Optimal taxation in the presence of tax evasion: Expected utility versus prospect theory.

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3 September 2009

Abstract

It is now well known that the predictions of expected utility theory (EUT) applied to tax evasion are quantitatively and qualitatively at variance with the evidence. Furthermore, prospect theory (PT) makes the correct predictions. In this literature the tax rate is exogenous. We endogenize the tax rate, and require that a successful theory should explain, jointly, the facts on the tax rate, tax gap and the level of government expenditure. When taxpayers have behavioural biases, misperceptions, and/or context dependent preferences, is the data best described by the government respecting these (liberalism) or correcting for these (paternalism)? We find that the data is best described by taxpayers using PT and the government using standard utility theory, EUT. The results are robust to several possible alternative specifications.

Keywords: Prospect theory, Expected utility theory, Tax evasion, Optimal taxation, Normative versus positive economics, Context dependent preferences, Liberalism, Paternalism.

JEL Classification: D81 (Criteria for Decision Making Under Risk and Uncertainty), H26 (Tax Evasion), K42 (Illegal Behavior and the Enforcement of Law).

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"*Even when an agent is perfectly rational in the sense that he systematically maximizes some function, it is not at all obvious that the utility function which explains his behavior should be inserted into the welfare considerations ... More generally, once psychological effects enter into the calculus, there is no escape from separating welfare and behavior.*" Rubinstein (2005).

"*paternalism* [is the] *power and authority one person or institution exercises over another to confer benefits or prevent harm for the latter regardless of the latter's informed consent ... paternalism ... increases the potential for the abuse of state power, arbitrary discrimination, tyranny, and civil strife ... liberalism ... provides the only way for the proponents of conflicting ways of life to live together ... liberalism* [is] *the only viable basis for peaceful coexistence in culturally and religiously plural societies.*" The Oxford Companion to Philosophy (2005, pp 515, 648).

"*... the tax context produces a rather unique behavioral framework which arguably requires different theoretical approaches.*" Graetz and Wilde (2001, pp 358, 359).

1. Introduction

Issues of tax evasion are extremely important for all countries. Losses to society from tax evasion are huge. For the USA, for example, based on the most recent data, the *tax gap*[1] is of the order of $300 billion per year (Slemrod, 2007).[2] An important feature of the existing analysis of tax evasion is that it is largely carried out in an expected utility theory (EUT) framework.

Recent research points to several serious problems in using an EUT approach to tax evasion. Dhami and al-Nowaihi (2007) apply Kahneman and Tversky's (1979) prospect theory[3] (PT) to the tax evasion decision facing a taxpayer. They show that while EUT gives the correct qualitative results for the effects of the probability of detection and the penalty rate, there are several problems. First, EUT makes the prediction that under reasonable attitudes to risk, namely, non-increasing absolute risk aversion, the taxpayer evades less as the tax rate goes up. The implication is that tax evasion will be at a minimum when the tax rate is 100 percent. This result, due to Yitzhaki (1974), is contradicted by the bulk of empirical evidence. Second, at existing penalty rates and detection probabilities, the quantitative predictions of EUT on the extent of tax evasion are wrong by a factor of

[1] The *tax gap* is the difference between the amount owed in taxes and the amount actually collected by the tax authorities. Income tax accounts for about two thirds of this gap; see Slemrod (2007).

[2] To this has to be added the cost of enforcement, the cost of misallocation of resources due to workers and firms diverting their efforts to less productive but easier to evade activities and the cost due to distorted prices, in particular between privately and publicly provided goods and services.

[3] The standard references for prospect theory are Kahneman and Tversky (1979) (which, incidentally, is the most cited Econometrica paper) and Tversky and Kahneman (1992). Section 4, below, gives a self-contained exposition of prospect theory.

about 100. On the other hand, PT gives the correct quantitative and qualitative results.[4] Given the magnitudes involved, the misleading welfare consequences of applying EUT to an analysis of tax evasion are potentially very large.

Dhami and al-Nowaihi (2007), however, treat the tax rate as exogenous. In this paper, we extend the analysis of Dhami and al-Nowaihi (2007) by making the tax rate endogenous. This, in turn, requires the specification of an objective function for the government. There are two standard approaches to the latter problem. The first is to select a positive political economy model that seeks to accurately describe actual government behavior. A difficulty in implementing this approach arises from the multifarious government objectives and the complexity of the constraints facing it. An additional difficulty is that there is no generally agreed upon positive model of government behavior. The second, but also standard approach, is to select on normative grounds a simple social welfare function for the government, then ask the question 'to what extent do the predictions conform to the stylized facts?' We follow the second approach. Although normative criteria considerably reduce the set of possible social welfare functions, they do not determine a unique social welfare function. Therefore, we consider three very different regimes (regimes PT1, PT2 and PT3, see Subsection 2.4). We find that our results are robust across these regimes. We also explain why we expect that a more general model would give substantially similar results (Subsection 5.1).

1.1. A brief description of the model

Our framework of analysis is as follows. We consider a model where the government levies taxes to finance public provision of goods and services. Individuals can choose to evade a fraction of their income. The government audits a fraction of the tax returns. If a taxpayer is caught evading, he pays back owed tax plus a penalty. Individuals gain utility from both private and public consumption. The government chooses the optimal tax rate, given society's preference between private and public expenditure, and taking the subsequent tax evasion behavior of taxpayers into account. In this simple framework, we assess the relative success of EUT and PT. We find that PT far outperforms EUT. We also highlight the importance of recognizing that preferences are context dependent.

1.2. Brief literature review

The literature on endogenous evasion and optimal taxation is fairly limited and, without exception, uses EUT. There are three main strands of the literature.

[4] But these are not the only problems that PT can rectify in the context of tax evasion. Empirical and experimental evidence show that *obligatory advance tax payments* reduce tax evasion, a fact that can be explained by PT but not by EUT; see Elffers and Hessing (1997) and Yaniv (1999).

The first strand is exemplified in the work of Reinganum and Wilde (1985)[5], Cremer and Gahvari (1996), Marhuenda and Ortuno-Ortin (1997) and Chandar and Wilde (1998). Here the approach is to choose, simultaneously, the optimal tax and the enforcement structure in order to induce truthful reporting of income. These models predict that, in equilibrium, there would be no tax evasion. Hence, these models fail to explain the existence of tax evasion.

In the second strand, associated with Cremer and Gahvari (1993) and Boadway, Marchand and Pestieau (1994), the problem is to find the optimal taxes set by a benevolent planner in the presence of *tax avoidance*, rather than *tax evasion*. So, by incurring a cost, the taxpayer can ensure that evasion is never discovered by the tax authorities. This in itself is an interesting problem and allows one to make a case for commodity taxation. Because, in these models, income tax can be avoided while commodity tax cannot be, it is efficient to have some commodity taxes.[6] However, this leaves open the relation between tax evasion and optimal taxation.

The third strand in the literature is exemplified by the recent work of Richter and Boadway (2006). They explicitly model the tax evasion decision in the standard Allingham and Sandmo (1972) model. They consider a representative household, hence, the focus is entirely on efficiency issues. The central question, as in the second strand, is the optimal mix of income and consumption taxes when the ease of evading different types of taxes is different. The welfare objective of the government is to maximize its total revenues (arising from income taxes, consumption taxes and penalties on those caught evading) subject to a taxpayer participation constraint. The main disadvantage of the income tax is that, by inviting tax evasion, it exposes the risk-averse taxpayer to income risk.[7] While the consumption tax cannot be evaded, its main disadvantage is that it distorts relative prices of goods. Hence, optimal taxation arises from a tradeoff between imposing income risk on the taxpayer and creating tax distortions.

In the context of our paper, the main criticisms of the existing literature are as follows. First, the behavioral approach to tax evasion shows that an EUT based analysis of tax eva-

[5] Reinganum and Wilde (1985) was the pioneering paper in the mechanism-design approach to tax evasion. However, as a piece of positive analysis it has its limitations. From their assumptions A1 to A7 and A9, Reinganum and Wilde show that the optimal audit and taxation strategy takes the following form. The government announces a cutoff point, T, and audits all taxpayers who report a pretax income less than T. An audit reveals the true income of the taxpayer, which the government then expropriates in total. The government audits no taxpayer who reports an income equal to, or greater than, T, and simply collects a lumpsum tax equal to T from that taxpayer. No attempt is made to compare the predictions of the model with the evidence on taxation, which is not surprising since they are grossly at variance with any known tax system.

[6] However, if tax bases are measured with error, then a case can be made for commodity taxes even in the absence of tax evasion; see Dhami and al-Nowaihi (2006).

[7] This exposure to risk is completely ignored by the second strand of the literature, which assumes a constant cost of such risky activity.

sion is seriously misleading. However, the existing literature is based entirely on the EUT framework. Second, the mechanism design approach to evasion, which typically searches for optimal tax/penalty schemes to completely eliminate evasion, seems at variance with the evidence. Third, the cost of risk to the taxpayer from the risky activity of tax evasion needs to be explicitly modelled. Fourth, one needs to explore welfare criteria other than revenue maximization in designing optimal tax schemes.

1.3. Welfare analysis in behavioral economics

Under uncertainty, when decision makers have expected utility preferences, the Pareto frontier can be found, subject to the standard regularity conditions, by solving for the optimization problem of a benevolent utilitarian planner who maximizes a weighted sum of the individuals' expected utilities.

Suppose now that individuals have *prospect theory preferences* (which we discuss in detail in Section 4, below). Under these preferences, decision makers (1) overweight small probabilities, (2) are loss-averse, (3) evaluate gains and losses relative to a reference point, and (4) have distinct risk preferences depending on whether they are in the domain of gains or losses, relative to the reference point.

Should a social planner respect the PT preferences of an individual? Or should the planner disregard these preferences and evaluate the well being of society on the basis of a standard social welfare function? These questions take us into relatively unchartered territory in economics. There is no clear consensus on the approach to be taken, but we illustrate some well known views on this matter, below.

Tversky and Kahneman (1986, abstract) state that "no theory of choice can be both normatively adequate and descriptively correct". Their argument is as follows. Since *invariance* and *dominance* are always obeyed when their application is transparent, they should be essential features of any normative theory. However, because of bounded rationality, they are often violated when the application is not transparent. A descriptively adequate theory must take account of this.

The modern literature on merit goods has stressed the bounded rationality of decision makers. Should policy makers respect boundedly rational decisions or try to alter them? This is the theme of recent work by Camerer et. al. (2003). These authors advocate the case for *asymmetric paternalism*. Essentially this is an attempt to steer boundedly rational individuals in the direction of avoiding costly mistakes but, at the same time, distorting the decisions of rational people as little as possible. In the words of the authors: "And a variety of researchers have shown that people exhibit systematic mispredictions about the costs and benefits of choices—for example, the degree of loss aversion exhibited in people's choices seems inconsistent with their actual experiences of gains and losses. It is such errors—apparent violations of rationality—that can justify the need for paternalistic policies to help people make better decisions and come closer to behaving in their own best interest."[8]

Several other forms of paternalism are advocated in the literature. Benjamin and Laibson (2003) introduce the concept of *benign paternalism.* The idea here is to encourage the individual to undertake socially desirable actions, without violating individual liberty to make the decision. The idea is made operational by introducing small hurdles in the way of harmful choices in order to shepherd individuals in a better direction.[9] Jolls et al. (1998) coin the term *anti-antipaternalism* which rejects the idea of pure libertarianism. O'Donoghue and Rabin (2003) advocate the idea of *optimal paternalism* which advocates taking account of all costs and benefits of paternalism. They reject the view that interventions should be minimal. In their words: "In some instances, even seemingly large deviations from the policy that is optimal for fully rational economic agents would not cause severe harm to those agents. In such cases, even a small probability of people making errors can have dramatic effects for optimal policy."

1.4. Liberalism, paternalism and context dependence of preferences

In Section 1.3 we have illustrated the increasing appeal of paternalism (as opposed to liberalism) in conducting welfare analysis when individuals have bounded rationality. However, *non-paternalism*[10] is a fundamental principle of liberalism: individuals are the best judges of their own welfare. It is also a fundamental assumption of social choice theory, welfare economics and mechanism design.[11] In its weakest and most general form, non-paternalism is expressed by the *Bergson-Samuelson* social welfare function:

$$W(\mathbf{x}) = F(u_1(\mathbf{x}), u_2(\mathbf{x}), ..., u_I(\mathbf{x})) \text{ (Bergson-Samuelson)}, \tag{1.1}$$

where $\mathbf{x}$ is a social state, I is the number of individuals in society and $u_i(\mathbf{x})$ is the utility of individual i *as seen by that individual.* The constant elasticity form is a special case of (1.1):

$$W(\mathbf{x}) = \frac{1}{1-\rho}\sum_{i=1}^{I}\beta_i\left[u_i(\mathbf{x})\right]^{1-\rho},\ \rho \neq 1,\ \beta_i \geq 1,\ u_i(\mathbf{x}) > 0 \text{ (constant elasticity)}. \tag{1.2}$$

[8] Camerer et. al. (2003) illustrate the usefulness of their approach for boundedly rational individuals in several contexts such as the following. Setting default options which encourage savings behavior and protect insurance rights, framing of contracts to include seemingly irrelevant information, disclosure issues, etc.

[9] For instance, gamblers are asked to choose up-front a level of liquidity which cannot be subsequently exceeded when they might be in a tempted state. Gamblers without subsequent self control problems will not need to be disciplined so they have no problems with setting liquidity limits, while gamblers with subsequent self control problems will clearly benefit from these up-front limits.

[10] Also known as *welfarism* or *individualism.*

[11] See, for example, chapters 21, 22 and 23 in Mas-Colell, Whinston and Green (1995).

In (1.2), the government exhibits a degree of paternalism in that (1) it may give different weights to different individuals, and (2) it may exhibit a degree of inequality aversion (captured by ρ) that is different from the one exhibited by an individual (as captured by $u_i(\mathbf{x})$). Further specialization of (1.2) gives the two forms we shall use in this paper[12]:

$$W(\mathbf{x}) = u_i(\mathbf{x}) \text{ (representative agent)}, \tag{1.3}$$

$$W(\mathbf{x}) = \sum_{i=1}^{I} u_i(\mathbf{x}) \text{ (utilitarian or Benthamite)}, \tag{1.4}$$

The social welfare function in the form (1.4) may be viewed as the most liberal of the above, as the government gives equal weight to all individuals and does not modify any of the individual utility functions. A number of axiomatizations lead to (1.4).[13]

We shall argue that the evidence on tax evasion is best explained by the following combination. We use PT to model the tax evasion decision of the taxpayers and derive the government budget constraint. The government then chooses the tax rate that maximizes social welfare. Although the government recognizes correctly that taxpayers use PT in making their tax evasion decision, and derives its budget constraint on that basis, when it chooses the tax rate it assumes that society's true preference over private and public provision, the $u_i(\mathbf{x})$ in (1.3) or in (1.4), is given by standard utility theory.

We could defend this methodology in either of two ways. The first is more traditional while the latter relies on the emerging empirical evidence. Our results *do not* hinge on which interpretation we adopt. Readers of different persuasions will have a preference for one or the other.

1. **Paternalism**: The government, when choosing the tax rate, behaves paternalistically, i.e., the government assumes that it has better knowledge of the true welfare of individuals than the individuals themselves.

2. **Liberalism plus context dependent preferences**: When taking their tax evasion decision, taxpayers exhibit behavior described by PT. But when these same individuals express their preferences over private and public provision (through, say,

[12] We prefer the term *representative agent* to *dictator*. First, individual i may be different from the decision maker. Second, even if individual i is the decision maker, he/she might be selected by a democratic process, say an election.

[13] For example, Harsanyi (1953, 1955, 1975) derived (1.4) using expected utility theory and a 'behind the veil of ignorance' argument. D'Aspremont and Gevers (1977) derived (1.4) from cardinal measurability with unit comparability, non-paternalism, the strong Pareto principle and anonymity (or symmetry). Maskin (1978) derived (1.4) from cardinal measurability, non-paternalism, the strong Pareto principle, anonymity, separability of indifferent households, continuity and strong equity. Other axiomatizations lead to other special forms of (1.1); see, for instance, Boadway and Bruce (1984).

surveys, referenda, elections, etc.), these preferences are described by a standard utility function, $u_i(\mathbf{x})$; and it is this, $u_i(\mathbf{x})$, that figures in the social welfare function (1.3) or (1.4).

Formal definitions of paternalism and liberalism plus context dependent preferences are provided in section 2 below. Our own preference is for the second interpretation, see our second quote at the beginning of the paper. However, this view entails that individuals do not have a complete preference ordering over all states. A very large body of empirical evidence that has been generated in behavioral economics strongly suggests that individuals do not have a complete preference ordering over all states.

In particular, the evidence suggests, very clearly, that the framing of choices can, for instance, have a very large impact on the outcome.[14] The problem does not go away once professionals are presented with choices that they must make on a regular basis.[15] The mental accounting literature pioneered by Richard Thaler raises similar issues of incomplete preferences.[16]

An individual, when buying an air ticket, might also buy travel insurance, thus exhibiting risk averse behavior. But, the same individual, when he reaches his holiday destination, may visit a gambling casino and exhibit risk loving behavior there. Individuals, when making a private consumption decision might act so as to maximize their selfish interest. But in a separate role as part of the government, as a school governor or as a voter, could act so as to maximize some notion of public well being. Individuals might, for instance, send their own children to private schools (self interest) but could at the same time vote for more funding to government run schools in local or national elections (public interest). Thus, individuals can put on different hats in different situations (a form of mental accounting).

Appealing to this body of evidence, we can assume that individuals, when taking their tax evasion decision, exhibit behavior described by PT. But when these same individuals express their preferences over private and public provision, these preferences are described by a standard utility function.[17]

[14] The following is just one example out of hundreds described in Kahneman and Tversky (2000). It is problems 9 and 10 from Quattrone and Tversky (1988). In a survey, 64% of respondents thought that an increase in inflation from 12% to 17% was acceptable if it lead to a reduction in unemployment from 10% to 5%. However, only 46% of the respondents thought that exactly the same increase in inflation (from 12% to 17%) was acceptable if it increased employment from 90% to 95%.

[15] In a well known example, Kahneman and Tversky find, in the context of medical decisions, that the choice between various programs depends on whether the choices are posed in terms of *lives saved* or *lives lost* (see page 5 in Kahneman and Tversky (2000)).

[16] See Part 4 of the book by Kahneman and Tversky (2000).

[17] However, completeness can be restored by the following ad hoc device. Introduce the state variable z. Let $z = 0$ in the state where the taxpayer is considering his tax evasion decision and $z = 1$ in the state where he is considering the desired mix of private/public provision. Let v be his utility function when considering his tax evasion decision and let u be his utility function when considering the desired mix of

1.5. Results and schematic outline

Under both EUT and PT, taxpayers evade less as the audit probability and the penalty rate increase. Under PT, additionally, one gets the plausible result that taxpayers evade more if the tax rate increases.

Using EUT to model both the tax evasion decision of a consumer-taxpayer and his preference over private and public consumption is unable to jointly account for the evidence on actual tax gaps and government expenditure. Given actual penalty rates and audit probabilities, if consumer preferences over private versus public consumption yield observed government expenditures, then, we find that EUT predicts far too big a tax gap. On the other hand, if consumer preferences over private and public consumption yield observed tax gaps, then EUT predicts far too much government expenditure. At the other extreme, using PT to model both the tax evasion decision of a consumer-taxpayer and his preference over private and public consumption gives empirically wrong and economically absurd results.

By contrast, using PT to model the tax evasion decision of a consumer-taxpayer and EUT to model his preference over private and public consumption, we have no difficulty reconciling observed tax gaps with observed government expenditures.

Section 2 describes the basic model. Sections 3 and 4 consider the tax evasion decision on the basis of EUT and PT, respectively. Sections 5 and 6 derive the resulting optimal tax rates under EUT and PT, respectively. Section 7 compares the success of PT in explaining tax evasion with that of EUT. Section 8 summarizes and concludes.

2. The model

We consider an economy consisting of a continuum of consumer-taxpayers located on the unit square $\Omega = [0,1] \times [0,1]$. Pretax income is exogenous and is given by the density function, $Y(x,s) \geq 0$, $x \in [0,1]$, $s \in [0,1]$, where s (explained in more detail below) captures the *stigma* faced by a tax evader when caught and x is purely a label that helps locate an individual taxpayer.[18] The government levies tax at the constant rate $t \in [0,1]$ on declared income and uses its tax revenue to finance the public provision of goods and services whose aggregate monetary value is G.[19]

Let the binary relation $\preceq$ give the consumer's preference over private consumption, C,

private/public provision. Let $w(z)$ be his 'overall' utility function. Then $w(0) = v$ and $w(1) = u$. This fix is purely ad hoc with the sole aim of formally restoring completeness. It has no behavioral consequences. In particular, there is no tradeoff between the states $z = 0$ and $z = 1$.

[18] In the discrete case, x could denote the individual's *social security number* or his *national insurance number* which uniquely locates the individual.

[19] Since Ω is of unit area, G can be interpreted as either the aggregate or the per capita monetary value of all publicly provided goods and services.

and public provision, G, *when the consumer is considering his tax evasion decision*. In general, $\preceq$, will depend on x and s. But, to simplify notation, we will not indicate this dependence. Let $\{\preceq\}$ be the profile of these consumer preferences.

Let the binary relation $\preceq^*$ give the government's preference over private and public provision. We allow for $\preceq^*$ to be different from any $\preceq \in \{\preceq\}$. Recall, from Subsection 1.4, that we could adopt a *paternalistic interpretation*: Although the behavior of taxpayers is determined by the profile of preference relations, $\{\preceq\}$, the government decides that society's welfare is best represented by the preference relation $\preceq^*$. Alternatively, we could adopt a *liberal-context-dependent interpretation*: When a taxpayer decides between public and private provision, his preference is given by $\preceq^*$, which is then adopted by the government. But when this same taxpayer takes his tax evasion decisions, his action is determined by $\preceq$ (which may or may not be the same as $\preceq^*$). This motivates the following definitions.

Definition 1 *(Paternalism): Let the binary relation $\preceq^*$ give the government's preference between private consumption, C, and public provision, G. Let the binary relation $\preceq$ give a taxpayer's preference between private consumption and public provision when this taxpayer is considering his tax evasion decision. We say that the government exhibits paternalism if $\preceq^* \neq \preceq$.*

Definition 2 *(Context dependent preferences): Let the binary relation $\preceq^*$ give a taxpayer's preference between private consumption, C, and public provision, G, when this taxpayer is considering public policy issues. Let the binary relation $\preceq$ give this same taxpayer's preference between private consumption and public provision when this taxpayer is considering his tax evasion decision. We say that the taxpayer has context dependent preferences if $\preceq \neq \preceq^*$.*

The ideal, both for society and for the government, is for none to evade. Hence the representative taxpayer in the social welfare function is taken to be in the state where he does not evade. We may call such a taxpayer an 'ideal' taxpayer. This ideal taxpayer takes no decisions, hence, has no budget constraint or incentive compatibility conditions to satisfy. Decisions are taken on his behalf by the government who, of course, has a budget constraint. The government derives its budget constraint knowing that the 'real' taxpayers of the economy, when deciding on how much income to declare, obey their own budget constraints.

The consumer-taxpayer located at (x, s) whose preferences between C, G are given by $\preceq$, declares income $D_{\preceq}(t, G, x, s)\, dxds$, such that $D_{\preceq}(t, G, x, s) \in [0, Y(x, s)]$. Thus $D_{\preceq}(t, G, x, s)$ is the density of declared income. Subsequent to the filing of tax returns, an exogenous fraction $p \in (0,1)$ of the taxpayers are audited, and the audit reveals the true taxable income. If caught, the dishonest taxpayer must pay the outstanding tax liabilities $t\left[Y(x,s) - D_{\preceq}(t,G,x,s)\right]dxds$ and a penalty proportional to unpaid taxes, $\lambda t\left[Y(x,s) - D_{\preceq}(t,G,x,s)\right]dxds$, where $\lambda > 0$ is the constant penalty rate. The density of tax revenue is then

$$T_{\preceq}(t,G,x,s) = tD_{\preceq}(t,G,x,s) + p(1+\lambda)t\left[Y(x,s) - D_{\preceq}(t,G,x,s)\right]. \qquad (2.1)$$

Clearly, $D_{\preceq}$ and $T_{\preceq}$ depend on p and λ as well as t, G, x and s.

2.1. Sequence of moves

The sequence of moves is as follows.

1. The tax authority announces the tax rate, t, the audit probability, p, the penalty rate, λ, and the monetary value of the provision of all publicly provided goods and services, G.
2. Taxpayers make the decision to either report their full income or evade a fraction of it, given t, p, λ and G.
3. The government audits a fraction, p, of the tax returns and dishonest taxpayers are required to give up a fraction, $1+\lambda$, of their unreported income.

2.2. Exogenous and endogenous variables

The exogenous variables of the model are the probability of an audit, p (which, here, is the same as the probability of detection), the penalty rate, λ, and the density function of income, Y. When considering tax evasion under prospect theory, we shall introduce and explain the further exogenous variables α, β and θ. The endogenous variables of the model are the tax rate, t, the density function of declared income, $D_{\preceq}$, and the tax revenue density function, $T_{\preceq}$. When the consumer-taxpayer makes his tax evasion decision, he takes as exogenous the model parameters, his location, (x,s), the tax rate, t, and level of public provision, G.

Penalty structures and audit strategies, as well as the tax rate, are (of course) chosen by the government and are important. We endogenize the tax rate because it is, by far, the most important of these variables, as it is the main source of government revenue. However, there are serious, well known, and unresolved problems associated with modelling penalty structures and audit strategies. For example, why do governments not make more use of fines and less use of prisons? Fines are far less costly than prisons to administer and generate revenue for the government. Why do governments not (anymore) hang evaders with probability (close to) zero? This would produce maximum deterrence at minimum cost. These, and many other issues, have been the subjects of ongoing research since, at least, Becker (1968). Furthermore, the penalty and audit rates are, unlike taxation, very often not free variables at all. Penalties on tax offenders must be consistent with similar cases of fraud and so are heavily dependent on the legal structure, legal history and legal norms of a country. Audit rates are often constrained by the budget constraints of revenue departments. It is equally unsatisfactory to endogenize these variables, while completing ignoring these constraints.[20]

There are further problems. For example, theoretically predicted audit probabilities are too high (compared to those observed). Furthermore, if the audit probability is a free decision variable for the government, then the equilibrium will be in mixed strategies. However, at the level of the individual, the evidence, very strongly, suggests that if people play mixed strategies, they do not do so in proportions that are predicted by the theory. This conclusion is unaltered even when players are provided with randomizing devices.[21]

Dealing in a satisfactory manner with all these issues is beyond the scope of this paper. We have, therefore, simply assigned empirically plausible values to the audit probability and the penalty rate.

Remark 1 *: To simplify notation, we shall suppress reference to the model parameters and drop the infinitesimal quantities, dx and ds, when such omission is not likely to lead to confusion.*

2.3. Government tax revenue

We assume that Y is integrable[22]. It will follow from the optimizing behavior of consumer-taxpayers that $D_{\preceq}$ is integrable. Hence, $T_{\preceq}$ is also integrable. Let $S \subset \Omega$ be measurable[23].

[20] To quote from Graetz and Wilde (2001, p358): "That an economic model analyzing the expected utility calculation of a would-be tax evader recommends large increases in the applicable sanction in light of the very low probability of its application quickly becomes irrelevant as a policy matter. In this country, at least, legal, moral and political constraints make this necessarily so. Coherence in our criminal law generally demands that 'punishment fit the crime'; regardless of any alleged potential theoretical advantages, life imprisonment is simply not within the feasible set of punishments for tax evasion. Moreover, no criminal sanction (nor even a substantial civil 'fraud' penalty) can be imposed absent satisfactory proof that the tax understatement was willful. The compliance problem will not be solved without taking into account these kinds of structural limitations on both detection and punishment. Neither the level of punishment nor of audits can realistically be greatly expanded."

[21] There is a very large literature on these issues. See, for instance, Brown and Rosenthal (1990) Sefton and Yavas (1996), and Shachat (2002).

[22] Integrability can be interpreted either in the sense of Riemann or Lebesgue. It does not matter for our purposes which interpretation is chosen.

[23] Measurable in the sense of Borel or Lebesgue. Any set of interest to us will be measurable in both senses.

Then the aggregate pretax income of all consumer-taxpayers in S is given by

$$\overline{Y}(S) = \int\int_{(x,s)\in S} Y(x,s)\, dsdx, \tag{2.2}$$

and the tax revenue collected from them is

$$\overline{T}_{\{\preceq\}}(t,G,S) = \int\int_{(x,s)\in S} tD_{\preceq}(t,G,x,s)\, dxds$$
$$+p(1+\lambda)\int\int_{(x,s)\in S} t\left[Y(x,s) - D_{\preceq}(t,G,x,s)\right] dxds. \tag{2.3}$$

In particular, let

$$\overline{Y} = \overline{Y}(\Omega) = \int_{x=0}^{1}\int_{s=0}^{1} Y(x,s)\, dsdx, \tag{2.4}$$

and

$$\overline{T}_{\{\preceq\}}(t,G) = \overline{T}_{\{\preceq\}}(t,G,\Omega) = \int_{x=0}^{1}\int_{s=0}^{1} tD_{\preceq}(t,G,x,s)\, dxds$$
$$+p(1+\lambda)\int_{x=0}^{1}\int_{s=0}^{1} t\left[Y(x,s) - D_{\preceq}(t,G,x,s)\right] dxds, \tag{2.5}$$

then, because Ω is of unit area, $\overline{Y}$ is both total income and average income. Likewise, $\overline{T}_{\{\preceq\}}(t,G)$ is both total and average tax revenue. We assume that $\overline{Y} > 0$.

Remark 2 : *Note that if S is of measure zero then $\overline{Y}(S) = 0$ and, hence, also $\overline{T}_{\{\preceq\}}(t,G,S) = 0$. In particular, the tax revenue collected from any single individual, (x,s), is zero, i.e., $\overline{T}_{\{\preceq\}}(t,G,\{(x,s)\}) = 0$. It follows from this that the sum of all tax revenue revenues, $\overline{T}_{\{\preceq\}}(t,G,\Omega - \{(x,s)\})$, collected from all taxpayers other than taxpayer, (x,s), equals total tax revenue, $\overline{T}_{\{\preceq\}}(t,G,\Omega)$, i.e., $\overline{T}_{\{\preceq\}}(t,G,\Omega - \{(x,s)\}) = \overline{T}_{\{\preceq\}}(t,G)$. This ensures that we can consistently assume that when a taxpayer decides how much income to declare, he can take public provision, G, as given.*[24]

Thus, the government's tax revenue, $\overline{T}_{\{\preceq\}}(t,G)$, comes from the following three sources:

1. Taxes on declared income, $\int_{x=0}^{1}\int_{s=0}^{1} tD_{\preceq}(t,G,x,s)\, dxds$.

2. Taxes recovered from those caught evading, $p\int_{x=0}^{1}\int_{s=0}^{1} t\left[Y(x,s) - D_{\preceq}(t,G,x,s)\right] dxds$.

[24] To be absolutely clear, $T_{\preceq}(t,G,x,s)$ is the *density* of tax revenue at (x,s). If $t > 0$ and $D(t,G,x,s) > 0$, then $T_{\preceq}(t,G,x,s) > 0$. However, the *tax revenue* collected from (x,s), $\overline{T}_{\{\preceq\}}(t,G,\{(x,s)\})$, is zero. A useful intuitive picture to retain in mind is $\overline{T}_{\{\preceq\}}(t,G,\{(x,s)\}) = T_{\preceq}(t,G,x,s)\, dxdy$, where $T_{\preceq}(t,G,x,s) > 0$ but $\overline{T}_{\{\preceq\}}(t,G,\{(x,s)\}) = 0$ because the 'infinitesimal rectangle', $dxdy$, has zero area. Such ideas can be made rigorous using Non-standard analysis, which was invented for such purpose; see, for example, Loeb and Wolff (2000).

3. Fines from those caught evading, $p\lambda \int_{x=0}^{1}\int_{s=0}^{1} t\left[Y\left(x,s\right)-D_{\preceq}\left(t,G,x,s\right)\right]dxds$.

From (2.4) and (2.5), we see that total tax revenue can be written in the slightly simpler form:

$$\overline{T}_{\{\preceq\}}\left(t,G\right)=tp\left(1+\lambda\right)\overline{Y}+t\left[1-p\left(1+\lambda\right)\right]\int_{x=0}^{1}\int_{s=0}^{1}D_{\preceq}\left(t,G,x,s\right)dxds. \tag{2.6}$$

It will be useful to distinguish between those who are unable to evade tax, and those who can evade but choose not to. Therefore, assume taxes are deducted at source for a fraction, ω, of the population, where

$$0\leq\omega\leq1, \tag{2.7}$$

so, these taxpayers cannot evade (if $\omega=1$, then nobody can evade).

We make the following two simplifying assumptions.

A1. *The opportunity to evade taxes does not depend on the taxpayer's income*[25]: Thus, $Y\left(x,s\right)$ does not depend on ω. Hence, the tax revenue collected from the consumer-taxpayers in $S\subset\Omega$ is given by

$$\overline{T}_{\{\preceq\}}\left(t,G,S\right)=\omega t\int\int_{(x,s)\in S}Ydxds$$
$$+\left(1-\omega\right)t\left[\int\int_{(x,s)\in S}D_{\preceq}dxds+p\left(1+\lambda\right)\int\int_{(x,s)\in S}\left[Y-D_{\preceq}\right]dxds\right], \tag{2.8}$$

which simplifies to

$$\overline{T}_{\{\preceq\}}\left(t,G,S\right)=\left[\omega+p\left(1+\lambda\right)\left(1-\omega\right)\right]t\int\int_{(x,s)\in S}Ydxds$$
$$+\left(1-\omega\right)\left[1-p\left(1+\lambda\right)\right]t\int\int_{(x,s)\in S}D_{\preceq}dxds. \tag{2.9}$$

Total tax revenue becomes

$$\overline{T}_{\{\preceq\}}\left(t,G\right)=\overline{T}_{\{\preceq\}}\left(t,G,\Omega\right)$$
$$=\left[\omega+p\left(1+\lambda\right)\left(1-\omega\right)\right]\overline{Y}t+\left(1-\omega\right)\left[1-p\left(1+\lambda\right)\right]t\int_{x=0}^{1}\int_{s=0}^{1}D_{\preceq}dxds. \tag{2.10}$$

[25] The evidence is that the most important variable determining the opportunity to evade is not the taxpayer's income but the source of income. For example it is almost impossible for a teacher to evade paying tax on his salaried income, but easy to evade paying tax on his income from private tutoring. Note that ω can also be interpreted the fraction of pretax income that is taxed at source and, therefore, cannot be evaded.

A2. *Stigma is unrelated to income*: Although different people suffer different rates of stigma, we do not know of any strong evidence that this is related to income, e.g., we do not know of any strong evidence that rich persons suffer higher, or lower, rates of stigma than poor persons, on account of their income.[26] Our second simplifying assumption is, therefore, that the income density function, $Y(x,s)$, is independent of s. From (2.4) we get

$$\overline{Y} = \int_{x=0}^{1}\int_{s=0}^{1} Y ds dx = \int_{x=0}^{1} Y \left(\int_{s=0}^{1} ds\right) dx = \int_{x=0}^{1} Y dx. \tag{2.11}$$

The independence of income, stigma and the ability to evade, will give rise to simple expressions for total tax revenue and aggregate utility.

2.4. Behavior of the government and consumer-taxpayers

The tax authority moves first, making an announcement of the tax rate, t, the audit probability, p, the penalty rate, λ, and the total monetary value of all publicly provided goods and services, G. Given t, p, λ and G, the taxpayer then makes the decision to either report full income ($D_{\preceq} = Y$) or evade a fraction of it ($D_{\preceq} < Y$). Let Y_{NC} be the after tax income of the taxpayer if he is not caught, then,

$$Y_{NC} = Y - tD_{\preceq}, \text{ with probability } 1-p. \tag{2.12}$$

If evasion is discovered, then the taxpayer also suffers some stigma, whose monetary value is $s(Y - D_{\preceq})$, where s is the stigma rate on evaded income, $s \in [0,1]$. As in Gordon (1989) and Besley and Coate (1992), such stigma enters linearly, as a monetary equivalent, into the payoff in that state of the world[27]. His after tax income is then Y_C, given by

$$Y_C = (1-t)Y - (s+\lambda t)(Y - D_{\preceq}), \text{ with probability } p. \tag{2.13}$$

The government spends the total tax revenue, $\overline{T}_{\{\preceq\}}(t,G)$, given by (2.6), on providing goods and services under the balanced budget constraint,

$$\overline{T}_{\{\preceq\}}(t,G) = G. \tag{2.14}$$

Consumers derive utility from private consumption and the publicly provided goods and services. However, when making the decision on how much income to declare, a consumer

[26] See Slemrod (2007, p30) for a review of this.

[27] Although a natural interpretation of stigma might include factors such as the tug on one's conscience and loss of face among family and community etc., other interpretations of stigma are possible. These might include, in an appropriately specified dynamic game, the reputational costs that impinge on current and future earnings. For a more detailed discussion of stigma in the context of tax evasion as well as a more general formulation, see Dhami and al-Nowaihi (2007).

takes the publicly provided goods and services as given. Thus we have a free-rider problem: each consumer derives utility from public provision, but hopes that others will pay for it. The government chooses the tax rate, t, so as to maximize social welfare, taking into account (i) the utility that individuals derive from private and public consumption, and (ii) the effect of the tax rate on tax evasion and, hence, on tax revenue.

Remark 3 : *The government chooses the tax rate that is optimal with respect to the preference relation, $\preceq^*$. This could be because the government is paternalistic (see Definition 1 and the discussion in Subsection 1.4) or because taxpayers have context-dependent preferences (see Definition 2 and the discussion in Subsection 1.4). However, it is important to note, from (2.14), that the government recognizes that subsequent tax revenues will depend on the profile of real taxpayer preferences $\{\preceq\}$, which might or might not be different from $\preceq^*$. Hence, asides from the issue of interpretation (paternalism or context dependent preferences), the method of solution and it's logic are identical to standard principal-agent problems.*[28]

We shall consider five regimes, summarized in Table 2.1. We use the shorthand notation CARA for constant absolute risk aversion and log for logarithmic utility.

Regime	Government Preferences	Taxpayer Preferences
EUT	EU, representative consumer, separable, log	EU, complete
PT1	EU, representative consumer, separable, log	PT, context dependent
PT2	EU, representative consumer, non-separable	PT, context dependent
PT3	EU, utilitarian, separable, CARA	PT, context dependent
PT4	PT, utilitarian	PT, complete

Figure 2.1: Classification of regimes

The description of the regimes will rely heavily on the arguments developed in the introduction; see in particular Section 1.4.

1. Regime EUT: The second row of Table 2.1 describes regime EUT. In this regime, consumer preferences, $\{\preceq\}$, over private provision, C, and public provision, G, are given by the tractable form:

$$U(C, G) = \mu \ln G + (1 - \mu) \ln C,\ 0 < \mu < 1,\ G > 0,\ C > 0, \tag{2.15}$$

[28]Invoking paternalism and/or context dependence preferences, in our sense, has a long history in public economics. For instance, maximizing the utility of a representative or average taxpayer (with 2.1 children, say) explicitly invokes this criterion. In other contexts, in maximizing a social welfare function, the utility of corrupt regulators, criminals or dishonest taxpayers may be ignored.

Expected utility is then given by,

$$EU = \mu \ln G + (1-\mu)\left[p \ln Y_C + (1-p)\ln Y_{NC}\right], \tag{2.16}$$

where Y_C and Y_{NC} are given by (2.13) and (2.12), respectively.

Maximizing (2.16) gives the density function, $D_{\preceq}$, of declared income and, hence, government tax revenue $\overline{T}_{\{\preceq\}}(t, G)$. We describe the taxpayer's preferences in this regime as EU.

The government uses (2.15) to maximize the utility of a representative consumer with average income, $\overline{Y}$, who does not evade (the 'ideal' taxpayer). Thus, in this case, $\preceq^* = \preceq$: the government's preference over private and public provision is the same as that of the representative consumer with average income who does not evade (but, of course, the 'real' taxpayers do evade and, in so doing, generate the government tax revenue, $\overline{T}_{\{\preceq\}}(t, G)$, which the government takes into account when forming its budget constraint). The details are given in Section 3, below. We describe the government preferences in this regime as "EU, representative consumer, separable and logarithmic".

This regime turns out to perform poorly in explaining, jointly, the tax rate, the tax gap and government expenditure.

2. Regime PT1: Government preferences, $\preceq^*$, over private and public provision are given by the same utility function (2.15) as in regime EUT. However, when consumers make tax evasion decisions, their preferences, $\{\preceq\}$, are described by prospect theory, the details of which are given in Section 4, below. Thus, the utility function describing tax evasion behavior is different from the utility function describing preferences between private and public provision: $\preceq^* \neq \preceq$.

   The government then maximizes (2.15) for a representative consumer with average income who does not evade (the 'ideal' taxpayer). The 'real' taxpayers do evade and have prospect theory preferences. The government takes these two facts into account when forming its budget constraint, $\overline{T}_{\{\preceq\}}(t, G) = G$.

   We describe the preferences of the taxpayer as "PT". Those of the government we describe as "EU, representative consumer, separable and logarithmic".

   In contrast to regime EUT, regime PT1 successfully explains the tax rates, observed tax gap and the level of government expenditure.

3. *Regime PT2*: The utility function (2.15) is additively separable over private and public provision. However, in actual practice, there would seem to be strong complementarities between the two. For example, utility derived from private car ownership heavily depends on the quality of publicly provided roads. We would like to investigate the consequences of recognizing this complementarity. The fourth row of Table

2.1 describes regime PT2. In this regime the preference, $\preceq^*$, over private and public provision is given by the utility function:

$$U(C,G) = F\left(C - \frac{B}{G}\right),\ F' > 0,\ F'' < 0,\ G > 0,\ 0 < \frac{B}{G} < C, \tag{2.17}$$

For example, $F\left(C - \frac{B}{G}\right) = \ln\left(C - \frac{B}{G}\right)$. An interpretation of this model is that public provision, G, has value only insofar as it facilitates private consumption, C. When making the tax evasion decision, consumer preferences, $\preceq$, are given by prospect theory, as in regime PT1. Thus, the consumer-taxpayer, again, exhibits context-dependent preferences (alternatively, the government is paternalistic).

The government uses the utility function (2.17) to maximize the utility of a representative consumer with average income, $\overline{Y}$, who does not evade. The details are given in Section 4, below. The results are very similar to that of regime PT2.

4. *Regime PT3*: The fifth row of Table 2.1 describes regime PT3. In this regime, the preference, $\preceq^*$, over private and public provision is given by the utility function:

$$U(C,G) = \begin{cases} \mu\frac{G^{1-\gamma}}{1-\gamma} + (1-\mu)\frac{C^{1-\gamma}}{1-\gamma}, & \gamma \neq 1, \\ \mu \ln G + (1-\mu)\ln C, & \gamma = 1, \end{cases} \tag{2.18}$$

where $0 < \mu < 1$, $G > 0$, $C > 0$. When making the tax evasion decision, consumer preferences, $\preceq$, are given by prospect theory, as in regimes PT1 and PT2. Thus, the consumer-taxpayer, again, exhibits context-dependent preferences (alternatively, the government is paternalistic). The government uses the utility function (2.18) to maximize the *sum* (in the form of an integral) of the utilities of all the consumers in the state in which they do *not* evade. The details are given in Section 4, below. Note that regime PT3 differs from regimes EUT and PT1 in two ways: the coefficient of relative risk aversion, γ, can now take any value (not just 1) and the government now maximizes the sum of all utilities, not that of a representative consumer. Yet the result is very close to those of regimes PT1 and PT2 and successfully explains the tax gap and government expenditure.

5. *Regime PT4*: The last row of Table 2.1 describes regime PT4. In this regime, when taking the tax evasion decision, consumer preferences, $\preceq$, are given by prospect theory, as in regimes PT1, PT2 and PT3. However, the government also uses these same prospect theory preferences to maximize the sum of the utilities of all the consumers. The details are given in Section 4, below. The results turn out not only to be empirically incorrect but also economically absurd.

We briefly summarize the performance of the five regimes as follows. Using the same EUT utility function to model both the consumer-taxpayer's tax evasion decision and his preference over private versus public consumption ($\preceq = \preceq^*$) cannot reconcile the observed tax gap and level of government expenditure. At the other extreme, modelling both using PT gives absurd results. The best results are obtained by using PT to model tax evasion behavior but EUT to model the consumer's preference over private versus public consumption ($\preceq \neq \preceq^*$), as in regimes PT1, PT2 and PT3. The key is to recognize the context dependence of preferences, or, alternatively, the paternalistic interpretation.

2.5. Game-theoretic formulation

The sequence of moves has already been described in Subsection 2.1. Here we outline the method of solution in our two stage game.

Consider first the second stage. In the second stage, the taxpayer takes as given, the public policy variables, the tax rate and the level of public good provision (i.e. t, G), that are determined in the first stage. The taxpayer located at $(x, s) \in [0, 1]^2$, with preferences between private and public consumption given by $\preceq$, chooses an amount of income to declare, $D^*_{\preceq}(t, G, x, s) \in [0, Y(x, s)]$, so as to maximize his/her payoff. The taxpayer takes as given t, G and the exogenous parameters p (audit probability) and λ (penalty rate). For each pair, $(t, G) \in [0, 1] \times \left[0, \overline{Y}\right]$, the profile of strategies, $D^*_{\preceq}(t, G, x, s)$, $(x, s) \in [0, 1]^2$, generates the total government revenue, $\overline{T}^*_{\{\preceq\}}(t, G)$, given by (2.6).

In the first stage, the government, whose preference between public and private consumption given by $\preceq^*$,[29] chooses the tax rate, $t \in [0, 1]$, and the monetary value of all publicly provided goods and services, $G \in \left[0, \overline{Y}\right]$, where $\overline{Y}$ is aggregate income given by (2.4). In choosing the optimal tax rate, the government takes into account that the second stage tax revenues, $\overline{T}^*_{\{\preceq\}}(t, G)$, are determined by taxpayers whose preferences, $\preceq$, over private and public consumption are possibly different from it's own, given by $\preceq^*$.

Each possible pair, $(t, G) \in [0, 1] \times \left[0, \overline{Y}\right]$, chosen by the government in the first stage, determines a (second stage) subgame. The profile of strategies, $D^*_{\preceq}(t, G, x, s)$, $(x, s) \in [0, 1]^2$, in the second stage forms a Nash noncooperative equilibrium in the subgame, $(t, G) \in [0, 1] \times \left[0, \overline{Y}\right]$. A subgame perfect equilibrium is the triple $\left(t^*, G^*, D^*_{\preceq}\right)$, where (t^*, G^*) maximizes the government's social welfare function subject to the government's budget constraint, $\overline{T}^*_{\{\preceq\}}(t, G) = G$.

3. The taxpayer's evasion problem: Expected utility theory

Consider the case of a consumer-taxpayer located at (x, s) who derives utility, $U(C, G)$, from private consumption, C, and the level of public provision, G. Given his income, $Y(x, s)$, the level of public provision, G, and the values of the parameters t, p, λ, and

[29] As pointed out above, $\preceq^*$ is not necessarily identical to $\preceq$. See our discussion on context dependent preferences and paternalism, above.

s, the consumer chooses the amount of income to declare, $D_{\preceq}$. If he is not caught (with probability $1-p$), then his disposable income, and, hence, private consumption, is Y_{NC}. However, if he is caught (with probability p) then his disposable income, and, hence, private consumption, is Y_C. His expected utility is thus

$$EU = pU\left(Y_C, G\right) + (1-p)\,U\left(Y_{NC}, G\right), \tag{3.1}$$

where Y_C and Y_{NC} are given by (2.13) and (2.12), respectively.[30]

3.1. The Yitzhaki result

Eliminate $D_{\preceq}$ from (2.13) and (2.12) to get

$$\frac{1}{1+\left(\frac{s}{t}+\lambda\right)}Y_C + \frac{\frac{s}{t}+\lambda}{1+\left(\frac{s}{t}+\lambda\right)}Y_{NC} = (1-t)\,Y. \tag{3.2}$$

For the special case of no stigma, $s = 0$, (3.2) reduces to

$$\frac{1}{1+\lambda}Y_C + \frac{\lambda}{1+\lambda}Y_{NC} = (1-t)\,Y. \tag{3.3}$$

We may view the problem as choosing Y_C and Y_{NC} so as to maximize expected utility (3.1) subject to the budget constraint (3.3), given income $(1-t)\,Y$ and prices $\frac{1}{1+\lambda}$ and $\frac{\lambda}{1+\lambda}$. Since prices do not depend on the tax rate, t, an increase in the tax rate has a pure income effect. Making the plausible assumption of constant or declining absolute risk aversion, we get that an increase in the tax rate reduces tax evasion.[31] This result, obtained by Yitzhaki (1974), is rejected by the bulk of experimental, econometric and survey evidence.[32]

In the more general case with stigma, (3.2), a change in the tax rate will have both income and substitution effects. However, simulations with plausible functional forms and parameter values indicate that in the presence of stigma, an increase in the tax rate causes a decline in evasion under EUT.[33]

[30] Obviously, EU depends on D, G, x, s, t, λ, σ, s, and p. We have omitted reference to these to reduce the burden of notation.

[31] For a formal proof see, for example, Dhami and al-Nowaihi (2007, Proposition 2). Along with constant or declining absolute risk aversion, we also need the assumptions $0 \leq D \leq Y$, $\left[\frac{\partial EU(D)}{\partial D}\right]_{D=0} > 0$, $\left[\frac{\partial EU(D)}{\partial D}\right]_{D=Y} < 0$, $\frac{\partial^2 EU(D)}{\partial D^2} < 0$, $p > 0$, $\lambda > 0$ and $t > 0$.

[32] See, for example, Friedland et al. (1978), Clotfelter (1983), Baldry (1987), Andreoni et al. (1998) and Pudney et al. (2000). However, a notable exception is Feinstein (1991).

[33] See below and Dhami and al-Nowaihi (2007).

3.2. Tax evasion under EUT

As described above in Section 2.4, in the regime EUT, when the taxpayer uses EUT, the preferences over private and public goods consumption are given by (2.15) and expected utility is given by (2.16). Note that expected utility (2.16) differs across consumer-taxpayers only in so far as they differ in income, $Y(x,s)$, and stigma, s. Differentiating (2.16) with respect to $D_{\preceq}$, using (2.12) and (2.13), gives

$$\frac{\partial EU}{\partial D_{\preceq}} = (1-\mu)\left[\frac{1-p}{Y_{NC}}\frac{\partial Y_{NC}}{\partial D_{\preceq}} + \frac{p}{Y_C}\frac{\partial Y_C}{\partial D_{\preceq}}\right], \tag{3.4}$$

$$or \ \frac{\partial EU}{\partial D_{\preceq}} = (1-\mu)\left[(s+\lambda t)\frac{p}{Y_C} - t\frac{1-p}{Y_{NC}}\right], \tag{3.5}$$

$$\frac{\partial^2 EU}{\partial D_{\preceq}^2} = -(1-\mu)\left[p\left(\frac{s+\lambda t}{Y_C}\right)^2 + (1-p)\left(\frac{t}{Y_{NC}}\right)^2\right] < 0. \tag{3.6}$$

If $t > 0$, then (3.6) holds everywhere. However, if $t = 0$, then $\frac{\partial^2 EU}{\partial D_{\preceq}^2} = 0$ at $s = 0$. Hence, (3.6) holds almost everywhere. Since EU is continuous on the compact interval, $0 \leq D_{\preceq} \leq Y(x,s)$, a maximum, $D_{\preceq}(t,G,x,s)$, exists. This maximum is unique except at $t = s = 0$. The first order conditions for a maximum are

$$\left[\frac{\partial EU}{\partial D_{\preceq}}\right]_{D_{\preceq}=0} \leq 0, \ \left[\frac{\partial EU}{\partial D_{\preceq}}\right]_{0<D_{\preceq}<Y(x,s)} = 0, \ \left[\frac{\partial EU}{\partial D_{\preceq}}\right]_{D_{\preceq}\geq Y(x,s)} \geq 0. \tag{3.7}$$

Let[34]

$$s_1 = \min\left[1, \max\left(0, \frac{(1-p)(t-t^2)}{(t-pt+p)} - \lambda t\right)\right], \tag{3.8}$$

$$s_2 = \max\left[s_1, \min\left(1, \frac{1-p}{p}t - \lambda t\right)\right]. \tag{3.9}$$

From (2.12), (2.13), (3.5), (3.7), (3.8) and (3.9) we get

$$\begin{array}{lll} 0 \leq s \leq s_1 & \Rightarrow & D_{\preceq} = 0 \\ s_1 < s < s_2 & \Rightarrow & D_{\preceq} = \left[1 + \frac{p}{t} - p - \frac{(1-p)(1-t)}{s+\lambda t}\right] Y \\ s_2 \leq s \leq 1 & \Rightarrow & D_{\preceq} = Y \end{array} \tag{3.10}$$

Thus, taxpayers who suffer low stigma if caught (the interval $0 \leq s \leq s_1$) hide all their income. Taxpayers who suffer moderate stigma if caught (the interval $s_1 < s < s_2$) hide some but not all income. Taxpayers who would suffer high stigma if caught (the interval $s_2 \leq s \leq 1$) declare all their income. Also note that declared income, $D_{\preceq}$, does not depend on the level of public provision, G. The latter result follows from the facts that utility (2.15) is additively separable in private and public consumption and the consumer-taxpayer takes public provision as given when deciding how much income to declare.

[34] For $p > 0$ and $0 \leq t \leq 1$, it can be shown that $s_1 \leq \frac{1-p}{p}t - \lambda t$.

3.3. Tax revenue under EUT

From (3.10) we get:

$$\begin{aligned}\int_{x=0}^{1}\int_{s=0}^{1} D_{\preceq}dxds &= \int_{x=0}^{1}\int_{s=0}^{s_1} D_{\preceq}dsdx + \int_{x=0}^{1}\int_{s=s_1}^{s_2} D_{\preceq}dsdx + \int_{x=0}^{1}\int_{s=s_2}^{1} D_{\preceq}dsdx, \\ &= \int_{x=0}^{1}\int_{s=s_1}^{s_2}\left(1+\frac{p}{t}-p-\frac{(1-p)(1-t)}{s+\lambda t}\right)Ydsdx \\ &\quad +\int_{x=0}^{1}\int_{s=s_2}^{1} Ydsdx. \end{aligned} \tag{3.11}$$

We now invoke the assumption that income distribution, $Y(x,s)$, is independent of stigma, s, so that $\int_{x=0}^{1} Y(x,s)\,dx = \int_{x=0}^{1} Y(x)\,dx = \overline{Y}$. Hence, (3.11) becomes

$$\begin{aligned}\int_{x=0}^{1}\int_{s=0}^{1} D_{\preceq}dxds &= \overline{Y}\int_{s=s_1}^{s_2}\left(1+\frac{p}{t}-p-\frac{(1-p)(1-t)}{s+\lambda t}\right)ds + (1-s_2)\overline{Y}, \\ &= \left[1-s_1+(s_2-s_1)\left(\frac{p}{t}-p\right)\right]\overline{Y} - (1-p)(1-t)\overline{Y}\int_{s=s_1}^{s_2}\frac{ds}{s+\lambda t}, \\ &= \left[1-s_1+(s_2-s_1)\left(\frac{p}{t}-p\right)\right]\overline{Y} \\ &\quad -(1-p)(1-t)\overline{Y}\left[\ln(s_2+\lambda t)-\ln(s_1+\lambda t)\right]. \end{aligned} \tag{3.12}$$

Letting

$$\overline{D_{\{\preceq\}}}(t) = \int_{x=0}^{1}\int_{s=0}^{1} D_{\preceq}dxds, \tag{3.13}$$

we get

$$\overline{D_{\{\preceq\}}}(t) = \left[1-s_1+(s_2-s_1)\left(\frac{p}{t}-p\right)\right]\overline{Y} - (1-p)(1-t)\overline{Y}\left[\ln(s_2+\lambda t)-\ln(s_1+\lambda t)\right]. \tag{3.14}$$

Substitute from (3.14) into (2.10), to get total tax revenue:

$$\overline{T}_{\{\preceq\}}(t,G) = \left[\omega + p(1+\lambda)(1-\omega)\right]\overline{Y}t + (1-\omega)\left[1-p(1+\lambda)\right]t\overline{D_{\{\preceq\}}}(t). \tag{3.15}$$

Now, impose the government budget constraint, $\overline{T}_{\{\preceq\}}(t,G(t)) = G(t)$, to get total public expenditure (which is also per capita public expenditure):

$$G(t) = \left[\omega + p(1+\lambda)(1-\omega)\right]\overline{Y}t + (1-\omega)\left[1-p(1+\lambda)\right]t\overline{D_{\{\preceq\}}}(t). \tag{3.16}$$

4. The taxpayer's evasion problem: Prospect theory

The preferences of the taxpayer under prospect theory are more complicated. Those familiar with prospect theory should just skim the material in this Section, skipping directly to (4.9) for the value function under PT. For those unfamiliar with PT, we provide a brief, self-contained, treatment below.

The basic building blocks of prospect theory can be heuristically explained as follows.[35] Prospect theory distinguishes between two phases in decision making: an *editing phase,* followed by an *evaluation phase.* In the editing phase, a complex problem is first simplified to facilitate decision making. In the evaluation phase, the highest value prospect is chosen. During the editing phase outcomes are coded as *gains* or *losses* relative to a *reference point.* The reference point is usually, but not necessarily, the status quo.[36]

While there is no general theory of the editing phase, prospect theory has a very precise theory of the evaluation phase. Suppose that a consumer faces a lottery (or prospect) with several possible outcomes. First, each outcome in the prospect is assigned a number, using a *utility function.* This number is a positive real number if it has been coded as a gain relative to the reference point, and a negative number if it has been coded as a loss (the reference point having been arrived at in the editing phase). The utility function under prospect theory has the following properties: *continuity*, *monotonicity*, *reference dependence*, *declining sensitivity* and *loss aversion.* Continuity and monotonicity are as in EUT.

Unlike EUT, where the carriers of utility are final levels of wealth (or incomes or consumption levels or commodities), under prospect theory the carriers of utility are gains and losses relative to the reference point. Declining sensitivity means that the utility function is concave in the domain of gains and convex in the domain of losses. Loss aversion is based on the idea that losses are more salient than gains. Given an amount of money, $y > 0$, and a utility function, $v(y)$, (to be specified precisely below) loss aversion implies that $v(y) > -v(-y)$.

Finally, the utilities of each outcome in a prospect are aggregated using *decision weights* into a *value function.* These decision weights are non-linear functions of the cumulative probabilities. Probabilities in the two domains (gains and losses) are cumulated separately. The *probability weighting function* used for the domain of gains need not be the same as that for losses. Decision weights are not probabilities and do not, necessarily, add up to one (unlike EUT). However, if all outcomes are either in the domain of gains or if all are in the domain of losses, then decision weights do add up to one and can be interpreted as probabilities. Agents facing uncertain situations overweight small probabilities but underweight large ones.[37]. In choosing among several prospects, an individual using PT

[35] There is a substantial body of evidence in support of these building blocks of prospect theory, as well as a mounting number of successful applications in economics; see, for instance, the collection of papers in Kahneman and Tversky (2000) and in Camerer et al. (2004).

[36] Also, in the editing phase, it is decided which low probability events to ignore and which high probability events to treat as certain.

[37] In Kahneman and Tversky (1979) decision weights are transformed probabilities. This proved unsatis-

chooses the one that gives rise to a higher number for the value function.

We now provide a more formal treatment of the building blocks of prospect theory.

4.1. Utility of an outcome under PT

Since the consumer takes public provision as given when deciding how much tax to evade, let us assume that public provision is ignored in the editing phase. Let the reference private consumption of the taxpayer be R. Then, private consumption relative to the reference point is

$$X_i = Y_i - R,\ i = C, NC. \tag{4.1}$$

As in Kahneman and Tversky (1979) and Tversky and Kahneman (1992), the utility, $v(X_i)$, associated with an outcome X_i is given by

$$v(X_i) = \begin{cases} X_i^{\beta} & \text{if } X_i \geq 0, \\ -\theta\left(-X_i\right)^{\beta} & \text{if } X_i < 0, \end{cases} \tag{4.2}$$

where $\theta > 1$ is the parameter of loss aversion; it ensures that a loss is more salient than a gain of equal monetarily value. Based on experimental evidence, Tversky and Kahneman (1992) suggest that $\beta \simeq 0.88$ and $\theta \simeq 2.25$.[38]

4.2. The reference point under PT

Although prospect theory does not provide sufficient guidance to determine the reference point in each possible situation, there is often a plausible candidate for a reference point. Indeed, specifying a suitable reference point is essential for a successful application of prospect theory.

As in Dhami and al-Nowaihi (2007), we take the legal after-tax income (which is also the level of private consumption expenditure in the absence of tax evasion) as the reference point in this paper.[39] Hence,

$$R = (1 - t)Y. \tag{4.3}$$

factory. So, in Tversky and Kahneman (1992), decision weights are transformed cumulative probabilities. This produced cumulative prospect theory (CPT). However, since we have exactly one outcome in the domain of gains and exactly one outcome in the domain of losses, CPT reduces to PT. If non-linear transformation of probabilities is combined with EUT, we get *rank dependent expected utility* (RDEU). The attraction of RDEU is that it can be regarded as EUT applied to the transformed cumulative probability distribution. Hence, the whole machinery of analysis of risk developed for EUT can be transferred to RDEU. See Quiggin (1993) for details. While RDEU can explain some anomalies of EUT it cannot explain others. For the latter, we need PT. For an application of RDEU to tax evasion, see Eide (2001).

[38] See al-Nowaihi, Bradley and Dhami (2007) for an axiomatic derivation of the form (4.2).

[39] Arguments for this particular reference point are given in Dhami and al-Nowaihi (2007). Alternative reference points are analyzed in Bernasconi and Zanardi (2004).

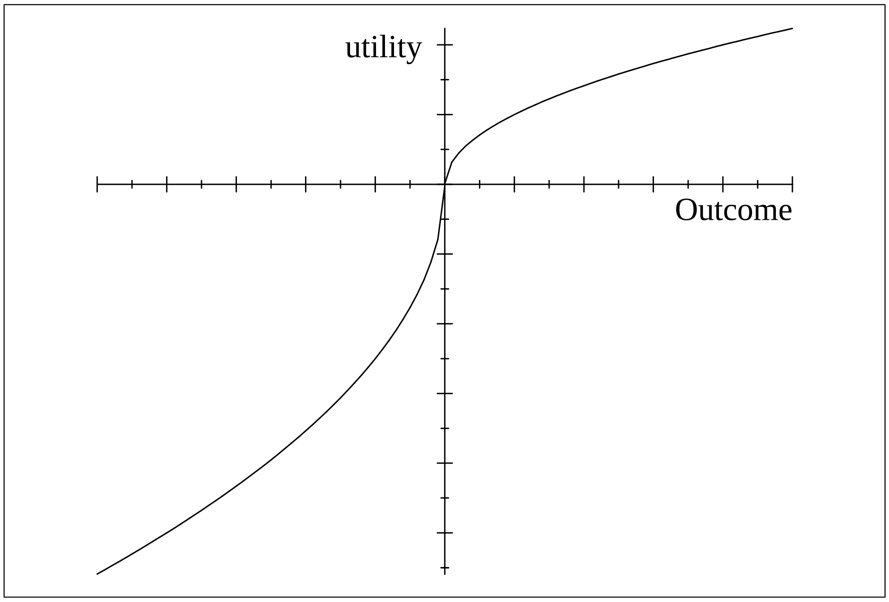

Figure 4.1: **Plot of prospect theory preferences for** $\beta = 0.5$, $\theta = 2.5$

4.3. The decision problem under PT

From (4.1), (4.3) $X_{NC} = Y_{NC} - (1-t)Y$ and $X_C = Y_C - (1-t)Y$. Then, using (2.12), (2.13) and recalling that $0 \leq D_{\preceq} \leq Y$, we get

$$X_{NC} = t(Y - D_{\preceq}) \geq 0, \tag{4.4}$$

$$X_C = -(s + \lambda t)(Y - D_{\preceq}) \leq 0. \tag{4.5}$$

Hence, the taxpayer is in the *domain of losses* if caught but in the *domain of gains* if not caught. Let v be the taxpayer's value function and w^+, w^- be her probability weighting functions for the domains of gains and losses, respectively.[40] Then, according to *prospect theory* (PT), the taxpayer maximizes:

$$V = w^-(p)v(X_C) + w^+(1-p)v(X_{NC}), \tag{4.6}$$

Comparing (4.6) with the analogous expression (3.1) for expected utility theory, we see the following differences. First, the carriers of utility in PT are gains and losses relative to the reference point rather than final levels. Second, one uses decision weights in PT to aggregate outcomes while one uses objective probabilities under expected utility theory.

[40] By a *probability weighting function* we mean a strictly increasing function $w : [0,1] \overset{onto}{\longrightarrow} [0,1]$. Note that a probability weighting function, w, has a unique inverse, $w^{-1} : [0,1] \overset{onto}{\longrightarrow} [0,1]$ and that w^{-1} is strictly increasing. Furthermore, it follows that w and w^{-1} are continuous and must satisfy $w(0) = w^{-1}(0) = 0$ and $w(1) = w^{-1}(1) = 1$. For an example of a probability weighting function, see below.

Third, the level of public provision, G, is present in (3.1) but absent for (4.6). This is because, in EUT, a decision maker has a complete preference relation over all outcomes.

From (4.2), (4.4), (4.5) and (4.6) we get:

$$V = t^{\beta}\left(Y - D_{\preceq}\right)^{\beta} w^{+}(1-p) - \theta\left(s + \lambda t\right)^{\beta}\left(Y - D_{\preceq}\right)^{\beta} w^{-}(p). \tag{4.7}$$

4.4. The probability weighting function

Empirical evidence is widely consistent with an inverted S shaped form for the weighting function; see for example Kahneman and Tversky (1979), Tversky and Kahneman (1992) and Prelec (1998). Denoting by p the cumulative probability, Prelec (1998) derives the following weighting function (see Figure 4.2).[41] By the *Prelec function* we mean the probability weighting function $w : [0,1] \xrightarrow{onto} [0,1]$ given by[42]:

$$w(p) = w^{+}(p) = w^{-}(p) = \exp\left[-(-\ln p)^{\alpha}\right], \tag{4.8}$$

where $0 < \alpha \leq 1$. The smaller α is, the more the overweighting of small probabilities

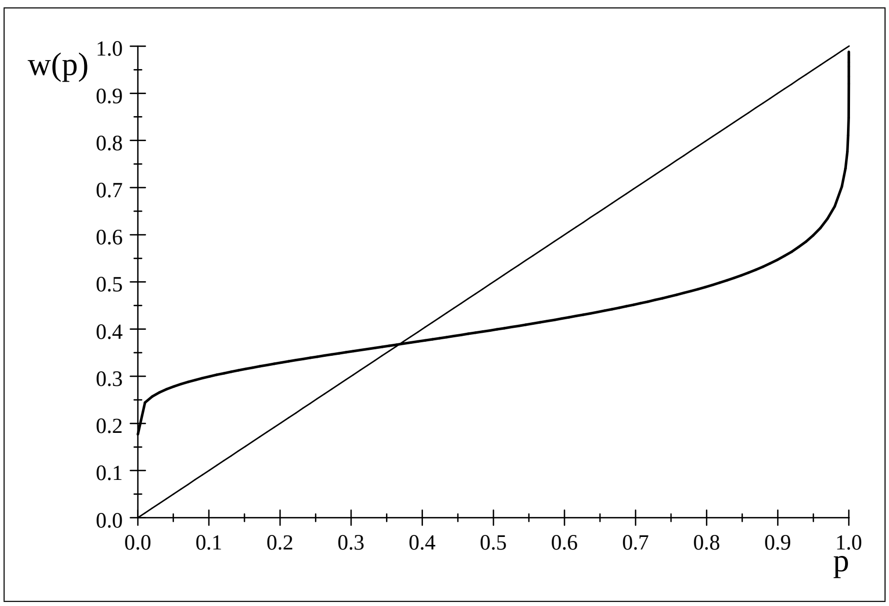

Figure 4.2: **A graph of the Prelec weighting function for** $\alpha = 0.225$

[41] There are several advantages of using the Prelec weighting function, relative to the others suggested in the literature. First, it has an inverted S shape which is consistent with experimental evidence. Second, it is based on axiomatic foundations; see Prelec (1998), Luce (2001) and al-Nowaihi and Dhami (2006). Third, it has the same form for gains and losses.

[42] To quote from Prelec (1998, last line of Appendix A): "Empirically ... one observes $w^{+}(p) = w^{-}(p)$." Therefore, in our calibration exercises, we shall take $w^{+}(p) = w^{-}(p)$.

and $w(p) \to p$ as $\alpha \to 1$. Thus, the weights approach objective probabilities as $\alpha \to 1$.[43] Figure 4.2 plots the Prelec function for $\alpha = 0.225$.[44]

4.5. Tax evasion under PT

Substituting (4.8) in (4.7) we get

$$V = t^{\beta} (Y - D_{\preceq})^{\beta} \exp\left[-(-\ln(1-p))^{\alpha}\right] - \theta (s + \lambda t)^{\beta} (Y - D_{\preceq})^{\beta} \exp\left[-(-\ln p)^{\alpha}\right]. \quad (4.9)$$

Recall that $s \in [0, 1]$. We are first interested in defining a critical level of stigma, s_c such that all taxpayers with stigma lower than s_c choose to evade taxes and all taxpayers with stigma higher than s_c choose to declare their full income. Let

$$\psi = \left(\frac{\exp\left[-(-\ln(1-p))^{\alpha}\right]}{\theta \exp\left[-(-\ln p)^{\alpha}\right]} \right)^{\frac{1}{\beta}} - \lambda, \quad (4.10)$$

$$\begin{array}{lll} s_c = 0 & if & \psi t \leq 0 \\ s_c = \psi t & if & 0 < \psi t < 1 \\ s_c = 1 & if & \psi t \geq 1 \end{array} \quad (4.11)$$

then (4.9) can be written as:

$$V = (Y - D_{\preceq})^{\beta} \theta \left[(s_c + \lambda t)^{\beta} - (s + \lambda t)^{\beta} \right] \exp\left[-(-\ln p)^{\alpha}\right]. \quad (4.12)$$

From (4.12), we see that the optimal values of $D_{\preceq}$ are

$$\begin{array}{lll} \text{Case-I:} & D_{\preceq} = 0 & \text{if } s < s_c, \\ \text{Case-II:} & \text{any } D_{\preceq} \in [0, Y] & \text{if } s = s_c, \\ \text{Case-III:} & D_{\preceq} = Y & \text{if } s > s_c. \end{array} \quad (4.13)$$

The solution to the tax evasion problem under PT, when the probability of detection is fixed, is a *bang-bang (or corner) solution*. We would argue that the *bang-bang solution* seems descriptive of several forms of tax evasion which take the form of hiding certain activities completely from the tax authorities while fully declaring other sources. For instance, an academic might not report income arising from an invited but paid lecture. A school teacher might not report tuition income for after-school lessons. A householder

[43] There is no reason to suppose that different uncertain situations should have the same α, hence, prospect theory does not impose an exact value on α. For instance, people might overweight the probability of dying in an air crash far higher relative to dying in a car accident.

[44] Based on experimental data, Prelec (1998) estimates $\alpha \simeq 0.65$. However, Bernasconi (1998) argues that, because of ambiguity aversion, taxpayers 'in the wild' would exhibit more overweighting of low probabilities than in the laboratory. Bernasconi (1998) reports that, while actual probabilities of audits are in the range 0.01 to 0.03, an average of USA taxpayers' assessments of the audit probability is 0.09. Taking a central value, this gives the value $\alpha = 0.225$ that we use.

might pay cash to a builder for a minor extension of the house. Line item reporting of tax returns might further encourage this behavior. This, bang-bang, implication of reporting taxable income can also be drawn from the experimental results of Pudney et al. (2000). Slemrod and Yitzhaki (2002) find, based on TCMP data for 1988, that "the voluntary reporting percentage was 99.5% for wages and salaries, but only 41.4% for self-employment income". Additional support comes from the behavior of non-profit organizations whose profits from activities unrelated to their primary tax exempt purpose are subject to federal and state tax. The reporting behavior of such organizations is also suggestive of the bang-bang solution; see Omer and Yetman (2002).[45]

Assuming non-increasing absolute risk aversion, EUT predicts (Yitzhaki, 1974) that individuals evade *less* income as the tax rate *increases*. On the other hand, PT predicts the more factual result that tax evasion increases with an increase in the tax rate. This is formally stated in Proposition 1 below. The proof of this, and other results, can be found in Dhami and al-Nowaihi (2007).

Proposition 1 : *Ceteris-paribus,* $\exists\, t = t_c \in [0, 1]$ *such that the individual does not evade taxes if* $t < t_c$ *but evades taxes if* $t > t_c$.

Consider the more general case of an endogenous probability of detection $p(D_{\preceq})$ such that $p\left(D_{\preceq}\right)$ is continuously differentiable and $p'\left(D_{\preceq}\right) \leq 0$, as in Dhami and al-Nowaihi (2007). In this case, the declared income can have an interior solution and varies continuously with the exogenous parameters. The comparative static results in this Section, including, in particular, the explanation of the Yitzhaki puzzle, can also be demonstrated for the more general case.[46] However, the general case is not very conducive for undertaking calibration exercises which is the main method we use for distinguishing among alternative theories.[47]

[45] Dhami and al-Nowaihi (2007) introduce an endogenous probability of detection $p(D)$ such that $p\left(D\right)$ is continuously differentiable and $p'\left(D\right) \leq 0$ i.e. the taxpayer is more likely to be caught if s(he) evades more. This induces enough curvature in the model for interior solutions. However, this does not alter the comparative static results.

[46] For the general case, Dhami and al-Nowaihi (2007, Proposition 4) prove the following: (a) At a regular interior optimum, tax evasion is strictly decreasing in the punishment rate, λ, the stigma rate, s, and the coefficient of loss aversion, θ. However, tax evasion is strictly increasing in the tax rate, t.
(b) At an optimum on the boundary ($D^* = 0$ or $D^* = Y$), tax evasion is non-increasing in the punishment rate, λ, the stigma rate, s, and the coefficient of loss aversion, θ. Tax evasion is non-decreasing in the tax rate, t.

[47] Even in macroeconomics where calibration is most prevalent, despite the non-linearities it is usually the simplified log-linearized version of the model that is used in the calibration exercises.

4.6. Tax revenue under PT

From (4.13) we get:

$$\begin{aligned}\overline{D_{\{\preceq\}}}(t) &= \int_{x=0}^{1}\int_{s=0}^{1} D_{\preceq}(t,G,x,s)\,dxds \\ &= \int_{x=0}^{1}\int_{s=0}^{s_c} D_{\preceq}dxds + \int_{x=0}^{1}\int_{s=s_c}^{1} D_{\preceq}dxds \\ &= \int_{x=0}^{1}\int_{s=s_c}^{1} Y dxds. \end{aligned} \tag{4.14}$$

Now, invoke our assumption that stigma is independent of pretax income, $Y(x,s) = Y(x)$. Then, **using** (2.11), (4.11) and (4.14), we get:

$$\overline{D_{\{\preceq\}}}(t) = \begin{cases} \overline{Y} & if \quad \psi t \leq 0 \\ (1-\psi t)\overline{Y} & if \quad 0 < \psi t < 1 \\ 0 & if \quad \psi t \geq 1 \end{cases} \tag{4.15}$$

From (2.10), (4.15) and the government budget constraint, $G = \overline{T}_{\{\preceq\}}(t,G)$, we get total tax revenue:

$$G(t) = \begin{cases} \overline{Y}t & if \quad \psi t \leq 0 \\ \overline{Y}t\left[1-(1-\omega)(1-p(1+\lambda))\psi t\right] & if \quad 0 < \psi t < 1 \\ \overline{Y}t\left[\omega + p(1+\lambda)(1-\omega)\right] & if \quad \psi t \geq 1 \end{cases} \tag{4.16}$$

5. Optimal tax under EUT (regime EUT)

We have, so far, considered the tax evasion decision of taxpayers when they respectively follow EUT and PT. We have also computed the associated tax revenues under each form of preferences. We now analyze the optimal tax decision of a central planner under EUT.

The objective function of the government is explained in Section 2.4 above, for the case of regime EUT. It chooses the tax rate so as to maximize the utility of a representative consumer, with average income, $\overline{Y}$, in the state where he does not evade[48],

$$U = \mu \ln\left[G(t)\right] + (1-\mu)\ln\left[(1-t)\overline{Y}\right], \tag{5.1}$$

subject to the government budget constraint (3.16), where $\overline{D_{\{\preceq\}}}(t)$, s_1 and s_2 are given by (3.14), (3.8) and (3.9), respectively. We find the solution to this maximization problem by a simple search over values of $t \in [0,1]$.

Calibration Values: We use the values $p = 0.015$ and $\lambda = 0.5$ that appear typical for the USA (Alm et al. (1992), Andreoni et al. (1998), Bernasconi (1998)). From Slemrod

[48]Note that, like the average family with 2.1 children, there may be no consumer in Ω with average income, who does not evade.

and Yitzhaki (2002) and Andreoni et al. (1998) we can infer the value $\omega = 0.4$. While the general view seems to be that stigma costs from evasion are low, for example see Brooks (2001), we are not aware of the exact magnitudes. Some evidence is available from stigma costs that arise from claiming welfare benefits. For Britain, Pudney et al. (2002) find that the total stigma costs (which they define as stigma, hassle, search costs, etc.) range from about 0.1 to 0.2. We do not know the appropriate value of μ, which determines society's preference for public compared to private consumption.

Calibration Strategy: Our strategy is as follows. For values of μ in the interval $(0, 1)$, we compute the optimal tax rate, the resulting level of tax revenue, $G(t)$, and the tax gap ratio, $\frac{t\overline{Y}-G(t)}{G(t)}$, where the denominator is total tax revenue and the numerator (the tax gap) is the difference between what is theoretically owed in taxes ($t\overline{Y}$) and what is actually collected in taxes ($G(t)$).

For the USA, the tax gap ratio for 1992 was 0.222 and for 1998 it was 0.199 (Americans for Fair Taxation). For the USA, total government tax revenue, as a percentage of GDP was 32% for 2004 (Laurin, 2006). We use the normalization $\overline{Y} = 100$.

Calibration Results: The calibration results are tabulated in Table-II.

Table-II (Optimal tax, tax gap and government spending under EUT)

μ	0.1	0.2	0.3	**0.4**	0.41	0.5	0.6	**0.65**	0.7	0.8	0.9
t	0.10	0.22	0.37	**0.52**	0.54	0.66	0.73	**0.77**	0.80	0.87	0.94
s_1	0.73	0.62	0.43	**0.21**	0.19	0.002	0	**0**	0	0	0
s_2	1	1	1	**1**	1	1	1	**1**	1	1	1
$G(t)$	4.39	10.11	18.91	**31.71**	33.23	48.23	58.22	**64.10**	68.60	80.87	90.34
$\frac{t\overline{Y}-G(t)}{G(t)}$	1.28	1.18	0.94	**0.64**	0.61	0.37	0.25	**0.20**	0.17	0.09	0.04

From Table-II we see that as society's desire for public provision (measured by μ) increases, so does the optimal tax rate, t, and government tax revenue, $G(t)$. This result is exactly what is expected. However, tax evasion as measured by the tax gap ratio, $\frac{t\overline{Y}-G(t)}{G(t)}$, drops very dramatically with an increase in the tax rate, from the very high value of 1.28 to the very low value of 0.04. Thus tax evasion can be practically eliminated by a tax rate of almost 100%. This is the *Yitzhaki puzzle* under EUT described above.

From the fourth row we see that, at all tax rates, $s_2 = 1$ (see (3.9), (3.10)), i.e., all taxpayers who can evade, evade at least some tax. From the first column, we see that $s_1 = 0.73$ (see (3.8), (3.10)), i.e., of taxpayers who can evade, 73% evade all taxes. This steadily declines as the tax rate increases so that for tax rates in excess of 73%, all tax payers pay at least some tax.

From the first highlighted column, we see that at the tax rate $t = 0.52$, government tax revenue is similar to actual values ($G(0.52) = 31.71\%$, compared to the actual value of

32%). However, the predicted tax gap ratio is far too high ($\frac{0.52\overline{Y}-G(0.52)}{G(0.52)} = 0.64$, compared to the actual value of about 0.2). From the second highlighted column, we see that the tax rate $t = 0.77$ gives a tax gap ratio close to that observed ($\frac{0.77\overline{Y}-G(0.77)}{G(0.77)} = 0.2$). However, the same tax rate gives a total tax revenue ($G(t) = 64.1\%$) that is about twice the observed value. To summarize, *our simple EUT optimal tax model with tax evasion can either get tax revenue right or the tax gap right but not both.*

5.1. Robustness of the results

We do not think that this result is merely the consequence of using a simple model, but appears to be a problem inherent in the EUT approach to tax evasion. We now give some supporting arguments.

1. (*Risk aversion*) The use of logarithmic utility entails a coefficient of relative risk aversion of 1, which is rather low. What results can we expect when a government assigns, in its social welfare function, a higher value for the coefficient of risk aversion? Bernasconi (1998) reports that the coefficient of relative risk aversion lies in the range 1 to 2. However, Skinner and Slemrod (1985) report that a coefficient of relative risk aversion of 70 is needed to square the extent of tax evasion under EUT with the evidence.[49] Our simulations (7) support this, suggesting that increasing the coefficient of relative risk aversion to 2 would not make much difference.

2. (*More general preferences*) The simulations in Section 7 also suggest that the additively separable form of the utility function for the representative taxpayer is not the problem, nor is the assumption of a representative consumer that is used to evaluate social welfare.

3. (*Stigma*) Increasing stigma cost would reduce tax evasion but, as discussed above, our assumed level of stigma appears already to be on the high side.

4. (*Other factors*) How about realistic features of the tax system that we have not included? For example, forms of taxes other than income taxes, fraudulent claims of benefits and costs of enforcing tax compliance. But in each of these cases we expect the paradox to reappear: at a tax rate that would generate the observed government tax revenue, EUT predicts too much evasion. Because of the Yitzhaki (1974) result, an increase in the tax rate that would reduce evasion to observed magnitudes would

[49] Mankiw and Zeldes (1991) calculated that a coefficient of relative risk aversion of 30 implies that the certainty equivalent of the prospect win $50,000$ or $100,00$ each with probability 0.5 is $51,209$. This is clearly absurd.

simultaneously increase tax revenue to well above observed values. Modelling labour supply has its own problems.[50]

6. Optimal tax under PT (regimes PT1 to PT4)

We now consider the four regimes, PT1 to PT4. In all four of these regimes, the taxpayer uses PT in making the tax evasion decision.

6.1. Regime PT1

For PT1, consumer-taxpayers' evasion decisions are described by PT, as in Section (4) above. In particular, tax revenues are now given by (4.16). However, their preferences over private and public consumption are given by the same utility function as in regime EUT.[51] The government's objective is still to maximize (5.1), the utility of a representative consumer-taxpayer with average income who does not evade. But now, the relevant budget constraint is (4.16). It is routine, though tedious, to show that this optimizing problem has a unique maximum, and that the resulting optimal tax rate is in the interior of the interval $[0,1]$. Hence, it can be found by solving the first order condition $U'(t)=0$. Some simple algebra show that this is equivalent to solving $\mu(1-t)\frac{\partial G(t)}{\partial t}=(1-\mu)G(t)$. The solution is:

$$t=\mu,\text{ for }\omega=1,\ \psi\mu\geq 1\text{ or }\psi\leq 0. \tag{6.1}$$

If $\omega=1$, then no one can evade. If $\psi\leq 0$ then all those who can evade decide not to. If $\psi\mu\geq 1$ then all those who can evade do so. In all these cases, the optimal tax rate is $t=\mu$.

However, the more empirically relevant case is when $\omega<1$ and $0<\psi\mu<1$. Here some taxpayers can evade but choose not to. In this case, the first order condition gives the quadratic equation in t:

$$(1+\mu)(1-\omega)(1-p(1+\lambda))\psi t^2-[1+2\mu(1-\omega)(1-p(1+\lambda))\psi]t+\mu=0 \tag{6.2}$$

The optimum is the solution with the negative square root. It is given by:

$$t=\frac{1+2\mu(1-\omega)(1-p(1+\lambda))\psi-Z}{2(1+\mu)(1-\omega)(1-p(1+\lambda))\psi},\ \omega<1,\ 0<\psi\mu<1,$$

[50] A large number of studies show a small *negative* aggregate labour supply elasticity of income (see, for example, Pencavel (1986) and Killingworth and Heckman (1986)). Clearly this would compound rather than solve the difficulties faced by EUT in explaining tax evasion. Many studies also show a small positive aggregate labour supply elasticity and some show a large positive elasticity. Many problems of applying EUT to labour supply are discussed in Bewley (1999).

[51] Recall the discussion in subsection 1.4, in paricular interpretations 1 (paternalism) and 2 (liberalism plus context dependent preferences). Our results do not depend on which interpretation is chosen. This comment applies to regimes PT2 and PT3 as well.

where,

$$Z=\sqrt{\left[1+2\mu\left(1-\omega\right)\left(1-p\left(1+\lambda\right)\right)\psi\right]^{2}-4\mu\left(1+\mu\right)\left(1-\omega\right)\left(1-p\left(1+\lambda\right)\right)\psi}.$$

6.2. Regime PT2

The utility function (5.1) is additively separable over private and public provision. However, in actual practice, there are strong complementarities between the two. To take account of this, the utility function is now given by (2.17), as explained in Section 2.4, under the regime PT2. In this model, we see that public provision has value only insofar as it facilitates private consumption. As with regime PT1, the government chooses the tax rate, t, so as to maximize the utility of a representative consumer with average income who does not evade, but now with the utility function:

$$U\left(t\right)=F\left(\left(1-t\right)\overline{Y}-\frac{B}{G\left(t\right)}\right),\ F'>0,\ F''<0,\ G\left(t\right)>0,\ 0<\frac{B}{G\left(t\right)}<\left(1-t\right)\overline{Y},\quad (6.3)$$

where $G\left(t\right)$ is given by the budget constraint (4.16). The condition $0<\frac{B}{G(t)}<\left(1-t\right)\overline{Y}$, together with (4.16), guarantees that any optimum must be an interior point. Hence, it must satisfy the first order condition $U'\left(t\right)=0$. Some simple algebra shows that this is equivalent to solving $B\frac{\partial G}{\partial t}=\overline{Y}G^{2}$. Using (4.16), the solution is

$$t=\begin{cases}\frac{\sqrt{B}}{\overline{Y}} & for\quad \omega=1 \text{ or } \psi\leq 0\\ \frac{1}{\overline{Y}}\sqrt{\frac{B}{\omega+p(1+\lambda)(1-\omega)}} & for\quad \psi t\geq 1\end{cases}\quad (6.4)$$

If $\omega=1$, then no one can evade. If $\psi\leq 0$ then all those who can evade decide not to. In both cases, the optimal tax rate is $t=\frac{\sqrt{B}}{\overline{Y}}$. For the case $\psi t\geq 1$, all those who can evade, do evade. However, the most empirically relevant case is when $0<\psi t<1$. Here some taxpayers can evade but choose not to. In this case, the first order condition gives the quartic equation in t:

$$\begin{aligned}&\left(1-\omega\right)^{2}\left(1-p\left(1+\lambda\right)\right)^{2}\psi^{2}\rho t^{4}-2\left(1-\omega\right)\left(1-p\left(1+\lambda\right)\right)\psi\rho t^{3}+\rho t^{2}\\ &+2\left(1-\omega\right)\left(1-p\left(1+\lambda\right)\right)\psi t-1=0;\ 0<\psi t<1 \text{ and } \rho=\frac{\left(\overline{Y}\right)^{2}}{B}.\end{aligned}\quad (6.5)$$

(6.5) has, of course, four roots. However, in simulations we always found the optimum to be the only root in the interval $(0,1)$.

6.3. Regime PT3

So far, we have considered a representative consumer. An alternative welfare criterion is for the government to maximize the sum (or average) utility. For this we need to specify an income distribution. We choose the gamma distribution because of its tractability and because it gives quite a good representation of income distribution for middle incomes (which are the most relevant for determining aggregate tax revenue: the poor do not have much income and the rich are few in number) see, for example, Cowell (2000, p146). The appendix gives all that we need to know about the gamma distribution.

As explained in Section 2.4, under the regime PT3, we generalize regime PT1 in two ways. First, society's preference over private and public provision is given by the more general utility function (2.18). Second, instead of a representative consumer, the government now chooses the tax rate, t, so as to maximize the sum of utilities (or the average utility). Thus, the social welfare function, $W(t)$, is given by

$$W(t) = \int_{x=0}^{1} \int_{s=0}^{1} \left[\mu \frac{[G(t)]^{1-\gamma}}{1-\gamma} + (1-\mu) \frac{[(1-t)Y]^{1-\gamma}}{1-\gamma} \right] dxds, \tag{6.6}$$

where $G(t)$ is given by (4.16). Thus, the government chooses the tax rate, t, so as to maximize society's total (or average) utility in the state where none evades tax. However, the government does allow for the effect on the tax revenue, $G(t)$, of tax evasion as described by PT. (6.6) simplifies to:

$$W(t) = \mu \frac{(G(t))^{1-\gamma}}{1-\gamma} + \frac{1-\mu}{1-\gamma}(1-t)^{1-\gamma} \int_{x=0}^{1} \int_{s=0}^{1} Y^{1-\gamma} dxds. \tag{6.7}$$

The combination of the power form (2.18) for utility with the gamma distribution for income (10.1) leads to very tractable math. In particular, we get

$$\begin{aligned} \int_{x=0}^{1} \int_{s=0}^{1} Y^{1-\gamma} dxds &= \frac{1}{b^a \Gamma(a)} \int_{Y=0}^{\infty} Y^{1-\gamma} Y^{a-1} e^{-\frac{Y}{b}} dY \\ &= \frac{b^{1-\gamma}\Gamma(1+a-\gamma)}{\Gamma(a)} \frac{1}{b^{1+a-\gamma}\Gamma(1+a-\gamma)} \int_{Y=0}^{\infty} Y^{(1+a-\gamma)-1} e^{-\frac{Y}{b}} dY \\ &= \frac{b^{1-\gamma}\Gamma(1+a-\gamma)}{\Gamma(a)} \end{aligned} \tag{6.8}$$

From (6.7) and (6.8), we get:

$$W(t) = \mu \frac{[G(t)]^{1-\gamma}}{1-\gamma} + \frac{1-\mu}{1-\gamma}(1-t)^{1-\gamma} \frac{b^{1-\gamma}\Gamma(1+a-\gamma)}{\Gamma(a)}. \tag{6.9}$$

Let

$$\varphi = \frac{1-\mu}{1-\gamma}(1-t)^{1-\gamma} \frac{b^{1-\gamma}\Gamma(1+a-\gamma)}{\Gamma(a)}.$$

From (4.16) and (6.9), we get:

$$W(t) = \begin{cases} \mu(1-\gamma)^{-1}(ab)^{1-\gamma} t^{1-\gamma} + \varphi & if \quad \psi \leq 0 \\ \mu(1-\gamma)^{-1}(ab)^{1-\gamma} \left[t - (1-\omega)(1-p(1+\lambda))\psi t^2\right]^{1-\gamma} + \varphi & if \quad 0 < \psi t < 1 \\ \mu(1-\gamma)^{-1}\left[\omega + p(1+\lambda)(1-\omega)\right]^{1-\gamma}(ab)^{1-\gamma} t^{1-\gamma} + \varphi & if \quad \psi t \geq 1 \end{cases} \tag{6.10}$$

Maximizing $W(t)$ in the two cases $\psi \leq 0$ and $\psi t \geq 1$ is simple and leads to the optimal tax rates:

$$t = \begin{cases} \left[1 + \left(\frac{1-\mu}{\mu}\right)^{\frac{1}{\gamma}} \left(\frac{\Gamma(1-\gamma+a)}{a^{1-\gamma}\Gamma(a)}\right)^{\frac{1}{\gamma}}\right]^{-1} & if \quad \psi \leq 0 \\ \frac{[\omega+p(1+\lambda)(1-\omega)]^{\frac{1-\gamma}{\gamma}}}{[\omega+p(1+\lambda)(1-\omega)]^{\frac{1-\gamma}{\gamma}} + \left(\frac{1-\mu}{\mu}\right)^{\frac{1}{\gamma}} \left(\frac{\Gamma(1-\gamma+a)}{a^{1-\gamma}\Gamma(a)}\right)^{\frac{1}{\gamma}}} & if \quad \psi t \geq 1 \end{cases} \tag{6.11}$$

When $\psi \leq 0$, no taxpayer evades. When $\psi t \geq 1$, all those who can evade, do evade. However, the empirically more relevant case is when $0 < \psi t < 1$, where some who can evade do so, but others who can evade choose not to. For this case, the first order condition, $W'(t) = 0$ gives the following non-linear equation:

$$\begin{aligned} & [1 - 2(1-\omega)(1-p(1+\lambda))\psi t]^{\frac{1}{\gamma}} (1-t) \\ = & \left(\frac{1-\mu}{\mu}\right)^{\frac{1}{\gamma}} \left(\frac{\Gamma(1-\gamma+a)}{a^{1-\gamma}\Gamma(a)}\right)^{\frac{1}{\gamma}} \left[t - (1-\omega)(1-p(1+\lambda))\psi t^2\right], \text{ if } 0 < \psi t < 1. \end{aligned} \tag{6.12}$$

Equation (6.12) can be easily solved numerically, given values for the parameters.

It is easy to check that, for $\gamma = 1$, (6.11) reduces to (6.1) and that (6.12) reduces to (6.2). Thus, the utilitarian regime, PT3, reduces to the representative consumer regime, PT1, for $\gamma = 1$.

6.4. Regime PT4

Finally, we consider a regime where tax evasion is described by PT and, also, the government chooses the tax rate, t, so as to maximize the sum (or average) of the PT utilities of the consumer-taxpayers. Thus social welfare is now given by:

$$W(t) = \int_{x=0}^{1} \int_{s=0}^{1} V dx ds. \tag{6.13}$$

Substituting from (4.12) into (6.13), we get:

$$W(t) = \int_{x=0}^{1} \int_{s=0}^{1} \theta w^{-}(p) \left[(s_c + \lambda t)^{\beta} - (s + \lambda t)^{\beta}\right] [Y - D_{\preceq}]^{\beta} dx ds. \tag{6.14}$$

From (4.13) and (6.14), we get

$$W(t) = \int_{x=0}^{1} \int_{s=0}^{s_c} \theta w^{-}(p) \left[(s_c + \lambda t)^{\beta} - (s + \lambda t)^{\beta}\right] Y^{\beta} dx ds. \tag{6.15}$$

Invoking our assumption of independence of stigma and income we get, from (6.15),

$$\begin{aligned} W(t) &= \int_{s=0}^{s_c} \theta w^{-}(p) \left[(s_c + \lambda t)^{\beta} - (s + \lambda t)^{\beta}\right] ds \int_{x=0}^{1} Y^{\beta} dx \\ &= \theta w^{-}(p) \left[s_c (s_c + \lambda t)^{\beta} - \frac{1}{(\beta+1)} \left[(s_c + \lambda t)^{\beta+1} - (\lambda t)^{\beta+1}\right]\right] \int_{x=0}^{1} Y^{\beta} dx. \end{aligned} \tag{6.16}$$

From (4.11) and (6.16), we get:

$$W(t) = \begin{cases} 0 & if \quad \psi \leq 0 \\ \theta w^{-}(p) t^{\beta+1} \left[\psi (\psi+\lambda)^{\beta} - \frac{(\psi+\lambda)^{\beta+1} - \lambda^{\beta+1}}{1+\beta} \right] \int\limits_{x=0}^{1} Y^{\beta} dx & if \quad 0 < \psi t < 1 \\ \theta w^{-}(p) \left[(\lambda t)^{\beta} - \frac{(\lambda t)^{\beta+1} - (\lambda t)^{\beta+1}}{1+\beta} \right] \int\limits_{x=0}^{1} Y^{\beta} dx & if \quad \psi t \geq 1 \end{cases} \tag{6.17}$$

We shall consider only the case $0 < \psi t < 1$, since this is the empirically relevant one. Here, some taxpayers who can evade do, but others who can evade do not. It can be checked that the optimal tax depends on the sign of

$$\Delta = \left[\psi (\psi+\lambda)^{\beta} - \frac{1}{1+\beta} \left[(\psi+\lambda)^{\beta+1} - \lambda^{\beta+1} \right] \right]. \tag{6.18}$$

If $\Delta < 0$, then welfare is strictly decreasing in t. Hence, the optimal tax rate is zero. This result is not surprising at all, since public provision does not figure in the utility functions, thus increasing the tax rate reduces private consumption without any gain. If $\Delta > 0$, then welfare is strictly increasing in the tax rate. Thus, the optimal tax rate is $t = 1$ and the 'optimum' is for the government to confiscate all the wealth of those who cannot evade or will not evade. This causes a huge loss in private consumption, but with no gain, since public provision does not figure in the utility functions.

The reason for this absurd result is that taxation shifts the reference point down. Those who cannot, or will not, evade are always at their reference point, so they always receive zero utility relative to the reference point. Those who evade, and are not caught, have a huge increase in utility, measured *relative* to their reference point. But this huge relative increase in relative utility has no real welfare significance. Surprisingly, it is this, the most absurd of cases, that is picked up by our simulations (see next Section). Hence, the message is that inferring preferences revealed in an inappropriate context can have disastrous policy effects.

7. Optimal taxation: PT and EUT compared

We now compare the relative success of PT and EUT in explaining tax evasion. We use the same parameter values as before, when taxpayers had expected utility preferences: $p = 0.015$, $\lambda = 0.5$, and $\omega = 0.4$. We take the values $\beta = 0.88$ and $\theta = 2.25$ from Tversky and Kahneman (1992). We adopt the Prelec probability weighting function, $w^{+} = w^{-} = e^{-(-\ln p)^{\alpha}}$, with the value $\alpha = 0.225$ implied by the data reported by Bernasconi (1998). The results are tabulated below, where *na* stands for 'not applicable'.

Table-III: Optimal taxes, spending and tax gaps in various regimes

Regime	μ	γ	B	t	s_1	s_2	s_c	$G(t)$	$\frac{t\overline{Y}-G(t)}{G(t)}$
EU	0.40	1	na	0.52	0.21	1	na	31.71	0.640
EU	0.65	1	na	0.77	0	1	na	64.10	0.200
PT1	0.44	1	na	0.387	na	na	0.282	32.28	0.198
PT2	na	na	1539	0.385	na	na	0.280	32.14	0.197
PT3	0.45	2	na	0.385	na	na	0.281	32.20	0.197
PT3	0.58	0.12	na	0.388	na	na	0.282	32.40	0.199
PT4	na	na	na	1	na	na	0.728	57.30	0.745

The first two rows are reproduced from Table-II, which gave the calibration results under regime EUT. Consumer-taxpayers are expected utility maximizers and use the same utility function to decide how much income to declare and also to express their preferences over private and public provision. The government uses this same utility function to determine the optimal tax rate. From the first row, we see that $\mu = 0.4$ gives approximately the correct level of total government tax revenues, $G(t)$, for the USA. However, the tax gap ratio, $\frac{t\overline{Y}-G(t)}{G(t)}$, is far too high, being about three times the correct value. On the other hand, from the second row, we see that $\mu = 0.65$ gives approximately the correct tax gap ratio for the USA. But then total government tax revenue is far too high, being about twice what it should be. The reason is that at the observed audit probabilities and penalty and stigma rates, evasion is very attractive to an expected utility maximizer. Hence, evasion is too high at observed levels of taxation. However, because of the Yitzhaki effect, *increasing* the tax rate *reduces* evasion. So, to get observed evasion rates, taxes have to be too high.

Rows 3 to 7 give the results when taxpayers use prospect theory to make their tax evasion decisions.

Row 3 gives the results for regime PT1. For PT1, consumer-taxpayers' evasion decisions are described by PT. However, their preferences over private and public consumption are given by the same utility function as in regime EUT.[52] The government chooses the tax rate so as to maximize the same welfare function as in regime EUT. However, the government takes into account the fact that subsequent tax revenues arise from the PT preferences of taxpayers.

Row 3 of the table shows that when consumer preferences over private and public consumption ($\mu = 0.44$) give a tax revenue ($G(t) = 32.280$) close to what is observed (32), we automatically get a tax gap ratio ($\frac{t\overline{Y}-G(t)}{G(t)} = 0.198$), close to what is observed (0.2).

[52] Recall the discussion in subsection 1.4, in paricular interpretations 1 (paternalism) and 2 (liberalism plus context dependent preferences). Our results do not depend on which interpretation is chosen. This comment applies to regimes PT2 and PT3 as well.

This suggests that PT provides the correct model for tax evasion.

The welfare function used in regime PT1 is additively separable over private and public consumption. By contrast, the welfare function used in regime PT2 exhibits strong complementarity between the two. From row 4, we see that when preferences between private consumption and public provision ($B = 1539$) give a tax revenue ($G(t) = 32.143$) close to the observed value (32), they also gives a value for the tax gap ratio ($\frac{t\overline{Y}-G(t)}{G(t)} = 0.197$) close to what is observed (0.2). Thus PT successfully explains tax evasion with two very different welfare functions.

For regime PT3, the criterion is to maximize aggregate (or average) utility, rather than that of a representative consumer of regimes PT1 and PT2. Also, regime PT3 generalizes PT1 in that it allows any (constant) coefficient of relative risk aversion, γ, while in PT1 this coefficient was $\gamma = 1$. In fact, for $\gamma = 1$, PT3 reduces to PT1. Since $\gamma = 1$ is at the low end of the range (between one to two) reported by Bernasconi (1998), in row 5 we choose $\gamma = 2$ to test the high end. The value $\mu = 0.45$ results in approximately the correct level of tax revenue ($G(t) = 32.197$) *and* gives a nearly correct tax gap ratio (0.197). Row 6 reports the results by setting $\gamma = 0.12$, which is consistent with the value of $\beta = 0.88$, reported by Tversky and Kahneman (1992).[53] The value $\mu = 0.575$ gives a near correct value for tax revenue (32.404) and, again, a near correct value for the tax gap ratio (0.199).

Using PT to model the tax evasion decision works for the following reasons. By taking the reference point to be the legal after-tax income, the taxpayer is in the domain of gains if not caught but in the domain of losses if caught.[54] This allows loss aversion, overweighting of the small probability of detection and underweighting of the high probability of non-detection to considerably increase the deterrence effect of punishment, despite low values of p and λ (and the mild convexity of the value function for losses). This facilitates the derivation of the correct government budget constraint. By contrast, under EUT we get the wrong government budget constraint.

A fundamental principle of liberalism is that people are the best judges of their own welfare. However, the regimes PT1-PT3 use one set of preferences, prospect theory, to model the consumer-taxpayer's tax evasion decision but another (standard utility theory) to describe the same consumer-taxpayer's preferences over private and public consumption (recall the discussion in Subsection 1.4). Furthermore, these regimes are successful in accounting for the observed facts on the tax gap and government expenditure.

[53] The low value, $\gamma = 0.12$, should not be interpreted to mean that consumers are nearly risk neutral. This is becaues, the attitude to risk in PT depends, not only on the curvature of the value function, but also on the coefficient of loss aversion and the probability weighting function.

[54] Proposition 3 of Dhami and al-Nowaihi (2007) establishes that a consumer is in the domain of gains if caught but in the domain of losses if not caught if, and only if, the reference point is the legal after tax income.

This conclusion is further reinforced by the results of regime PT4. Regime PT4 uses prospect theory to model the tax evasion decision (as in PT1-PT3), but *also* uses prospect theory as a welfare criterion. Row 7 shows that PT4 gives wildly wrong values for all the variables. Worse, it gives the economically absurd result that when neither consumers nor government care about public provision, then the optimal tax rate is 100%. This absurd result comes about because PT4 works by pushing down the reference point, so giving very high relative utility. But that does not correspond to any sensible measure of welfare. Thus, although PT gives the correct budget constraint in PT4, it also gives a completely wrong welfare criterion. The message is clear: inferring preferences from the wrong context can have disastrous policy effects.

8. Summary

At observed audit probabilities, penalty and stigma rates and attitudes to risk, tax evasion is very attractive to an expected utility maximizer. Hence, expected utility theory (EUT) predicts levels of evasion that are too high[55] at observed tax rates. On the other hand, because of the Yitzhaki effect, increasing the tax rate, under EUT, reduces evasion. So, to get observed evasion rates, taxes have to be too high. In addition, we found that EUT can either get tax revenue right or the tax gap right but not both. In particular, if consumer preferences are such, so as to yield the correct level of tax revenue, then the tax gap ratio is too high by a factor of over 3. On the other hand, if consumer preferences are such, so as to yield the correct tax gap ratio, then tax revenue is too high by a factor of almost exactly 2 (recall Table II and the discussion following it). We argued (Subsection 5.1) that we do not think that these results are merely the consequence of using simple models, but appear to be problems inherent in the EUT approach to tax evasion.

Under prospect theory (PT), taking the reference point to be the legal after-tax income ensures that the taxpayer is always in the domain of losses if caught and always in the domain of gains if not caught. Loss aversion, overweighting of low probabilities and the concavity of the value function for losses then ensure that punishment hurts more under PT than under EUT. Because of the underweighting of high probabilities, the prospect of not being caught is less attractive under PT than under EUT. Hence, the deterrence effect of punishment is much stronger under PT. Thus, using PT to model tax evasion produces the observed levels of tax evasion. Moreover, under PT, increasing the tax rate increases evasion, as is consistent with the bulk of empirical evidence.

Because of bounded rationality, consumer-taxpayers have to simplify considerably, concentrating on the salient features of the problem at hand. Hence, we cannot take their tax evasion behavior as revealing their preferences over private versus public consumption.

[55] By a factor of about 100, according to Dhami and al-Nowaihi (2007).

Thus, while PT gives the correct government budget constraint, it gives a completely wrong welfare criterion. As an illustration, we considered a regime (regime PT4) where the government maximizes the sum of the PT utilities of the consumers. The results turned out to be empirically incorrect and economically absurd. Thus, inferring preferences from the wrong context can have disastrous policy effects.

A fundamental assumption of neoclassical economics is that decision makers have a complete transitive ordering over all possible outcomes. Research over the last 60 years has shown that this is not valid, not even as a rough approximation. This research has also shown that consumer preferences are context-dependent. In line with this research, we assume that individuals, when making their tax evasion decision, exhibit behavior described by PT. But when these same individuals express their preferences over private and public provision through, say, surveys, referenda, elections, etc., these preferences are described by a standard utility function (alternatively, and as we show, one might invoke arguments based on paternalism). It is this standard utility function that figures in the social welfare function. With a simple general equilibrium model of optimal taxation in the presence of tax evasion, we were able to account well jointly for the observed magnitudes of important fiscal variables (tax rate, tax gap and tax revenues). We investigated the robustness of our model by considering three very different regimes (regimes PT1, PT2 and PT3, see Subsection 2.4). We have to stress that not only does PT perform better than EUT at explaining tax evasion, the predictions of EUT are grossly incorrect, while those of PT are almost exactly right (recall Table III and the discussion that followed it).

9. Acknowledgements

We are grateful for comments from an anonymous referee and discussions during invited seminar presentations at Cambridge, Keele, Leicester and Warwick. We thank Joel Slemrod, Toke Aidt and Erling Eide for comments. We thank J. Barkley Rosser Jr. for encouraging us to write this paper and for guiding us through its several revisions.

10. Appendix: The gamma distribution

Most of the definitions, below, can also be found in Cowell (2000). The *gamma distribution* is given by

$$G\left(Y;a,b\right)=\frac{1}{b^{a}\Gamma\left(a\right)}\int_{y=0}^{Y}y^{a-1}e^{-\frac{y}{b}}dy,\; a\geq 1,\; b>0, \tag{10.1}$$

where $\Gamma\left(a\right)$ is the *gamma function*, given by:

$$\Gamma\left(a\right)=\int_{x=0}^{\infty}x^{a-1}e^{-x}dx,\; a\geq 1. \tag{10.2}$$

If n is a positive integer, then

$$\Gamma(n) = (n-1)! \tag{10.3}$$

The mean and variance of the gamma distribution (10.1) are given by:

$$\overline{Y} = ab, \text{ var}Y = ab^2. \tag{10.4}$$

The *Gini index* for the gamma distribution is given by:

$$\begin{aligned} Gini(a,b) &= \frac{1}{\overline{Y}} \int_{Y=0}^{\infty} (2G(Y;a,b) - 1) Y dG(Y;a,b) \\ &= \frac{2b}{\overline{Y}(\Gamma(a))^2} \int_{Y=0}^{\infty} Y^a e^{-Y} \left(\int_{X=0}^{Y} X^{a-1} e^{-X} dX \right) dY - 1. \end{aligned} \tag{10.5}$$

The simplest gamma distribution that has the correct shape is:

$$G(Y;2,1) = \int_{y=0}^{Y} y e^{-y} dy, \tag{10.6}$$

for which

$$\begin{aligned} Gini(2,1) &= 0.375, \\ \text{mean} &= 2, \text{ median} = 1.6783, \\ \frac{\text{median}}{\text{mean}} &= \frac{1.6783}{2} = 0.83915. \end{aligned} \tag{10.7}$$

Ryu and Slottje (2001, p303) report that estimates of Gini coefficients typically lie in the range $0.34 - 0.43$, so $Gini(2,1) = 0.375$ is not unreasonable.

For $G(Y;2,1)$, the *quantal functional*, $Q(p)$, the *cumulative income functional*, $C(p)$, and the *Lorenz curve*, $L(p)$, are given by, respectively,

$$\begin{aligned} p &\in [0,1], Y \in [0,\infty), p = G(Y;2,1) \Rightarrow Q(p) = Y, \\ C(p) &= \int_{y=0}^{Q(p)} y^2 e^{-y} dy = 2 - 2e^{-Q(p)} - 2Q(p) e^{-Q(p)} - (Q(p))^2 e^{-Q(p)}, p \in [0,1], \\ L(p) &= \frac{C(p)}{\overline{Y}}, p \in [0,1]. \end{aligned} \tag{10.8}$$

$Q(p)$ can be found by solving the equations $(1-p) e^{x^2} = 1 + x^2$, $Q(p) = x^2$, where the square has been chosen to force the computer to the positive solution.

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