, while the Fehr-Schmidt specification is directly in terms of monetary payoffs, it is also consistent with comparison of payoffs in utility terms. These and related issues are more fully discussed in Fehr and Schmidt (1999). From (3.11), β_j is bounded below by 0 and above by 1 and α_j. On the other hand, there is no upper bound on α_j.[13]

We make the standard assumption that $\frac{\partial \widetilde{u}_j}{\partial c_j} > 0$. Since $\alpha_j \geq 0$ and $0 \leq \beta_j < 1$ it follows, from (3.9), that $\widetilde{U}_j\left(c_j, l_j; \mathbf{c}_{-j}, \mathbf{l}_{-j}, t, b, \alpha_j, \beta_j, \mathbf{s}\right)$ is also a strictly increasing function of c_j.[14] Hence, the budget constraint (3.8) holds with equality. Substituting $c_j = (1-t)\, s_j l_j + b$, from (3.8), into the utility function, $\widetilde{u}_j(c_j, 1-l_j)$, gives the following form for utility

$$u_j\left(l_j; t, b, s_j\right) = \widetilde{u}_j((1-t)\, s_j l_j + b, 1-l_j) \tag{3.12}$$

Correspondingly, the FS-utilities take the form

$$\begin{aligned} U_j\left(l_j; \mathbf{l}_{-j}, t, b, \alpha_j, \beta_j, \mathbf{s}\right) &= u_j\left(l_j; t, b, s_j\right) \\ &\quad -\frac{\alpha_j}{n-1}\sum_{k\neq j}\max\left\{0, u_k\left(l_k; t, b, s_k\right) - u_j\left(l_j; t, b, s_j\right)\right\} \\ &\quad -\frac{\beta_j}{n-1}\sum_{i\neq j}\max\left\{0, u_j\left(l_j; t, b, s_j\right) - u_i\left(l_i; t, b, s_i\right)\right\}, \end{aligned} \tag{3.13}$$

[13] $\beta_j \geq 1$ would imply that individuals could increase utility by simply destroying all their wealth; this is counterfactual. The restriction $\beta_j < \alpha_j$ is based on experimental evidence. Finally the lack of an upper limit on α_j implies that 'envy' is unbounded.

[14] We are grateful to Referee 2 for this observation.

Remark 1 *: (Weighted utilitarian preferences) First define the sets A_j and D_j as the set of voters with respect to whom voter j has respectively, advantageous and disadvantageous inequity. So*

$$A_j = \{i : i \neq j \text{ and } u_i(l_i; t, b, s_i) \leq u_j(l_j; t, b, s_j)\}, \tag{3.14}$$

$$D_j = \{k : k \neq j \text{ and } u_k(l_k; t, b, s_k) > u_j(l_j; t, b, s_j)\}. \tag{3.15}$$

Denote the respective cardinalities of these sets by $|A_j|$ and $|D_j|$. Then FS-utility (3.13) can be written in a way that is reminiscent of the weighted utilitarian form:

$$U_j\left(l_j; \mathbf{l}_{-j}, t, b, \alpha_j, \beta_j, \mathbf{s}\right) = \omega_{jj} u_j(l_j; t, b, s_j) + \sum_{i \neq j}^{n} \omega_{ji} u_i(l_i; t, b, s_i), \tag{3.16}$$

where

$$\begin{array}{lll} i \in A_j & \Rightarrow & \omega_{ji} = \frac{\beta_j}{n-1} > 0, \\ i = j & \Rightarrow & \omega_{jj} = 1 - \frac{|A_j|\beta_j}{n-1} + \frac{|D_j|\alpha_j}{n-1} > 0, \\ k \in D_j & \Rightarrow & \omega_{jk} = -\frac{\alpha_j}{n-1} < 0. \end{array} \tag{3.17}$$

Furthermore, the weights sum up to one i.e.

$$\sum_{i=1}^{n} \omega_{ji} = 1. \tag{3.18}$$

In particular, for selfish voters (3.10) and (3.17) give

$$\textit{If voter } j \textit{ is selfish, then } \omega_{jj} = 1 \textit{ and } \omega_{ji} = 0 \;\; (i \neq j). \tag{3.19}$$

3.2. Sequence of moves

We consider a two-stage game. In the first stage all voters vote directly and sincerely on the redistributive tax rate. Should a median voter equilibrium exist, then the tax rate preferred by the median voter is implemented. In the second stage, all voters make their labor supply decision, conditional on the tax rate chosen by the median voter in the first stage. On choosing their labor supplies in the second stage, the announced first period tax rate is implemented and transfers made according to (3.6).

In the second stage, the voters play a one-shot Nash game. Each voter, j, chooses his/her labor supply, l_j, given the vector, $\mathbf{l}_{-j}$, of labor supplies of the other voters, so as to maximize his/her FS-utility (3.13). In the first stage, each voter votes for his/her preferred tax rate, correctly anticipating second stage play.

The solution is by backward induction. We first solve for the Nash equilibrium in labor supply decisions of voters conditional on the announced tax rates and transfers. The second stage decision is then fed into the first stage FS-utilities to arrive at the indirect utilities of voters, which are purely in terms of the tax rate. Voters then choose their most desired tax rates which maximize their indirect FS-utilities, with the proposal of the median voters being the one that is implemented.

3.3. Labor supply decision of taxpayers (second stage problem)

Given the tax rate, t, and the transfer, b, both determined in the first stage (see Section 3.5, below), the voters play a one-shot Nash game (in the subgame determined by t and b). Each voter, j, chooses own labor supply, l_j, so as to maximize his/her FS-utility (3.13), given the labor supplies, $\mathbf{l}_{-j}$, of all other voters.

Since in (3.16), $u_i(l_i; t, b, s_i)$, $i \neq j$, enter additively, and $\omega_{jj} > 0$, it follows that maximizing the FS-utility, $U_j(l_j; \mathbf{l}_{-j}, t, b, \alpha_j, \beta_j, \mathbf{s})$, with respect to l_j, given $\mathbf{l}_{-j}$, t, b and $\mathbf{s}$, is equivalent to maximizing own-utility, $u_j(l_j; t, b, s_j)$, with respect to l_j, given t, b and s_j. We summarize this in the following proposition.[15]

Proposition 1 : *In the second stage of the game, voter j, whether fair or selfish, chooses own labor supply, l_j, so as to maximize own-utility, $u_j(l_j; t, b, s_j)$, given t, b and s_j.*

3.4. Simplifying assumptions

To derive the comparative static results of section 5 we need a specific functional form for the utility function, $\widetilde{u}_i$, of voter i. We adopt a functional form that is standard in the literature.

In common with the literature, we assume that all voters have the same own-utility function, $\widetilde{u}$, although, of course, their realized utility will depend on their realized consumption, c_i, and their realized leisure, $1 - l_i$. Thus

$$\widetilde{u}_i(c_i, 1 - l_i) = \widetilde{u}(c_i, 1 - l_i). \tag{3.20}$$

Furthermore, we assume that the own-utility function is quasi-linear, with constant elasticity of labor supply, which is the most commonly used functional form in various applications of the median voter theorems.

$$\widetilde{u}(c, 1 - l) = c - \frac{\epsilon}{1 + \epsilon} l^{\frac{1+\epsilon}{\epsilon}}, \tag{3.21}$$

where ϵ is the constant elasticity of labor supply, assumed positive.[16] The case $\epsilon = 1$ has special significance in the literature. In this case,

$$\widetilde{u}(c, 1 - l) = c - \frac{1}{2} l^2. \tag{3.22}$$

Meltzer and Richard (1981) use (3.22) to derive the celebrated result that the extent of redistribution varies directly with the ratio of the mean to median income. Piketty (1995)

[15] Proposition 1 would not be true if the u_i, $i \neq j$, entered non-additively into the FS-utility function. However, the empirical evidence strongly supports the adopted form for the FS-utility function.

[16] A large number of studies suggest positive labour supply elasticities (see, for example, Pencavel (1986) and Killingworth and Heckman (1986)). Negative labour supply elasticities may be due to estimating misspecified models (see Camerer and Loewenstein (2004), Chapter 1, 'Labor Economics', pp33-34).

restricts preferences to the quasi-linear case with disutility of labor given by the quadratic form, (3.22). Benabou and Ok (2001) do not actually consider a production side and their model has exogenously given endowments which evolve stochastically. Benabou (2000) considers the additively separable case with log consumption and disutility of labor given by the constant elasticity case, (3.21).

Substituting $c_j = (1-t)\, s_j l_j + b$ in (3.21), the own-utility function of voter j, we get

$$u_j\left(l_j; t, b, s_j\right) = u\left(l_j; t, b, s_j\right) = (1-t)\, s_j l_j + b - \frac{\epsilon}{1+\epsilon} l_j^{\frac{1+\epsilon}{\epsilon}}. \tag{3.23}$$

We list, in lemmas 1, 2, below, some useful results.

Lemma 1 *(Labor supply): Given t, b and s_j, the unique labor supply for voter j, $l_j = l\left(t, b, s_j\right)$, that maximizes utility (3.23), is given by*

$$l_j = l\left(t, b, s_j\right) = (1-t)^{\epsilon} s_j^{\epsilon},$$

and is independent of b.

Substituting labor supply, $l\left(t, b, s_j\right)$, given by Lemma 1, into (3.3), gives the before-tax income:

$$y_j\left(t, b, s_j\right) = (1-t)^{\epsilon} s_j^{1+\epsilon}. \tag{3.24}$$

Define $\overline{S}$ to be the 'weighted average of skills' when there are n voters, in the following sense.

$$\overline{S} = \frac{1}{n}\Sigma_{i=1}^{n} s_i^{1+\epsilon} \tag{3.25}$$

From (3.4), (3.5), (3.24) and (3.25) we get that for the median skill level, s_m,

$$s_m^{1+\epsilon} < \overline{S}, \tag{3.26}$$

Substituting labor supply in (3.6) we get,

$$b\left(t, \mathbf{s}\right) = t\,(1-t)^{\epsilon}\, \overline{S}. \tag{3.27}$$

Substituting labor supply in (3.23) we get the indirect utility function corresponding to the own-utility of voter j:

$$v\left(t, b, s_j\right) = u\left(l\left(t, b, s_j\right); t, b, s_j\right) = b + \frac{(1-t)^{1+\epsilon}}{1+\epsilon} s_j^{1+\epsilon}. \tag{3.28}$$

Lemma 2 *(Properties of the indirect utility function corresponding to own-utility):*
(a) $\frac{\partial v(t,b,s)}{\partial b} = 1$,
(b) $\frac{\partial v(t,b,s)}{\partial s} = (1-t)^{1+\epsilon} s^{\epsilon}$. *Hence,* $\left[\frac{\partial v(t,b,s)}{\partial s}\right]_{t=1} = 0$ *and* $t \in [0,1) \Rightarrow \frac{\partial v(t,b,s)}{\partial s} > 0$.

Lemma 2 shows that an increase in transfer payment, b, increase utility one for one and that for any interior tax rate, indirect utility is strictly increasing in the level of skill.

3.5. Preferences of voters over redistribution (the first stage problem)

Given the second stage choice of labor supplies by the voters (Proposition 1 and Lemma 1), the first stage problem is to choose the redistributive tax rate, t (and, consequently, the transfer, b, given by (3.3), (3.4) and (3.6)). For this purpose, we calculate the voters' indirect utility functions corresponding to their FS-preferences.

To find the indirect utility function, for voter j, $V_j = V_j\left(t, b, \alpha_j, \beta_j, \mathbf{s}\right)$, that corresponds to his/her FS-preferences, substitute labor supply (Lemma 1) into (3.13), and take account of (3.28) and Lemma 2b to get

$$\begin{aligned} V_j &= u\left(l\left(t,b,s_j\right);t,b,s_j\right) - \frac{\alpha_j}{n-1}\sum_{k\neq j}\max\left\{0, u\left(l\left(t,b,s_k\right);t,b,s_k\right) - u\left(l\left(t,b,s_j\right);t,b,s_j\right)\right\} \\ &\quad -\frac{\beta_j}{n-1}\sum_{i\neq j}\max\left\{0, u\left(l\left(t,b,s_j\right);t,b,s_j\right) - u\left(l\left(t,b,s_i\right);t,b,s_i\right)\right\}, \\ &= v\left(t,b,s_j\right) - \frac{\alpha_j}{n-1}\sum_{k\neq j}\max\left\{0, v\left(t,b,s_k\right) - v\left(t,b,s_j\right)\right\} \\ &\quad -\frac{\beta_j}{n-1}\sum_{i\neq j}\max\left\{0, v\left(t,b,s_j\right) - v\left(t,b,s_i\right)\right\}, \\ &= v\left(t,b,s_j\right) - \frac{\alpha_j}{n-1}\sum_{k>j}\left[v\left(t,b,s_k\right) - v\left(t,b,s_j\right)\right] - \frac{\beta_j}{n-1}\sum_{i<j}\left[v\left(t,b,s_j\right) - v\left(t,b,s_i\right)\right], \\ &= b + \frac{(1-t)^{1+\epsilon}}{1+\epsilon}\left[s_j^{1+\epsilon} - \frac{\alpha_j}{n-1}\sum_{k>j}\left(s_k^{1+\epsilon} - s_j^{1+\epsilon}\right) - \frac{\beta_j}{n-1}\sum_{i<j}\left(s_j^{1+\epsilon} - s_i^{1+\epsilon}\right)\right]. \end{aligned} \tag{3.29}$$

For any voter j, define the following three useful constants, S_j^-, S_j^+ and ϕ_j:

$$S_n^- = 0 \text{ and } S_j^- = \frac{1}{n-1}\sum_{k>j}\left(s_k^{1+\epsilon} - s_j^{1+\epsilon}\right) > 0, \text{ for } j < n, \tag{3.30}$$

$$S_1^+ = 0 \text{ and } S_j^+ = \frac{1}{n-1}\sum_{i<j}\left(s_j^{1+\epsilon} - s_i^{1+\epsilon}\right) > 0, \text{ for } j > 1, \tag{3.31}$$

$$\phi_j = s_j^{1+\epsilon} - \alpha_j S_j^- - \beta_j S_j^+. \tag{3.32}$$

The constants S_j^- and S_j^+ are function of the exogenous parameters, n (number of voters) and $\mathbf{s}$ (the skills vector). In addition to these, ϕ_j is also a function of ϵ (elasticity of labor supply), α_j (disadvantageous inequity parameter) and β_j (advantageous inequity parameter).

Remark 2 : *From (3.29), (3.30) and (3.31), we see that S_j^- and S_j^+ are, respectively, directly proportional to disadvantageous and advantageous inequality experienced by voter j.*

Substitute from (3.30) - (3.32) into (3.29) to get

$$V_j\left(t, b, \alpha_j, \beta_j, \mathbf{s}\right) = b + \frac{(1-t)^{1+\epsilon}}{1+\epsilon}\left[s_j^{1+\epsilon} - \alpha_j S_j^- - \beta_j S_j^+\right], \\ = b + \frac{(1-t)^{1+\epsilon}}{1+\epsilon}\phi_j. \tag{3.33}$$

When voter j votes on the tax rate, t, and the transfer, b, he/she takes into account the government budget constraint (3.27). Hence, substitute $b(t, \mathbf{s})$, given by (3.27), into (3.33), to get

$$W_j\left(t, \alpha_j, \beta_j, \mathbf{s}\right) = t(1-t)^{\epsilon}\,\overline{S} + \frac{(1-t)^{1+\epsilon}}{1+\epsilon}\phi_j. \tag{3.34}$$

Voter j votes for a tax rate, t, that maximizes social welfare from his/her own point of view, as given by his/her FS-indirect utility function (3.34).

In Proposition 2, below, we give some results on the existence of optimal (or most preferred) taxes for any individual voter who at the first stage is asked to state his/her choice of the most preferred tax rate. The next section, Section 4, will look at the equilibrium tax rate that is actually implemented by society.

Proposition 2 *(Existence of optimal tax rates):*
(a) Given α_j, β_j and $\mathbf{s}$, $W_j\left(t, \alpha_j, \beta_j, \mathbf{s}\right)$ attains a maximum at some $t_j \in [0, 1)$.
(b) If $\overline{S} - \phi_j \leq 0$, then
(i) $W_j\left(t, \alpha_j, \beta_j, \mathbf{s}\right)$ is strictly decreasing for $t \in [0, 1]$,
(ii) the tax rate preferred by voter j is $t_j = 0$.
(c) If $\overline{S} - \phi_j > 0$, then
(i) the tax rate, t_j, preferred by voter j, is unique, satisfies $0 < t_j < 1$ and is given by

$$t_j = \frac{\overline{S} - \phi_j}{(1+\epsilon)\,\overline{S} - \phi_j}, \tag{3.35}$$

(ii) $W_j\left(t, \alpha_j, \beta_j, \mathbf{s}\right)$ is strictly increasing on $[0, t_j)$ and strictly decreasing on $(t_j, 1]$,
(iii) t_j is non-decreasing in α_j, β_j.
(iv) t_j is strictly increasing in α_j for $j < n$.
(v) t_j is strictly increasing in β_j for $j > 1$.
(d) $t_n < t_1$.

The results in Proposition 2 are mainly technical results, most of which follow directly from the first order conditions of the voters. Proposition 2 c(iii-v) are, however, deserving of further comment. The intuition is that an increase in α_j increases disutility arising from disadvantageous inequity. By increasing the redistributive tax rate, the voter reduces, relatively, the utility of anyone who is richer, hence, reducing disadvantageous inequity. On the other hand, an increase in β_j increases disutility arising from advantageous inequity. An increase in the redistributive tax benefits everyone poorer than the voter relatively more, thus, reducing advantageous inequity.

Recall, from Proposition 1, that voter j, whether fair or selfish, chooses own labor supply so as to maximize own-utility. By contrast, from Proposition 2(c), we see that the tax rate preferred by voter j will depend on his/her fairness parameters, α_j and β_j. We elaborate on this point more fully in Section 3.6, below.

3.6. An explanation of (apparent) inconsistent behavior, based on Fehr-Schmidt preferences

Contrary to the assumption in standard economics, a large and emerging body of empirical evidence clearly suggest that individuals do not have a complete and consistent preference ordering over all states; see, for instance Kahneman and Tversky (2000). We *do not* assume inconsistent preferences. Rather, a decision maker, despite having identical FS-preferences in two different domains, behaves *fairly* in one domain but *selfishly* in the other.

In particular, from Proposition 1, a fair voter, despite having social preferences, chooses labor supply exactly like a selfish voter who does not have social preferences. However, when making a decision on the redistributive tax rate, the same fair voter uses social preferences to choose the tax rate in a manner that the selfish voter does not. In other words, in two separate domains, labor supply and redistributive choice, the fair voter behaves *as if* he had selfish preferences in the first domain and social preferences in the second. We emphasize 'as if' because, of course, the voter has identical underlying social preferences in both domains. This opens up yet another dimension to the literature on inconsistency of preferences. To the best of our knowledge, this point has not been recognized previously.

For example, individuals, when making a private consumption decision might act so as to maximize their selfish interest. But in a separate role as part of the government, as a school governor or as a voter, could act so as to maximize some notion of public well being. Individuals might, for instance, send their own children to private schools (self interest) but could at the same time vote for more funding to government run schools in local or national elections (public interest). Thus, individuals can put on different hats in different situations. Future research might further demonstrate the link between these situations and FS-preferences.

4. Existence of a Condorcet winner when voters have heterogenous social preferences

We now ask if a median voter equilibrium exists when voters have heterogenous social preferences?[17] Tools for answering this question were provided by Black (1948a,b,c).[18] First, we start with some standard concepts and results.

Definition 1 : *(Majority voting) Let* $\prec_j$ *be the strong ordering*[19] *on* $[0,1]$ *defined by* $x \prec_j y$ *if, and only if, voter* j *strictly prefers tax rate* y *to tax rate* x*. Let* $\preceq_j$ *be the corresponding weak ordering. Let* $\preceq$ *be the binary relation on* $[0,1]$ *defined by* $x \preceq y$ *if, and only if, the number of voters such that* $x \preceq_j y$ *is, at least, as great as the number of voters such that* $y \preceq_j x$*. The relation* $\preceq$ *is called majority voting.*

Let t_j be the tax rate most preferred by voter j. The existence and uniqueness of t_j was established by Proposition 2 (bii) and (ci).

Definition 2 : *(Condorcet winner) Let* $\preceq$ *be the majority voting relation (Definition 1). Suppose* $t \preceq t_j$*, for every* $t \in [0,1]$*. Then the tax rate,* t_j*, is called the Condorcet winner. In this case, we also refer to voter* j *as a Condorcet winner.*

Definition 3 : *(Single-peakedness) A set of preferences* $\{\preceq_j \mid j = 1, 2, ..., n\}$ *is single-peaked if the following holds. Let* $x, y, z \in [0,1]$ *such that* $x < y < z$ *or* $z < y < x$ *(i.e.,* y *is strictly between* x *and* z*)*[20]*. Then, for all* j*,* $y \preceq_j x \Rightarrow z \prec_j y$*.*

Lemma 3 : *(Transitivity of majority voting) Suppose that the number of voters is odd and that the set of preferences* $\{\preceq_j \mid j = 1, 2, ..., n\}$ *is single-peaked (Definition 3). Then majority voting,* $\preceq$ *(Definition 1), is transitive.*[21]

[17] For example, when j voters are selfish while $n - j$ voters are fair.

[18] Alternatively see, for example, Black (1958, chapters I-V), Arrow (1963, p46, pp75-80) or Luenberger (1995, section 10.8). For the connections between single-peakedness and more recent concepts see, for example, Gans and Smart (1996).

[19] Recall that $\prec$ is a strong ordering on $X \neq \emptyset$ if it satisfies: (1) for no $x \in X$, $x \prec x$, (2) for all $x, y \in X$ either $x \prec y$ or $y \prec x$ or $x = y$ and (3) $x \prec y$ and $y \prec z \Rightarrow x \prec z$. Given a strong ordering, $\prec$, the corresponding weak ordering, $\preceq$, is defined by: $x \preceq y$ if, and only if, $y \prec x$ is not the case. Hence, (1) for all x, y : $x \preceq y$ or $y \preceq x$ and (2) for all x, y and z : $x \preceq y$ and $y \preceq z \Rightarrow x \preceq z$.

[20] We have taken $<$ to be the usual (strong) ordering of real numbers. But any convenient strong order on the set of feasible tax rates, $[0,1]$, will do.

[21] See Arrow (1963, Theorem 4, p78) or Luenberger (1995, Proposition 10.1, p357). In fact, more can be shown. Under these conditions, Arrow (1963, Theorem 4, p78) shows that majority voting satisfies (1) the Pareto criterion, (2) independence of irrelevant alternatives, (3) monotonicity and (4) no dictatorship. What is *not* satisfied is, of course, (5) unrestricted domain: we only considered single-peaked preferences.

If voters have purely selfish preferences then we can order their most preferred tax rates by their skill levels. The richest voter (the voter with the highest skill level, s_n) prefers the lowest redistributive tax rate, t_n. The poorest voter (the voter with lowest skill level, s_1) prefers the highest tax rate, t_1. All other voters in between are ordered monotonically, i.e., $t_1 > t_2 > ... > t_{n-1} > t_n$.[22] A consequence of introducing fairness concerns among voters, as noted by the experimental results of Ackert et al. (2007), Tyran and Sausgruber (2006) and Bolton and Ockenfels (2006), is that we cannot order the tax rates by skill levels. For instance, it is quite possible that a rich but fair voter prefers a higher tax rate than a middle income but selfish voter. Because tax rates now depend on skill as well as fairness, when tax rates are arranged in descending order they may not correspond to ascending order of skill, as they did for purely selfish voters. As before, let t_{j_k} be the tax rate most preferred by voter j_k, i.e., the tax rate most preferred by the voter with skill level s_{j_k}. Arrange these tax rates in decreasing size: $t_{j_1} \geq t_{j_2} \geq ... \geq t_{j_n}$. Then $(j_1, j_2, ..., j_n)$ will be a permutation of $(1, 2, ..., n)$, with j_k being the image of k under this permutation.

We now use this notation to introduce the remaining definitions and results for this section.

Definition 4 : *(Median tax rate, median voter) Order the voters in decreasing size of their most preferred tax rate t_j: $t_{j_1} \geq t_{j_2} \geq ... \geq t_{j_n}$. Let $\widehat{m} = j_{\frac{1}{2}(n+1)}$. Thus, $t_{\widehat{m}}$ is the median tax rate, $\widehat{m}$ is the median voter (i.e., the voter with skill level $s_{\widehat{m}}$) and $t_{\widehat{m}}$ is the tax rate preferred by the median voter.*[23]

Lemma 4 : *Suppose that the number of voters is odd and that the set of preferences $\{\preceq_j \mid j = 1, 2, ..., n\}$ is single-peaked (Definition 3). Then the median tax rate, $t_{\widehat{m}}$ (equivalently, the median voter, voter $\widehat{m}$), is a Condorcet winner (Definition 2).*[24]

Example 1 : *Suppose that there are three skill levels $s_1 < s_2 < s_3$. Suppose the highest skill level is fair but the other two are selfish. Suppose that the most preferred tax rates of the three voters are, respectively, $t_1 = 0.5$, $t_2 = 0.3$ and $t_3 = 0.4$. Clearly, $t_3 = 0.4$ is the median tax rate. Hence, if preferences are single peaked, then t_3 will be the Condorcet winner. Equivalently, since $t_1 > t_3 > t_2$, we have $j_1 = 1$, $j_2 = 3$ and $j_3 = 2$. The median voter is voter $\widehat{m} = j_{\frac{1}{2}(3+1)} = j_2 = 3$, i.e., the voter with skill level $s_{\widehat{m}} = s_3$: the highest skill voter. If preferences are single peaked, then this highest skill voter will be the Condorcet winner.*

[22] We could also have the case $t_1 > t_2 > ... > t_j = ... = t_n = 0$, for some $j > 1$.

[23] Unless all the inequalities are strict: $t_{j_1} > t_{j_2} > ... > t_{j_n}$, the ordering may not be unique. Hence, the median voter, $\widehat{m}$, may not be unique. However, the median tax rate, $t_{\widehat{m}}$, is unique; and this is what is important.

[24] See, for example, Arrow (1963, p80, third paragraph).

Lemma 5 *: The set of preferences* $\{\preceq_j \mid j = 1, 2, ..., n\}$, *given by Definition 1, is single-peaked (Definition 3).*

Proposition 3 *(Existence of a Condorcet winner): In each pairwise contest, a majority chooses the tax rate that is optimal for the voter with skill level* $s_{\widehat{m}}$ *(Definition 4).*[25,26]

In light of the emerging evidence, it increasingly appears that issues of fairness and concern for others are important human motives that play a significant part in the actual design of redistributive tax policies. Insofar as applications of a direct democracy framework largely use quasi-linear preferences and constant elasticity of labor supply, Proposition 3 establishes the existence of a Condorcet winner when voters have heterogenous social preferences. Hence, the result in Proposition 3 is of potential significance for political economy models that seek to incorporate social preferences.

5. Voter heterogeneity in a three-voting-classes model

Experimental evidence indicates the following:[27] (1) There is a large fraction (roughly 40-60 percent depending on the experiment) of purely self-interested individuals. (2) The behavior of these self-interested individuals accords well with the predictions of the selfish preferences model, even in bilateral interactions. Therefore, an important and interesting issue for theoretical and empirical research is to examine the implications of heterogeneity of preferences in the population. A range of theoretical and experimental work indicates that even a minority of individuals with social preferences can significantly alter the standard predictions[28].

The techniques developed in this paper apply to an n-voter model (for any positive odd integer, n)[29]. However, to keep the analysis simple, we concentrate on a 3-voter economy ($n = 3$). If we can interpret the three voters as being representative voters belonging to three classes: low-income, middle-income and high-income, then our analysis is more than

[25] Note that $\widehat{m}$ need not be equal to $\frac{1}{2}(n+1)$. Hence, $s_{\widehat{m}}$ need not be the median skill level.

[26] Both single-peakedness and single-crossing require that the policy space be unidimensional (totally ordered by, e.g., the tax rate). If, in addition, voters are heterogenous in one dimension only (e.g., skills only), then single-crossing is applicable. In this case, and as Referee 2 pointed out, single-crossing is a more general sufficient condition than single peakedness. However, if voters are heterogeneous in more than one dimension (e.g., skills and fairness), then single-crossing need not be applicable. Dhami and al-Nowaihi (2008) got over this problem by constructing an abstract ordering with the specific aim of proving existence. We are grateful to Referee 1 for suggesting that we use single-peakedness instead.

[27] See for intance, Fehr and Fischbacher (2002).

[28] For one (of many) such result, see, for instance, Fehr et al. (2007). They show that when selfish workers are paired with reciprocal firms who are expected to reward good performance with bonus contracts, then such workers behave (out of pure self interest) as if they have social preferences.

[29] In particular, and as Referee 1 pointed out, analogues of Propositions 4, 5 and Corollary 1 hold for any odd number of voters, n. However, the number of subcases grows exponentially as n increases.

merely illustrative. These classes correspond to the low-skill, middle-skill and high-skill voters and so we shall use the skill and income ranking interchangeably.

In subsection 5.1 we consider the general 3-voter model (α_j, β_j are general). To derive sharper results, in subsection 5.2 we restrict α_j and β_j for fair voters so that $\alpha_j = \alpha$ and $\beta_j = \beta$ (as before, for selfish voters $\alpha_j = \beta_j = 0$).

5.1. General results

Suppose that there is heterogeneity among the voters in the sense that the fairness parameters α_j, β_j, $j = 1, 2, 3$ could be different across the three voters. Proposition 3 guarantees the existence of a Condorcet winner. But there is no guarantee that the median skill voter is decisive in making the redistributive policy choice. Proposition 4, below, establishes that in this case, all three outcomes, i.e., the decisive median voter is respectively a low-income, middle-income and a high-income voter, are possible.

Proposition 4 *:*
(a) If $\frac{S_2^+}{S_2^-} > \frac{\alpha_2 - \alpha_1}{2+2\alpha_1-\beta_2}$ *and* $\frac{S_2^-}{S_2^+} > \frac{\beta_3 - \beta_2}{2+\alpha_2-2\beta_3}$*, then* $t_3 < t_2 < t_1$ *and the median-skill voter is decisive.*
(b) If $\frac{S_2^+}{S_2^-} < \frac{\alpha_2 - \alpha_1}{2+2\alpha_1-\beta_2}$*, then* $t_3 < t_1 < t_2$ *and the low-skill voter is decisive.*
(c) If $\frac{S_2^-}{S_2^+} < \frac{\beta_3 - \beta_2}{2+\alpha_2-2\beta_3}$*, then* $t_2 < t_3 < t_1$ *and the high-skill voter is decisive.*

The results in Proposition 4 are fairly intuitive. Consider, for instance, part (b). If α_2 is high enough then the poorest skill voter becomes decisive.[30] The intuition is that if α_2 is high enough, then the middle income voter derives a huge negative disutility from envy (which arises from having a lower income relative to the high income voter). In order to moderate the effect of envy, the middle income voter's optimal tax rate is even higher than that chosen by the low income voter. Thus, one gets the following ordering of the most preferred tax rates, $t_3 < t_1 < t_2$. In a pairwise contest, the tax rate preferred by the low income voter will defeat the tax rate preferred by any of the other two voters.

Analogously, in part (c), in a pairwise contest, the tax rate preferred by the high income voter will defeat the tax rate preferred by any of the other two voters. Part (a) of the proposition shows the (standard) case where the median skill voter is also the decisive median voter.

These, and other issues, will be discussed in more detail in subsection 5.2, below.

[30] Other factors conducive to this result holding are as follows. Low α_1, high β_2 and low $\frac{S_2^+}{S_2^-}$. The latter is the ratio of advantageous to disadvantageous inequality for voter 2.

5.2. Illustrative cases

For illustrative purposes, we restrict the heterogeneity among fair voters in the following manner:

$$\alpha_j = \alpha \text{ and } \beta_j = \beta,\ j = 1, 2, 3. \tag{5.1}$$

However, for selfish voters $\alpha_j = \beta_j = 0$ (see 3.10), as before. Hence, there is intra-group homogeneity of preferences within the groups of fair and selfish voters but inter-group heterogeneity across the two groups. A second source of heterogeneity across all voters is, of course, the level of skill.

For the n-voter model, there are 2^n cases to consider. Here, there are $2^3 = 8$ possible combinations of voters. Denoting by S and F respectively, a selfish and a fair voter, the 8 possible combinations of the voters (each combination arranged in order of increasing skill level from left to right) are: SSS, SSF, SFS, FSS, SFF, FSF, FFS, FFF.[31] Since the experimental evidence is that 1/3 to 2/3 of the participants are selfish, the most important cases are SSF, SFS, FSS, SFF, FSF, FFS.

In any mixture of the two types of voters, the redistributive outcome is altered if and only if, relative to the case of purely selfish or purely fair voters, the identity of the median voter alters. The various cases are examined in Proposition 5, below, which is a restatement of Proposition 4 applied to the particular type of economy discussed here.

To facilitate exposition, use (3.30), (3.31) to define two new constants θ_1, θ_2:

$$\theta_1 = \frac{S_2^+}{S_2^-} = \frac{s_2^{1+\epsilon} - s_1^{1+\epsilon}}{s_3^{1+\epsilon} - s_2^{1+\epsilon}},\ \theta_2 = \frac{2}{2+\theta_1}. \tag{5.2}$$

The constant θ_1 is, for the three voter case, directly proportional to the ratio of advantageous to disadvantageous inequality for voter 2. θ_2 depends inversely on θ_1.

Proposition 5 *: For the three voter model,*
(a) The median-skill voter is decisive, and $t_3 < t_2 < t_1$, in the following cases:
(i) SSS, FSS, FFS and FFF,
(ii) SFS and SFF, if $\frac{\alpha}{2-\beta} < \theta_1$,
(iii) SSF and FSF, if $\beta < \theta_2$,
(b) The low-skill voter is decisive, and $t_3 < t_1 < t_2$, in cases SFS and SFF, if $\theta_1 < \frac{\alpha}{2-\beta}$,
(c) The high-skill voter is decisive, and $t_2 < t_3 < t_1$, in cases SSF and FSF, if $\theta_2 < \beta$.

In cases SSS, FSS, FFS and FFF the median-skill voter is decisive in the redistributive tax choice (Proposition 5 a(i)). Hence, in these four cases, the redistributive outcome in the case of a mixture of voter types is identical to an economy in which all voters are

[31] For instance, SFF denotes an economy in which the lowest skill voter is selfish and the middle and high skill voters are both fair.

of the same type as the median-skill voter. So, for instance, the redistributive outcome in the FSS economy is the same as that in a SSS economy while that in the FFS economy is identical to the FFF economy.

The same is also true for the other four cases SFS, SFF, SSF and FSF provided that the relevant upper bounds on α and/or β are satisfied (Proposition 5a(ii) and a(iii)). In each of these eight cases, the tax rate implemented is the tax rate, t_2, preferred by the median-skill voter.

However, if the upper bounds in Proposition 5 a(ii) and a(iii) are not satisfied, then the median-skill voter is no more decisive. In cases SFS and SFF, the low-skill (and selfish) voter becomes decisive, and the implemented tax rate is t_1, (Proposition 5b). In cases SSF and FSF the high skill (and fair) voter becomes decisive, and the implemented tax rate is t_3, (Proposition 5c). Note that in all cases, *the implemented tax rate is the median tax rate and not, necessarily, the tax rate preferred by the median-skill voter.*

Before we proceed to discuss the intuition, consider the four particularly striking cases of the following corollary to Proposition 5

Corollary 1 : *(a) For SFS, the majority of voters are* selfish *yet, if $\frac{\alpha}{2-\beta} < \theta_1$, then a majority vote for the tax rate preferred by the median-skill* fair *voter (Proposition 5a(ii)).*
(b) For SFF, the majority of voters are fair *yet, if $\theta_1 < \frac{\alpha}{2-\beta}$, then a majority vote for the tax rate preferred by the low-skill* selfish *voter (Proposition 5b).*
(c) For SSF, the majority of voters are selfish *yet, if $\theta_2 < \beta$, then a majority vote for the tax rate preferred by the high-skill* fair *voter (Proposition 5c).*
(d) For FSF, the majority of voters are fair *yet, if $\theta_2 < \beta$, then a majority vote for the tax rate preferred by the median-skill* selfish *voter (Proposition 5 a(iii)).*

In order to understand the intuition behind the results more fully, consider the SFS economy when the restriction $\theta_1 < \frac{\alpha}{2-\beta}$ is satisfied (see Proposition 5b). We show the relevant information in Figure 5.1.

Denote by t_j^S and t_j^F to be the most preferred tax rates of voter j if he is respectively, selfish and fair.[32] From Proposition 5a(i), in an SSS economy, the median skill voter is decisive and so $t_3^S < t_2^S < t_1^S$. This is shown in figure 5.1 by the curve labelled *abc*.

Now suppose that we replace the representative *selfish* median skill voter in an SSS economy by a representative *fair* voter to generate the SFS economy. Since we assume that the inequality $\theta_1 < \frac{\alpha}{2-\beta}$ holds, from Proposition 5b, for the SFS economy, we get the ranking of most preferred tax rates to be $t_3^S < t_1^S < t_2^F$. This is shown in figure 5.1 by the curve *adc*. The tax rate of the low skill voter, t_1^S, is now able to defeat any of the other two tax rates in a pairwise contest (either of the other two classes of voters finds it optimal to

[32]These are the tax rates that would maximize (3.34) for respectively, a selfish and fair voter.

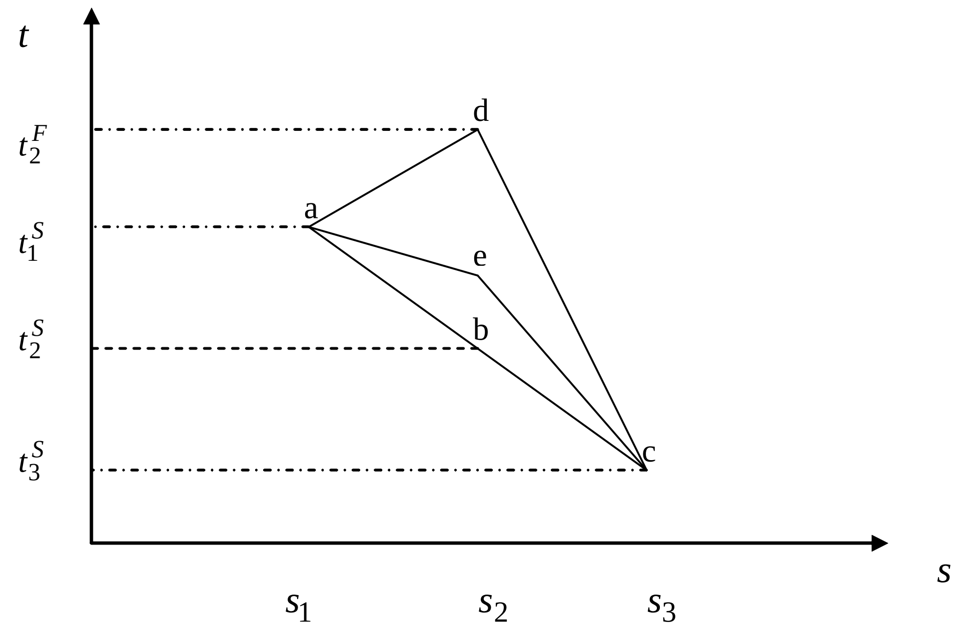

Figure 5.1: The SSS and SFS economies

align with the low skill voter against a tax proposal of the third voter). The optimal tax rate of the lowest skill voter becomes the decisive redistributive choice. Hence, in moving from the SSS to the SFS economy, there is a potentially large jump in the tax rate from t_2^S to t_1^S.

If the restriction $\theta_1 < \frac{\alpha}{2-\beta}$ is not satisfied, say, $\frac{\alpha}{2-\beta} < \theta_1$, then we get the case in Proposition 5a(ii). In this case, the ranking of tax rates, for the SFS economy, is $t_3^S < t_2^F < t_1^S$ and so the median skill voter is also the decisive median voter. In the figure, this ranking of tax rates is shown by the curve *aec*.

From Proposition 5b, the crucial inequality in this case is $\theta_1 < \frac{\alpha}{2-\beta}$. To see why this inequality is important, suppose that there is pairwise voting between the two tax rates, t_1^S, t_2^F. For the high skill voter (whose skill level is s_3) to prefer the tax rate of the low skill voter, t_1^S, the optimal tax rate of the middle skill voter, t_2^F, should be 'too high' in the sense that $t_1^S < t_2^F$. In this case, a majority (the rich and the poor) will prefer the tax rate t_1^S to t_2^F.[33] The following factors help to satisfy the inequality $\theta_1 < \frac{\alpha}{2-\beta}$, and, hence, are conducive to t_2^F being 'too high' as we now show.

(a) High inequity aversion, as captured by the magnitudes of α, β: From Proposition 2c, higher magnitudes of the inequity aversion parameters, α, β, increase the optimal tax rate

[33] Analogously, in a pairwise contest between the tax rates t_1^S, t_3^S a majority (the middle class and the poor) prefer the tax rate t_1^S to t_3^S.

for fair voters and so increase t_2^F.
(b) High skill inequality at the upper end of the skill distribution $(s_3 - s_2)$ and low inequality at the lower end of the income distribution $(s_2 - s_1)$ reduce θ_1 and increase t_2^F: To see, this, suppose that $s_3 - s_2$ increases because of an increase in s_3. Selfish voters would like to redistribute more when the rich get richer because average incomes increase and so the lumpsum available for redistribution is higher. Fair voters have an additional motive to redistribute more, namely, that it reduces disadvantageous inequity. This facilitates an increase in t_2^F.
For selfish voters, a decrease in the low skill level reduces the redistributive tax rate. The intuition is that reduction in low skill income reduces average income available for redistribution, hence, reducing the marginal benefits of increasing the tax rate. For fair voters, however, the results can go either way. The reason is that on the one hand, the fair voter is influenced by very similar considerations to the selfish voter (because the fair voter also cares about 'own' payoff). However, on the other hand, the empathy/concern for poorer voters, on account of the disutility arising from advantageous inequity induces the fair voter in the opposite direction i.e. greater redistribution. The interplay between these two opposing factors determines if the fair voter will respond, unlike the selfish voter, by redistributing more in response to poverty. The restriction $\theta_1 < \frac{\alpha}{2-\beta}$ ensures that overall, one gets the ranking $t_3^S < t_1^S < t_2^F$ for the optimal tax rates of the three voting classes.

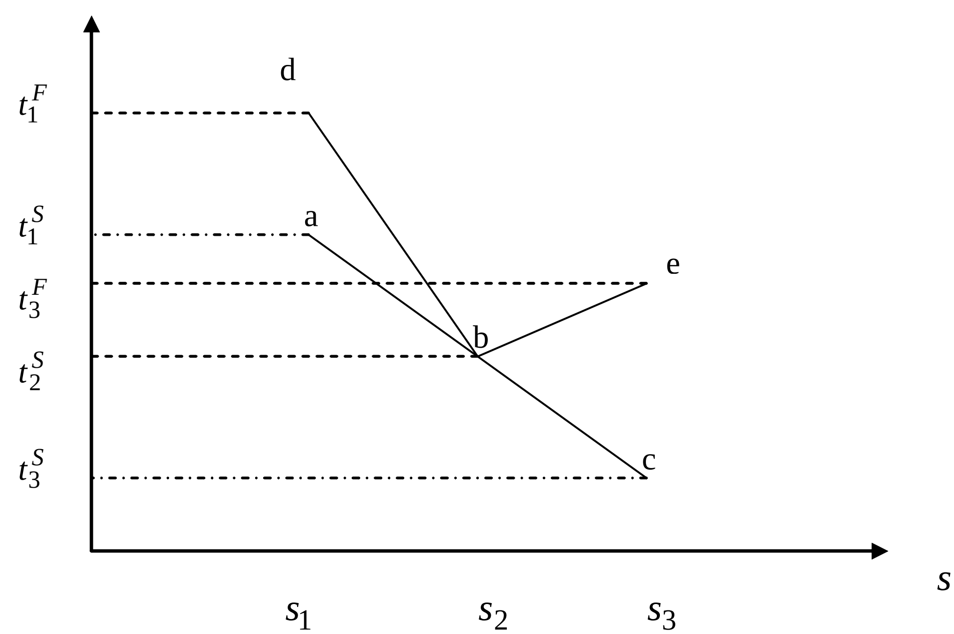

Figure 5.2: The SSS, SSF, and FSF economies

In Figure 5.2, we illustrate the second generic case when $\beta > \theta_2$ (see Proposition 5c). The benchmark SSS economy is represented by the line abc. For the SSS economy, we know from Proposition 5a(i), that the median skill voter, of skill s_2, is decisive and so $t_3^S < t_2^S < t_1^S$. Under the restriction specified in Proposition 5c ($\beta > \theta_2$), the economy SSF is shown in Figure 5.2 by the curve *abe* and one gets the ordering of tax rates to be $t_2^S < t_3^F < t_1^S$. In this case, the rich but fair voter is decisive because the proposed tax rate, t_3^F, can beat any of the other two proposed tax rates in a pairwise contest.

Under the restriction $\beta > \theta_2$, the FSF economy is also shown in Figure 5.2, by the curve *dbe*. The decisive median voter is again the rich-fair voter, who is not the median skill voter.

Why does the restriction $\beta > \theta_2$ not involve α? The reason is that for the cases shown in Figure 5.2 to arise, we need the most preferred tax rate of the rich-fair voter to be high enough, in the sense that $t_2^S < t_3^F$. The rich-fair voter faces no disadvantageous inequality, hence, the absence of the parameter α in the critical condition. However, if β is high enough, then the rich-fair voter suffers sufficiently highly from advantageous inequality with respect to the other two class of voters. Raising the tax rate, which ensures that the utility of the rich is affected relatively more as compared to the poorer voting classes is the optimal outcome. In particular, if $\beta > \theta_2$ then $t_2^S < t_3^F$, which leads to the surprising outcome in Proposition 5c.

6. Conclusions

We replace the self-interested voters (*selfish voters*) in the Romer-Roberts-Meltzer-Richard (RRMR) framework with voters who have the Fehr-Schmidt (1999) preferences for fairness (*fair voters*) and ask the following questions. Does a median voter equilibrium exist? What are the features of the equilibrium redistributive policy when there is a mixture of fair and selfish voters?

Some of our findings are as follows. We demonstrate the existence of a Condorcet winner[34] when n voters (where n is any odd positive integer) have heterogenous social preferences. Other things remaining fixed, greater fairness induces greater redistribution. This is true in two senses. In the first, a greater fairness preference of the existing fair voters increases redistribution. In the second, an increase in the proportion of fair voters relative to selfish voters may also increase redistribution. When voters differ in their degree of fairness, then the Condorcet winner need not necessarily be the one with the median skill level. Changes in the composition of the types (selfish or fair) of a minority of voters can induce large redistributive consequences.

[34] A Condorcet winner in our context is the unique redistributive tax policy that, under majority voting, wins in a pairwise contest over all other feasible tax policies.

The implications of having a mixture of selfish and fair voters within the same economy has interesting, important and hitherto unnoticed implications.[35] A flavour of the results is as follows. For the three voting classes model, and for the 6 interesting cases[36], parameter configurations allow changes involving no more than a third of the voters to have potentially large redistributive effects. In each of these four cases (SFS, SFF, SSF, FSF)[37] the Condorcet winner need not be the median skill voter. For instance, introducing selfish poor (fair rich) voters in an economy populated by fair (selfish) voters can result in a *reduction* (*increase*) in redistribution from rich to poor. On the other hand, introducing rich selfish (poor fair) voters in an economy populated by fair (selfish) voters will have no effect. Fair voters, when voting, exhibit a preference for fairness while in another domain, say, labor choice, they behave as if they have selfish preferences. We provide a resolution of this apparently inconsistent behavior.

Dhami and al-Nowaihi (2008), sketch some potential applications of this framework to the size of the welfare state, regional integration and issues of culture, identity and immigration. These may have potentially important implications that future research could explore.

7. Proofs

Proof of Proposition 1[38]: Let A_j, D_j and ω_{ji} be as in Remark 1. Consider voter j. In the second stage, voter j maximizes his/her own FS-utility $U_j(l_j; \mathbf{l}_{-j}, t, b, \alpha, \beta, \mathbf{s})$, recall (3.13), given the vector, $\mathbf{l}_{-j}$, of labor supplies of all other voters (recall subsection 3.2). Hence $u_i(l_i; t, b, s_i), i \neq j$, are fixed numbers. Since $u_j(l_j; t, b, s_j)$ is continuous in l_j, and since $\max\{0, x\}$ is continuous in x, it follows that $U_j(l_j; \mathbf{l}_{-j}, t, b, \alpha, \beta, \mathbf{s})$, as given by (3.13), is a continuous function of $l_j \in [0, 1]$. Hence, $U_j(l_j; \mathbf{l}_{-j}, t, b, \alpha, \beta, \mathbf{s})$ attains a maximum at some $l_j^* \in [0, 1]$. From (3.16), it follows that

$$U_j\left(l_j^*; \mathbf{l}_{-j}, t, b, \alpha, \beta, \mathbf{s}\right) = \omega_{jj} u_j\left(l_j^*; t, b, s_j\right) + \sum_{i \in A_j} \omega_{ji} u_i(l_i; t, b, s_i) + \sum_{k \in D_j} \omega_{jk} u_k(l_k; t, b, s_k).$$

We shall argue that l_j^* must maximize own-utility, $u_j(l_j; t, b, s_j)$. Suppose l_j^* does not maximize own-utility $u_j(l_j; t, b, s_j)$. Then we can find an l_j^{**}, sufficiently close to l_j^* (by the continuity of u_j), so that $u_j\left(l_j^{**}; t, b, s_j\right) > u_j\left(l_j^*; t, b, s_j\right)$ and the set D_j is unchanged (by the continuity of u_j and the definition of D_j). Hence, the set A_j is also unchanged, since

[35] We give the results for three voting classes, however, the results are in principle capable of being extended to more than three classes.

[36] The case of all selfish and all fair voters (which comprise the remaining two cases) are less interesting because experimental evidence indicates that about 40%-60% of subjects are selfish.

[37] The symbols S, F respectively denote a selfish and a fair voter. The voters are arranged in increasing levels of skills.

[38] Added at the request of Referee 2.

A_j is the complement of D_j. Then

$$U_j\left(l_j^{**};\mathbf{l}_{-j},t,b,\alpha,\beta,\mathbf{s}\right)=\omega_{jj}u_j\left(l_j^{**};t,b,s_j\right)+\sum_{i\in A_j}\omega_{ji}u_i\left(l_i;t,b,s_i\right)+\sum_{k\in D_j}\omega_{jk}u_k\left(l_k;t,b,s_k\right).$$

Hence, $U_j\left(l_j^{**};\mathbf{l}_{-j},t,b,\alpha,\beta,\mathbf{s}\right) > U_j\left(l_j^{*};\mathbf{l}_{-j},t,b,\alpha,\beta,\mathbf{s}\right)$, which cannot be, since l_j^* maximizes $U_j\left(l_j;\mathbf{l}_{-j},t,b,\alpha,\beta,\mathbf{s}\right)$. ■

Proof of Lemma 1 (derivation of labor supply): From (3.23) we see that, given t,b and s_j, $u\left(l_j;t,b,s_j\right)$ is a continuous function of l_j on the non-empty compact set $[0,1]$. Hence, a maximum exists. From (3.23), we also get

$$\begin{aligned}\frac{\partial u}{\partial l_j}\left(l_j;t,b,s_j\right) &= (1-t)\,s_j-l_j^{\frac{1}{\epsilon}}, &(7.1)\\ \frac{\partial u}{\partial l_j}\left(0;t,b,s_j\right) &= (1-t)\,s_j, &(7.2)\\ \frac{\partial u}{\partial l_j}\left(1;t,b,s_j\right) &= (1-t)\,s_j-1, &(7.3)\\ \frac{\partial^2 u}{\partial l_j^2}\left(l_j;t,b,s_j\right) &= -\frac{1}{\epsilon}l_j^{\frac{1-\epsilon}{\epsilon}}. &(7.4)\end{aligned}$$

First, consider the case $t=1$. From (3.23), or (7.1), we see that $u\left(l_j;1,b,s_j\right)$ is a strictly decreasing function of l_j for $l_j>0$. Hence, the optimum must be

$$l_j=0 \text{ at } t=1. \qquad (7.5)$$

Now, suppose $t\in[0,1)$. From (3.1), (3.2), (7.2) and (7.3) we get that $\frac{\partial u}{\partial l_j}\left(0;t,b,s_j\right)>0$ and $\frac{\partial u}{\partial l_j}\left(1;t,b,s_j\right)<0$. Hence an optimal value for l_j must lie in $(0,1)$ and, hence, must satisfy $\frac{\partial u}{\partial l_j}\left(l_j;t,b,s_j\right)=0$. From (7.1) we then get

$$l_j=(1-t)^{\epsilon}\,s_j^{\epsilon}, \qquad (7.6)$$

which, therefore, must be the unique optimal labor supply (this also follows from (7.4)). For $t=1$, (7.6) is consistent with (7.5). Hence, for each consumer, j, (7.6) gives the optimal labor supply for each $t\in[0,1]$. ■

Proof of Lemma 2 (Properties of the indirect utility function corresponding to own-utility): The proof follows from (3.28) by direct calculation. ■

Proof of Proposition 2 (existence of optimal tax rates)[39]: (a) From (3.34), we see that $W_j\left(t,\alpha_j,\beta_j,\mathbf{s}\right)$ is a continuous function of $t\in[0,1]$, for $j=1,2,...,n$. Hence, $W_j\left(t,\alpha_j,\beta_j,\mathbf{s}\right)$ attains a maximum at some $t_j\in[0,1]$. We will now show that $t_j<1$. From (3.34), we get

$$W_j\left(1,\alpha_j,\beta_j,\mathbf{s}\right)=0, \qquad (7.7)$$

[39] We are grateful to Referees 1 and 2 for alerting us to errors in our draft proof of Proposition 2, and for suggesting simplifications.

$$W_j\left(0,\alpha_j,\beta_j,\mathbf{s}\right)=\frac{\phi_j}{1+\epsilon}. \tag{7.8}$$

If $\phi_j>0$ then, from (7.7) and (7.8), we get $W_j\left(0,\alpha_j,\beta_j,\mathbf{s}\right)>W_j\left(1,\alpha_j,\beta_j,\mathbf{s}\right)$. Hence, $t_j<1$. Now, suppose $\phi_j\leq 0$. Let

$$t_0=\frac{1}{2}\left[1+\frac{-\phi_j}{(1+\epsilon)\overline{S}-\phi_j}\right]. \tag{7.9}$$

Clearly, $0<t_0<1$. Substituting for $t=t_0$ from (7.9) into (3.34), gives

$$W_j\left(t_0,\alpha_j,\beta_j,\mathbf{s}\right)=\frac{1}{4}\left(\frac{(1+\epsilon)}{(1+\epsilon)\overline{S}-\phi_j}\right)^{\epsilon}\left(\overline{S}\right)^{1+\epsilon}>0. \tag{7.10}$$

From (7.7) and (7.10) we get, again, that $t_j<1$.

(b) From (3.34), for the case $0\leq t<1$, we get

$$\begin{aligned}\frac{\partial W_j}{\partial t}\left(t,\alpha_j,\beta_j,\mathbf{s}\right) &= -\epsilon t(1-t)^{\epsilon-1}\overline{S}+(1-t)^{\epsilon}\left(\overline{S}-\phi_j\right), && (7.11)\\ &= (1-t)^{\epsilon-1}\left[(1+\epsilon)\overline{S}-\phi_j\right]\left[\frac{\overline{S}-\phi_j}{(1+\epsilon)\overline{S}-\phi_j}-t\right], && (7.12)\end{aligned}$$

(bi) If $\overline{S}-\phi_j\leq 0$, we see, from (7.11), that $\frac{\partial W_j}{\partial t}<0$ for all $t\in[0,1)$.

(bii) From (bi) it follows that the optimal tax rate for voter j must be $t_j=0$.

(ci and cii) If $\overline{S}-\phi_j>0$, we see, from (7.11), that $\frac{\partial W_j}{\partial t}>0$ at $t=0$. Hence, the optimal tax rate, t_j, for voter j satisfies $t_j>0$. Combining this with $t_j<1$ (from (a)), we get that, necessarily, $\frac{\partial W_j}{\partial t}=0$ at $t=t_j$. From (7.12) we then get $t_j=\left[\overline{S}-\phi_j\right]/\left[(1+\epsilon)\overline{S}-\phi_j\right]$, establishing (3.35). From (7.12) we also get that W_j is strictly increasing on $[0,t_j)$ and strictly decreasing on $(t_j,1]$. This establishes part (cii) and shows that t_j is the unique global maximum of W_j, completing the proof of part (ci).

Assume $\overline{S}-\phi_j>0$. Differentiating (3.35) with respect to α_j, β_j we get:

$$\frac{\partial t_j}{\partial\alpha_j}=\frac{\epsilon\overline{S}S_j^-}{\left[(1+\epsilon)\overline{S}-\phi_j\right]^2}\geq 0 \text{ (and } >0 \text{ for } S_j^->0), \tag{7.13}$$

$$\frac{\partial t_j}{\partial\beta_j}=\frac{\epsilon\overline{S}S_j^+}{\left[(1+\epsilon)\overline{S}-\phi_j\right]^2}\geq 0 \text{ (and } >0 \text{ for } S_j^+>0). \tag{7.14}$$

(ciii) From (7.13) and (7.14), we see that t_j is non-decreasing in α_j, β_j.

(civ) From (3.30) we know that $S_j^->0$ for $j<n$. Hence, from (7.13), t_j is strictly increasing in α_j for $j<n$.

(cv) From (3.31) we know that $S_j^+>0$ for $j>1$. Hence, from (7.14), t_j is strictly increasing in β_j for $j>1$.

(d) From (3.32), we have

$$\phi_1 = s_1^{1+\epsilon} - \alpha_1 S_1^-, \tag{7.15}$$

$$\phi_n = s_n^{1+\epsilon} - \beta_n S_n^+. \tag{7.16}$$

From (3.1), (3.25), (3.10), (3.11), (3.30) and (7.15), we get

$$\overline{S} - \phi_1 = \overline{S} - s_1^{1+\epsilon} + \alpha_1 S_1^- > 0. \tag{7.17}$$

From (3.1), (3.10), (3.11), (3.30), (3.31), (7.15) and (7.16)

$$\begin{aligned}
\phi_n - \phi_1 &= s_n^{1+\epsilon} - \beta_n S_n^+ - \left[s_1^{1+\epsilon} - \alpha_1 S_1^-\right], \\
&= s_n^{1+\epsilon} - s_1^{1+\epsilon} - \beta_n S_n^+ + \alpha_1 S_1^-, \\
&> s_n^{1+\epsilon} - s_1^{1+\epsilon} - S_n^+, \\
&= s_n^{1+\epsilon} - s_1^{1+\epsilon} - \left[\frac{1}{n-1}\sum_{i<n}\left(s_n^{1+\epsilon} - s_i^{1+\epsilon}\right)\right], \\
&= \frac{1}{n-1}\left[(n-1)\left(s_n^{1+\epsilon} - s_1^{1+\epsilon}\right) - \sum_{i<n}\left(s_n^{1+\epsilon} - s_i^{1+\epsilon}\right)\right], \\
&> \frac{1}{n-1}\left[(n-1)\left(s_n^{1+\epsilon} - s_1^{1+\epsilon}\right) - \sum_{i<n}\left(s_n^{1+\epsilon} - s_1^{1+\epsilon}\right)\right], \\
&= \frac{1}{n-1}\left[(n-1)\left(s_n^{1+\epsilon} - s_1^{1+\epsilon}\right) - (n-1)\left(s_n^{1+\epsilon} - s_1^{1+\epsilon}\right)\right], \\
&= 0.
\end{aligned}$$

Hence,

$$\phi_n - \phi_1 > 0. \tag{7.18}$$

From (7.17) and Proposition 2(ci), we get $t_1 > 0$ and

$$t_1 = \frac{\overline{S} - \phi_1}{(1+\epsilon)\overline{S} - \phi_1}. \tag{7.19}$$

If $\overline{S} - \phi_n \leq 0$ then, from Proposition 2(bii), it follows that $t_n = 0$. Hence, $t_n < t_1$. Now, suppose $\overline{S} - \phi_n > 0$. It then follows, from Proposition 2(ci) that

$$t_n = \frac{\overline{S} - \phi_n}{(1+\epsilon)\overline{S} - \phi_n}. \tag{7.20}$$

From (7.19) and (7.20), we get

$$\begin{aligned}
t_1 - t_n &= \frac{\overline{S} - \phi_1}{(1+\epsilon)\overline{S} - \phi_1} - \frac{\overline{S} - \phi_n}{(1+\epsilon)\overline{S} - \phi_n}, \\
&= \frac{\epsilon\overline{S}\left[\phi_n - \phi_1\right]}{\left[(1+\epsilon)\overline{S} - \phi_1\right]\left[(1+\epsilon)\overline{S} - \phi_n\right]}. \qquad (7.21)
\end{aligned}$$

From (7.18) and (7.21), we get that $t_n < t_1$. ■

Proof of Lemma 5: Let $x, y, z \in [0,1]$ such that $x < y < z$ and let $y \preceq_j x$. If $\overline{S} - \phi_j \leq 0$, then $W_j\left(t, \alpha_j, \beta_j, \mathbf{s}\right)$ is strictly decreasing for $t \in [0,1]$ (Proposition 2(bi)). Since $y < z$, it follows that $z \prec_j y$. Now suppose that $\overline{S} - \phi_j > 0$. Then the tax rate, t_j, preferred by voter j, is unique, satisfies $0 < t_j < 1$ and $W_j\left(t, \alpha_j, \beta_j, \mathbf{s}\right)$ is strictly increasing on $[0, t_j)$ and strictly decreasing on $(t_j, 1]$ (Proposition 2(ci and cii)). If $y \in [0, t_j]$, then it would follow that $x \prec_j y$. But this is not the case, since $y \preceq_j x$. Hence, $y \in (t_j, 1]$. Since $y < z$, it follows, again, that $z \prec_j y$. Similarly, we can show that if $x, y, z \in [0,1]$ such that $z < y < z$, then, for all j, $y \preceq_j x \Rightarrow z \prec_j y$. ■

Proof of Proposition 3 (existence of a Condorcet winner): By assumption the number of voters, n, is odd. By Lemma 5, the set of preferences, $\{\preceq_j \mid j = 1, 2, ..., n\}$, is single-peaked. Hence, by Lemma 4, the median voter, voter $\widehat{m}$, is a Condorcet winner. ■

Proposition 4 follows from Lemma 6, below, (and is actually equivalent to it).

Lemma 6 *:*
(a) If $\phi_1 < \phi_2 < \phi_3$, then $t_3 < t_2 < t_1$ and the median-skill voter is decisive.
(b) If $\phi_1 > \phi_2$, then $t_3 < t_1 < t_2$ and the low-skill voter is decisive.
(c) If $\phi_2 > \phi_3$, then $t_2 < t_3 < t_1$ and the high-skill voter is decisive.

Proof of Lemma 6:[40] From (3.1), (3.10), (3.11), (3.25), (3.26), (3.30), (3.31) and (3.32), we get:

$$\overline{S} - \phi_1 = \frac{1}{6}\left(2 + 3\alpha_1\right)\left(s_2^{1+\epsilon} + s_3^{1+\epsilon} - 2s_1^{1+\epsilon}\right) > 0, \tag{7.22}$$

$$\overline{S} - \phi_2 = \frac{1}{6}\left[2\left(s_1^{1+\epsilon} + s_3^{1+\epsilon} - 2s_2^{1+\epsilon}\right) + 3\alpha_2\left(s_3^{1+\epsilon} - s_2^{1+\epsilon}\right) + 3\beta_2\left(s_2^{1+\epsilon} - s_1^{1+\epsilon}\right)\right] > 0, \tag{7.23}$$

$$\overline{S} - \phi_3 = \frac{2s_3^{1+\epsilon} - s_1^{1+\epsilon} - s_2^{1+\epsilon}}{6}\left(3\beta_3 - 2\right) \lesseqqgtr 0 \Leftrightarrow \beta_3 \lesseqqgtr \frac{2}{3}. \tag{7.24}$$

From (7.22), (7.23), (7.24) and Proposition 2, we get:

$$t_1 = \frac{\overline{S} - \phi_1}{(1+\epsilon)\,\overline{S} - \phi_1} > 0, \tag{7.25}$$

$$t_2 = \frac{\overline{S} - \phi_2}{(1+\epsilon)\,\overline{S} - \phi_2} > 0, \tag{7.26}$$

$$\beta_3 \leq \frac{2}{3} \Rightarrow t_3 = 0, \tag{7.27}$$

$$\beta_3 > \frac{2}{3} \Rightarrow t_3 = \frac{\overline{S} - \phi_3}{(1+\epsilon)\,\overline{S} - \phi_3} > 0. \tag{7.28}$$

Parts (a), (b) and (c) then follow from (7.25)-(7.28). ■

[40] We are extremely grateful for the input of Referee 1 into this proof.

Proof of Proposition 4: Proposition 4 follows from (3.30), (3.31), (3.32) and Lemma 6. ■

Proof of Proposition 5: Using (5.2), parts (a), (b) and (c) follow, by simple calculations, from parts (a), (b) and (c) of Proposition 4, respectively. ■

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