International Tax and Public Finance, 9, 673–685, 2002

Optimal Consumption Taxes and Social Security Under Tax Measurement Problems and Uncertainty

SANJIT DHAMI Sanjit.Dhami@ncl.ac.uk
Department of Economics, University of Newcastle Upon Tyne, Newcastle NE1 7RU, UK

Abstract

A representative individual lives for two periods; works when young and depends on savings and a government operated social security system when old—the returns on both sources of income, when old, are random. Due to administrative problems the returns to savings are observed with some measurement error. Two alternative consumption tax systems are considered; the Registered Asset Treatment (RAT) and the Non-Registered Asset Treatment (NRAT). The advantage of the RAT is that it can perform a "social insurance role" while the disadvantage is that it imposes "measurement error risk." Correlation between the random return on saving and its measurement error can provide a "risk-hedging role" that can be further strengthened by the RAT version. The NRAT version neither provides "social insurance" nor imposes "measurement error risk." Both tax systems hedge against the uncertainties in the social security system. The taxpayer engages in precautionary saving in response to future uncertainty.

Keywords: income uncertainty, measurement problems, risk-hedging

JEL Code: D82, H21

1. Introduction

Issues in the theory of optimal commodity taxes, as well as proposals for tax reform, are generally informed by efficiency and equity considerations.[1] Despite their importance in real world tax policy, administrative issues are typically ignored. However, general commentaries on optimal commodity taxation often focus on administrative issues, such as the relative ease of measuring alternative commodity tax bases; for instance, Gale and Holtzblatt (2000), Mintz (1997), Devereux (1996) and Bradford (1980).

Surveys of optimal taxation often stress the lack of real world applicability of tax theory on administrative grounds. Heady (1996, p. 33) writes that "One way in which many models are unrealistic has already been mentioned: their neglect of administrative costs." Burgess and Stern (1993, p. 798) make similar remarks in the context of developing countries. Slemrod (1990, p. 157) has argued that "Differences in the ease of administrating various taxes have been and will continue to be a critical determinant of appropriate tax policy."

The meaning of the generic term "administrative issues" depends on the context. If all tax bases were costlessly and publicly observed, then administrative issues would not be very important. Hence, one implication of administrative issues must be that tax bases are measured with some measurement error. It is the "measurement error" aspect of administrative issues that this paper concentrates on.

Theoretical models that incorporate measurement issues in the theory of optimal taxation are rare. Mirrlees (1990) derives some "approximately optimal taxes" when the income tax base is observed with some error, however, measurement problems with the consumption tax base are completely ignored. Skinner (1996) looks specifically at land taxes and argues that one reason for their limited usage is that land quality is inherently hard to measure. Stern (1982) shows that when taxpayers can potentially be classified in a wrong category (a form of administrative problem) then optimal income taxes are lower.

The main objective of a consumption tax (and its major distinguishing feature relative to the income tax) is to avoid distorting the intertemporal decisions of households, essentially by omitting from the tax base, saving, inclusive of its rate of return. Actual consumption taxes, and proposals for the reform of consumption taxes take two main forms; the "direct" and the "indirect" methods.

Under "direct methods," a consumption tax takes two forms, the VAT and the retail sales tax, and in each of these forms it operates as a business tax; businesses and vendors effectively act as tax collectors for the government. In a VAT, the government taxes the value-added at each stage; in the absence of uncertainty, measurement errors and tax evasion activities of the vendors, the VAT is equivalent to a retail tax.

Under "indirect methods" the consumption tax takes the form of an "expenditure tax" and is applied in two distinct versions; the Registered Asset Treatment (RAT) and the Non-Registered Asset Treatment (NRAT). In the RAT version, the taxpayer can deduct savings allocated to certain registered assets from her taxable income and the income earned from such assets is not taxed as it accrues. However, when savings are withdrawn from the plan, the principal plus the realized interest or capital gains are taxed. In the NRAT version, the taxpayer cannot deduct savings from taxable income, nor is the principal plus the realized interest or capital gains taxed when savings are withdrawn. The present value of the tax on savings is zero in both versions.

One of the well known drawbacks of the income tax system is that it requires taxes to be levied on an "accrual basis" hence making difficult the taxation of capital gains, depreciation, inflation adjusted income and random rates of returns; see for instance Boadway and Wildasin (1996). Proponents of consumption taxes argue that the main administrative benefit of consumption taxes is that they are levied on a "realized based."

However, the consumption tax is faced with other, potentially severe, administrative problems. Mintz (1997, p. 461) shows that measurement problems with the "indirect methods" include the appropriate unit (individual, family or business) of taxpayers and issues of consumption versus business expenses, taxation of real versus financial transactions, treatment of wage versus self-employed income, treatment of losses and tracking of revenues, expenses, fixed assets and financial transactions etc. Measurement problems are also associated with indirect methods such as a VAT system, these often arise from the exemption of certain goods and services and are compounded by the operation of multiple rates that reduce the tax rate on lower income consumers. Gale and Holtzblatt (2000) consider similar issues, and show, in considerable detail, that measurement problems are further compounded by the deliberate or unintentional tax evasion activities of taxpayers.

The measurement problems in commodity tax bases could also result from certain features of the tax administration system; these problems are especially severe for developing countries. Burgess and Stern (1993, p. 798–799) identify some of these factors: insufficient staff with the appropriate skills, equipment, motivation or honesty, complex legal tax structures, poor and inconsistent records that are often under the control of different tax authorities and lack of incentive based remuneration etc. Related problems can also arise in developed countries, for instance, Fortin (1995, p. 2) writes: "A substantial portion of Revenue Canada employees fails elementary tests of the knowledge of the tax system. Even our best experts find it very hard to keep up."

The paper focusses on "indirect methods" such as the RAT and the NRAT because these figure prominently in recent proposals for tax reform and are more amenable to analysis within the representative-agent framework that is standard in optimal tax theory. A "generic" but reduced-form approach to measurement problems is taken, hence, the focus of the paper is not on specific institutional details nor on the precise sources of measurement errors; these are taken as given.

A representative individual lives for two periods; works only when young (in the first period) and earns a fixed income which can either be consumed or invested towards retirement (the second period) at some random rate of return. Tax authorities observe savings inclusive of the random return with some measurement errors which can be correlated with the size of the random return. Tax revenues are used to operate a social security system which takes the form of provision towards pensions for the old who earn no current income. The returns on the government operated social security system are allowed to be random. Two alternative consumption tax systems are considered; the Registered Asset Treatment (RAT) and the Non-Registered Asset Treatment (NRAT).

A summary of the results is as follows. Measurement errors are shown to reduce optimal taxes. In other contexts, Stern (1982), Mirrlees (1990) and Skinner (1996) report the same theoretical result. Consumption taxes can also provide a "social insurance role" similar to the one identified in the context of income taxes by Varian (1980) and Eaton and Rosen (1980). The advantage of the RAT is that it can perform the "social insurance role" while the disadvantage is that it imposes "measurement error risk." Correlation between the random return and measurement errors, depending on its sign, can either increase risk or provide a "risk-hedging role" for the taxpayer. In its RAT version, the consumption tax can strengthen the risk-hedging role. Under the NRAT version, the consumption tax does not perform the "social insurance role" but nor does it subject the taxpayer to "measurement errors." Both tax systems hedge against uncertainty that might arise from the operation of the social security system. The taxpayer engages in precautionary saving in response to uncertainty about second period income.

Section 2 describes the model. Sections 3 and 4 respectively derive and describe the optimal consumption tax in its NRAT and RAT versions. Some equivalence results are presented in Section 5 followed by the conclusions.

2. Model

The underlying model is a two-period model of consumption-saving choice, as in Varian (1980). A representative taxpayer lives for two periods, inelastically supplies a unit of labour when young (in the first period) and earns income I which can either be expended on first period consumption C_1 or on savings S. When old (in the second period), the taxpayer does not work and her (accrued) second period income is $S + \eta$, where η is a random return on savings such that $\eta \sim N(0, \sigma_\eta^2)$. Varian (1980) interprets η as "luck" or "idiosyncratic income uncertainty"; other possible interpretations include randomness in the storage technology, and "capital gains."[2] There is no bequest motive, hence, second period income is fully expended on second period consumption, C_2. In the absence of taxes, the first and the second period budget constraints of the taxpayer are $C_1 = I - S$ and $C_2 = S + \eta$ respectively.

2.1. Tax Base Measurement Technology

Due to tax administration problems, the government observes an imperfect signal ψ of second period income $S + \eta$. Measurement problems can arise, in principle, from any variety of factors, such as the tax evasion activities of the taxpayer, inherent difficulties in measuring the realization of η,[3] or the corrupt activities of the tax authorities; the paper does not compromise on generality by focusing on any particular source of measurement error. The signal ψ is defined as $\psi = (S + \eta) + \varepsilon$, where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ is the measurement error and $\frac{1}{\sigma_\varepsilon^2}$ captures the precision with which the government observes the savings tax base. The covariance in the random terms, η and ε, is σ_{12}, thus, the size of the measurement error depends on the size of the capital gains; the sign of σ_{12} is not specified.

2.2. Alternative Tax Systems

The government has at its disposal, two alternative consumption tax systems; the "registered asset treatment" (RAT), also known as the "individual cash flow approach" and the "non-registered asset treatment" (NRAT), also known as the "individual tax prepayment approach." Denote the (constant per unit) tax rates under these versions as respectively τ and θ. Abstracting from public goods, the maintenance of a social security system, in the form of a pension scheme, seems to be the most plausible usage of government revenues in a two-period model where the old earn no income. Thus, in each of the two consumption tax systems, the government invests the expected tax revenues for the taxpayer, respectively α_R and α_N, at some rate of return $\omega \sim N(\bar{\omega}, \sigma_\omega^2)$ and returns the proceeds to the taxpayer when old.

2.2.1. Non-Registered Asset Treatment (NRAT) In this version of the consumption tax, savings (and the returns thereof) are completely excluded from the ambit of taxation, thus, the budget constraints of the taxpayer are:

$$C_1 = (1 - \theta)I - S \tag{1}$$

$$C_2 = (S + \eta) + \alpha_N \tag{2}$$

The expected tax revenue is θI, which when invested by the government at the rate of return ω, gives $\alpha_N = \theta I(1 + \omega)$. Thus, the second period budget constraint is:

$$C_2 = (S + \eta) + \theta I(1 + \omega) \tag{3}$$

2.2.2. Registered Asset Treatment (RAT) In this version of the consumption tax, savings are exempted from taxation in the first period while the observed value of the realized return plus the principal, ψ, is taxed on withdrawal in the second period, thus, the budget constraints of the taxpayer are:

$$C_1 = (1 - \tau)\{I - S\} \tag{4}$$

$$C_2 = (S + \eta) - \tau\psi + \alpha_R \tag{5}$$

The expected tax revenue is τI, which when invested by the government at the rate of return ω, gives $\alpha_R = \tau I(1 + \omega)$. Substituting α_R and ψ, the second period budget constraint is:

$$C_2 = (1 - \tau)\{S + \eta\} - \tau\varepsilon + \tau I(1 + \omega) \tag{6}$$

2.3. Preferences

The taxpayer has CARA preferences that are additively separable in C_1 and C_2; the coefficient of absolute risk aversion equals ρ. Denoting by E, the expectation operator with respect to the joint distribution of η, ε, and ω, the taxpayer's expected utility is:

$$E[U[C_1, C_2]] = E[-e^{-\rho C_1}] + E[-e^{-\rho C_2}] \tag{7}$$

In both versions of the consumption tax, C_1 is non-random while C_2 is normally distributed (because it is a linear and additive function of normally distributed terms). Denote the expected value and variance of C_2 by $E[C_2]$ and $V[C_2]$, respectively. Using a standard result in statistics,[4] the taxpayer's preferences can be rewritten as:

$$E[U[C_1, C_2]] = -\exp\{-\rho C_1\} - \exp\{-\rho\zeta\} \tag{8}$$

where $\zeta = E[C_2] - \frac{\rho}{2}V[C_2]$ is the certainty equivalent; it is increasing in $E[C_2]$ and decreasing in $V[C_2]$. Hence, the joint assumptions of normality of the random terms and CARA preferences allows for the simplification of a fairly complicated problem; an identical approach is followed in the multitask agency literature, for instance Holmstrom and Milgrom (1990, 1991).

The objective of the (benevolent) government is to maximize the indirect utility of the representative taxpayer.

2.4. Sequence of Moves

The government acts as the Stackelberg leader and announces the consumption tax rate, prior to learning about the realizations of the random variables η, ε, and ω. The taxpayer then allocates the exogenous first period income between C_1 and S. Following this, the random terms and the payoffs are realized in that order.[5] The solution to the optimal tax rates is found by backward induction.

3. The Optimal Consumption Tax (NRAT Version)

Since the solution is by backward induction, the first step is to solve for the savings reaction function of the taxpayer for all possible announcements of the tax rate θ. The savings reaction function of the taxpayer is found by maximizing (8) subject to the budget constraints given in (1) and (3). Check, using equation (3), that $E[C_2] = S + \theta I(1 + \bar{\omega})$ and $V[C_2] = \sigma_\eta^2 + \theta^2 I^2 \sigma_\omega^2$. Denoting by $\zeta_N = E[C_2] - \frac{\rho}{2} V[C_2]$, the certainty equivalent under the NRAT version, and substituting (1) in the objective function, the taxpayer's unconstrained maximization problem is:

$$S_N^* \in \arg\max E[U[S]] = -\exp\{-\rho\{(1-\theta)I - S\}\} - \exp\{-\rho\zeta_N\} \tag{9}$$

The first order condition is:

$$\frac{\partial E[U[S]]}{\partial S} = -\rho \exp\{-\rho C_1\} + \rho \exp\{-\rho \zeta_N\} = 0 \tag{10}$$

The first term in equation (10) is the marginal utility of sacrificing C_1 by a unit, while the second is the marginal utility of gaining C_2 by a unit; its solution gives the savings reaction function $S_N^* = S_N^*(\theta, I, \rho, \sigma_\eta^2, \sigma_\omega^2)$:

$$S_N^* = \frac{1}{2}\{I(1 - \theta(2 + \bar{\omega}))\} + \frac{\rho}{4}\left\{\sigma_\eta^2 + \theta^2 I^2 \sigma_\omega^2\right\} \tag{11}$$

Since the taxpayer earns no income when old the first term in equation (11) captures the "consumption smoothing" role of savings. The second term in equation (11) illustrates the "precautionary motive" of the risk-averse taxpayer. Thus, greater second period uncertainty, reflected in σ_η^2 and σ_ω^2, increases savings.

Letting $\zeta_N^* = \{S_N^* + \theta I(1 + \bar{\omega})\} - \frac{\rho}{2}\{\sigma_\eta^2 + \theta^2 I^2 \sigma_\omega^2\}$, the government's unconstrained optimization problem in the first stage is:

$$\theta^* \in \arg\max E[U[\theta]] = -\exp\{-\rho\{(1-\theta)I - S_N^*\}\} - \exp\{-\rho\zeta_N^*\} \tag{12}$$

Using the envelope theorem, and the Kuhn-Tucker condition, the first order condition is:

$$\frac{\partial E[U[\theta]]}{\partial \theta} = \bar{\omega} I - \rho\theta I^2 \sigma_\omega^2 \leq 0; \quad \theta \geq 0 \tag{13}$$

The first and the second term show respectively the marginal benefits and costs of levying the consumption tax; higher taxes allow for greater second period pension income when the taxpayer is old (benefit) but accentuate the random fluctuations in the return to the pension scheme (cost). Equation (13) can be explicitly solved for the optimal tax rate:

$$\theta^* = \left(\frac{\bar{\omega}}{\sigma_\omega^2}\right)\left(\frac{1}{\rho I}\right) \tag{14}$$

Thus, the optimal tax is positive and directly proportional to the ratio of the expected return to the variance of the pension scheme. It is also inversely proportional to ρ because a more risk averse taxpayer bears a higher cost from the accentuation of the random fluctuations in the return to the pension scheme caused by a higher tax. This result is recorded, without proof, in Lemma 1 below.

Lemma 1 *The optimal consumption tax under the NRAT version is (1) always positive, (2) directly proportional to* $(\frac{\bar{\omega}}{\sigma_\omega^2})$*, and (3) indirectly proportional to* ρ*.*

In particular, in its NRAT version, by omitting savings and the returns thereof from the ambit of taxation completely, the optimal consumption tax is not influenced either by the administrative problems of measuring savings (and the returns thereof) nor is it influenced by (or can influence) uncertainty in the returns to savings.

4. The Optimal Consumption Tax (RAT Version)

As in Section 3, the first step is to solve for the savings reaction function, conditional on the consumption tax rate τ. The taxpayer maximizes expected utility in (8) subject to the budget constraints given in (4) and (6). From equation (6), $E[C_2] = (1-\tau)S + \tau I(1+\bar{\omega})$ and $V[C_2] = (1-\tau)^2\sigma_\eta^2 + \tau^2\sigma_\varepsilon^2 - 2\tau(1-\tau)\sigma_{12} + \tau^2 I^2\sigma_\omega^2$. Defining $\zeta_R = E[C_2] - \frac{\rho}{2}V[C_2]$ as the certainty equivalent under the RAT version and substituting (4) in the objective function, the taxpayer's unconstrained maximization problem is:

$$S_R^* \in \arg\max E[U[S]] = -\exp\{-\rho(1-\tau)\{I-S\}\} - \exp\{-\rho\zeta_R\}$$

The first order condition is:

$$\frac{\partial E[U[S]]}{\partial S} = -\rho\exp\{-\rho C_1\} + \rho\exp\{-\rho\zeta_R\} = 0$$

The first order condition has the usual interpretation in terms of marginal benefits and costs; it can be solved for the savings reaction function $S_R^* = S_R^*(\tau, I, \rho, \sigma_\eta^2, \sigma_\varepsilon^2, \sigma_{12}, \sigma_\omega^2)$:

$$S_R^* = \frac{1}{2(1-\tau)}\{(1-\tau\,(2+\bar{\omega}))I\} + \frac{\rho}{4(1-\tau)}V[C_2] \tag{15}$$

As before, the first term in (15) captures the "consumption smoothing" role of savings, while the second illustrates the "precautionary motive" of the risk-averse taxpayer. Relative to equation (11), under the RAT version precautionary savings are influenced, in addition, by the terms corresponding to σ_ε^2 and σ_{12}.

Letting $\zeta_R^* = (1-\tau)S_R^* + \tau I(1+\bar{\omega}) - \frac{\rho}{2}V[C_2]$, the government's unconstrained optimization problem is:

$$\tau^* \in \arg\max E[U[\tau]] = -\exp\{-\rho(1-\tau)\{I-S_R^*\}\} - \exp\{-\rho\zeta_R^*\} \tag{16}$$

Using the envelope theorem, and the Kuhn-Tucker condition, the first order condition is:

$$\frac{\partial E[U[\tau]]}{\partial \tau} = \frac{\bar{\omega}I}{\rho} + (1-\tau)\sigma_\eta^2 - \tau\sigma_\varepsilon^2 + (1-2\tau)\sigma_{12} - \tau I^2\sigma_\omega^2 \le 0; \quad \tau \ge 0 \tag{17}$$

The first term shows the "social security benefit" of taxation arising from the pension scheme; higher taxes allow for greater second period income when the taxpayer is old. The second term is the "social insurance effect" of taxation; an increase in τ dampens the uncertain returns on saving. The third term is the "administrative risk effect"; higher taxation compounds the risk arising from the measurement errors in taxation. The fourth term is the "risk hedging effect"; correlation in the returns to saving with measurement errors can either accentuate or dampen the risk facing the taxpayer, depending on the sign of σ_{12}. The final term captures the "social security risk effect"; higher taxes accentuate the random fluctuations in the return to the pension scheme. Equation (17) can be explicitly solved for the optimal tax rate:

$$\tau^* = \frac{\rho^{-1} I \bar{\omega} + \left(\sigma_\eta^2 + \sigma_{12}\right)}{\sigma_\eta^2 + \sigma_\varepsilon^2 + 2\sigma_{12} + I^2 \sigma_\omega^2} \tag{18}$$

4.1. Comparative Static Properties of the RAT Version

The consumption tax in the NRAT version is always positive (see (14)), however, under the RAT version, it can be positive or negative (a subsidy). Lemma 2 below formalizes the conditions under with τ^* is positive or negative.

Lemma 2 *When $\sigma_{12} > 0$, then the consumption tax is always positive i.e., $\tau^* > 0$. However, when $\sigma_{12} < 0$, then τ^* is positive (resp. negative) as $\sigma_\eta^2 + \sigma_{12}$ is positive (resp. negative).*

Proof: The second order condition is $\frac{\partial^2 E[U[\tau]]}{\partial \tau^2} = -\rho\{\sigma_\eta^2 + \sigma_\varepsilon^2 + 2\sigma_{12} + I^2\sigma_\omega^2\} \leq 0$, which always holds when $\sigma_{12} > 0$, and using equation (18) it is obvious that in this case, $\tau^* > 0$. When $\sigma_{12} < 0$, the second order condition requires that $2^{-1}\{\sigma_\eta^2 + \sigma_\varepsilon^2 + I^2\sigma_\omega^2\} \geq -\sigma_{12}$. Subject to this condition, and using equation (18), the numerator is positive (resp. negative) when $\sigma_\eta^2 + \sigma_{12}$ is positive (resp. negative); this proves the second part of the Lemma. ■

Lemma 2 shows that a consumption subsidy is only optimal when $\sigma_{12} < 0$ and $\sigma_\eta^2 < -\sigma_{12}$. In this case, imposition of any positive tax implies that the social insurance role of taxation (see Subsection 4.1.1 below) is swamped by the negative risk-hedging role of taxation (see Subsection 4.1.3 below).

4.1.1. Social Insurance Role of Consumption Taxes In the context of income taxation, Varian (1980) and Eaton and Rosen (1980) show that under income uncertainty the imposition of an income tax reduces the risk facing the taxpayer. A similar result also holds under the RAT version (but not the NRAT version) of consumption taxes; Proposition 1 below formalizes the argument.

Proposition 1 *The optimal consumption tax rate is non-decreasing in the variance of income i.e., $\frac{\partial \tau^*}{\partial \sigma_\eta^2} \geq 0$ and as $\sigma_\eta^2 \to \infty$, $\tau^* = 1$.*

Proof: Implicitly differentiating (18) one gets:

$$\frac{\partial \tau^*}{\partial \sigma_\eta^2} = \left\{\sigma_\eta^2 + \sigma_\varepsilon^2 + 2\sigma_{12} + I^2\sigma_\omega^2\right\}^{-1} \{1 - \tau^*\} \geq 0$$

The sign follows because, by definition, $\tau^* \leq 1$. Now taking the limits in equation (17), as $\sigma_\eta^2 \to \infty$, the complementary slackness condition is satisfied only when $\tau^* = 1$. ■

Proposition 1 shows that consumption taxes can perform the social insurance role of taxation, previously ascribed only to income taxes,[6] and moreover when the taxpayer faces extreme income uncertainty ($\sigma_\eta^2 \to \infty$), the consumption tax attains its highest possible value ($\tau^* = 1$) so as to completely offset this uncertainty.

4.1.2. Administrative Problems and the Consumption Tax In different contexts, Stern (1982), Mirrlees (1990) and Skinner (1996) have demonstrated that measurement problems reduce the optimal tax rate. A similar result is derived below in Proposition 2 (recall from Section 3 that in its NRAT version, the consumption tax is unaffected by measurement problems).

Proposition 2 *The optimal consumption tax is non-increasing in the imprecision, σ_ε^2, with which the government measures the tax base i.e., $\frac{\partial \tau^*}{\partial \sigma_\varepsilon^2} \leq 0$ and as $\sigma_\varepsilon^2 \to \infty$, $\tau^* = 0$.*

Proof: Implicitly differentiating (18) one gets:

$$\frac{\partial \tau^*}{\partial \sigma_\varepsilon^2} = \left\{\sigma_\eta^2 + \sigma_\varepsilon^2 + 2\sigma_{12} + I^2\sigma_\omega^2\right\}^{-1}(-\tau^*) \leq 0$$

which proves the first part of the proposition. Now taking the limits in the first order condition (17), as $\sigma_\varepsilon^2 \to \infty$, the complementary slackness condition is satisfied only when $\tau^* = 0$.■

In reality, governments have exogenous revenue requirements (chiefly, the enforcement of property rights), over and above the need to finance a social security system (as in this paper). Hence, the reduction in the optimal tax on account of administrative problems (as in Proposition 2) also, in a more general model, has implications for general tax revenues. Therefore, in actual tax systems, assets whose returns are extremely hard to measure are generally excluded from the ambit of the RAT version; examples include equity of various forms such as the purchase of a house, a car or a firm and bequests and inheritances.

4.1.3. Risk-Hedging and Consumption Taxes When $\sigma_{12} > 0$, measurement errors are high when the state of the world is good for the taxpayer (high η) and are low when the state of the world is bad. This provides a built-in insurance system for the taxpayer; conversely, when $\sigma_{12} < 0$, risks facing the taxpayer are accentuated. In the expression for $V[C_2]$, the relevant risk-hedging term is $-2\tau(1-\tau)\sigma_{12}$. Clearly, an increase in σ_{12} reduces $V[C_2]$; since the coefficient of σ_{12} depends on τ, consumption taxes can be used to reduce the uncertainty facing the taxpayer. Focusing on the "risk hedging effect" term, $(1-2\tau)\sigma_{12}$, in the first order condition (17), if $1-2\tau < 0$ then an increase in σ_{12} decreases the benefit to the taxpayer of increasing τ slightly; conversely if $1-2\tau > 0$ then it is optimal to increase τ.

Proposition 3 *The optimal consumption tax rate is increasing (resp. decreasing) in σ_{12} as $1 - 2\tau$ is positive (resp. negative).*

Proof: Implicitly differentiating (18) one gets:

$$\frac{\partial \tau^*}{\partial \sigma_{12}} = \left\{\sigma_\eta^2 + \sigma_\varepsilon^2 + 2\sigma_{12} + I^2\sigma_\omega^2\right\}^{-1} (1 - 2\tau^*)$$

The statement in the proposition now follows directly by observing that the sign of $\frac{\partial \tau^*}{\partial \sigma_{12}}$ is identical to the sign of $(1 - 2\tau^*)$. ■

The "risk-hedging role" is very similar to the idea of "yardstick competition" in the multitask agency literature. As pointed out by Holmstrom (1982) and Holmstrom and Milgrom (1990, 1991), the observation of two correlated signals of an agent's effort level allows the principal to filter away some of the risk facing the agent. One consequence of such correlation in that literature is more high-powered incentives, however, Proposition 3 shows that in the tax context the resulting optimal tax can be high or low-powered depending on its original value (sign of $1 - 2\tau$).

When $\sigma_{12} = 0$, the measurement errors are independent of the size of the random return to savings. Normalizing $\bar{\omega} = 0$, the optimal tax rate can be written as:

$$\tau^* = \left\{1 + \frac{\sigma_\varepsilon^2}{\sigma_\eta^2} + \frac{\sigma_\omega^2}{\sigma_\eta^2} I^2\right\}^{-1} > 0$$

In this case, the optimal tax rate is strictly positive and its magnitude is directly proportional to the importance of the social insurance role relative to the (1) imprecision in the measurement of the tax base (the term $\sigma_\eta^2/\sigma_\varepsilon^2$), and (2) risk associated with the operation of the social security system (the term $\sigma_\eta^2/\sigma_\omega^2$).

4.1.4. Social Security Risk and Consumption Taxes The consumption tax aids to hedge the uncertainty, σ_η^2, in the return to private saving (see Subsection 4.1.1 above), however, it accentuates uncertainty arising from the random return, σ_ω^2, to the government's pension scheme; see the term $\tau^2 I^2 \sigma_\omega^2$ in the expression for $V[C_2]$. Hence by reducing τ, the variance of C_2 can be reduced; this is checked formally in Proposition 4 below.

Proposition 4 *The optimal consumption tax rate is non-increasing in the variance of the return to social security i.e., $\frac{\partial \tau^*}{\partial \sigma_\omega^2} \leq 0$ and as $\sigma_\omega^2 \to \infty$, $\tau^* = 0$.*

Proof: Implicitly differentiating (18) one gets:

$$\frac{\partial \tau^*}{\partial \sigma_\omega^2} = \left\{\sigma_\eta^2 + \sigma_\varepsilon^2 + 2\sigma_{12} + I^2\sigma_\omega^2\right\}^{-1} \{-\tau^* I^2\} \leq 0$$

which proves the first part of the proposition. Now taking the limits in the first order condition (17), as $\sigma_\omega^2 \to \infty$, the complementary slackness condition is satisfied only when $\tau^* = 0$.
■

5. Some Equivalence Results

Using different methods, both versions of the consumption tax, exempt savings and the returns thereof from the ambit of taxation. However, given the differences in the timing of the two taxes, one would not in general expect the two taxes to be equivalent in their magnitude or on the effect that they have on savings. However, in one special case, the two taxes are equivalent; this is formalized in Proposition 5 below.

Proposition 5 *When the only uncertainty arises from the return to pensions i.e., $\sigma_\eta^2 = \sigma_\varepsilon^2 = \sigma_{12} = 0$ and $\sigma_\omega^2 > 0$, then $\tau^* = \theta^*$ and $S_N^* \leq S_R^*$. However, in this case, the two taxes are equivalent in the sense that the expected utility of the taxpayer is identical under both tax systems.*

Proof: Substitute $\sigma_\eta^2 = \sigma_\varepsilon^2 = \sigma_{12} = 0$ in (18) and use (14) to get $\tau^* = \theta^* = (\frac{\bar{\omega}}{\sigma_\omega^2})(\frac{I}{\rho})$. To prove the second part of the proposition, subtract (11) from (15) to get $\frac{S_R^*}{S_N^*} = \frac{1}{1-\theta^*} \geq 1$. It is straightforward to check that in this case, $E[U[\tau^*]] = E[U[\theta^*]]$. ■

The idea that uncertainty has important implications for the equivalence of the two tax systems has been recognized by Ahsan (1989) and Zodrow (1995). Proposition 5 shows that at least under one form of uncertainty i.e., that relating to social security, the two tax systems generate identical revenues and utilities. In general (when $\sigma_\eta^2 \neq 0$ or $\sigma_\varepsilon^2 \neq 0$), the two taxes will not generate identical expected revenues (which equal the tax rate multiplied by I), nor will they generate identical utilities; this is easily seen by comparing (14) and (18):

$$\theta^* \gtrless \tau^* \quad \text{as } 1 + \frac{\sigma_\varepsilon^2 + \sigma_{12}}{\sigma_\eta^2 + \sigma_{12}} \gtrless \frac{\rho I \sigma_\omega^2}{\bar{\omega}}$$

Thus, in general, τ^* is likely to be larger relative to θ^* if:

1. The relative variance $\frac{\sigma_\varepsilon^2}{\sigma_\eta^2}$ is small i.e., the returns from savings have large variance but are relatively easily measured. The intuition is that the benefit of the RAT version is the "social insurance role" but the cost is the greater "measurement error risk" imposed on the taxpayer.
2. The social security system is risky i.e., σ_ω^2 is high. The intuition can be seen from the first order conditions (13) and (17); a unit increase in σ_ω^2 raises the marginal 'cost to benefit ratio' of the NRAT tax by one unit while it raises the marginal 'cost to benefit ratio' of the RAT tax by less than a unit.

The second part of Proposition 5 ($S_N^* \leq S_R^*$) follows because when $\tau^* = \theta^*$, both tax systems have identical $V[C_2]$, however, $E[C_2]$ is lower under the RAT version. Hence, consumption smoothing across the two periods requires the taxpayer to save relatively more under the RAT version. Thus, when $\sigma_\eta^2 = \sigma_\varepsilon^2 = \sigma_{12} = 0$, the RAT version will be more effective in case the objective is to generate greater savings.

6. Conclusions

This paper considers two alternative consumption tax systems; the Registered Asset Treatment (RAT) and the Non-Registered Asset Treatment (NRAT) in the presence of uncertainty and administrative problems. Tax revenues are used to operate a social security system. In the absence of income uncertainty and administrative problems, the two tax systems are equivalent, however, in general, they are not equivalent.

The advantage of the RAT is that it can perform the "social insurance role of taxation" (a role previously ascribed only to income taxes) while the disadvantage is that it imposes "measurement error risk" on account of the difficulties in measuring the returns to savings. The random return to savings and the measurement errors can be correlated; for instance, a positive correlation implies that measurement errors are high in good states and low in bad states. This provides a "risk-hedging role" for the taxpayer, which, under the RAT version, can be further strengthened by using the tax system.

The disadvantage of the NRAT version is that it cannot perform the social insurance role, nor can it exploit the risk-hedging property of the government's measurement technology. However, the advantage of the NRAT version is that, by exempting savings and the returns thereof from taxation, it is not subject to measurement error problems.

The return on the government operated social security system is random; each of the two tax systems hedges against this uncertainty by lowering the optimal tax. Under each of the two versions of the consumption tax, the taxpayer engages in precautionary saving in response to future uncertainty.

The paper deals with measurement errors in a fairly general framework, without focussing on the specific sources of such errors. In specific situations, the exact nature of these errors will be important in determining the optimal choice. For instance if errors arise from corrupt tax collectors or tax evaders, then there might be important issues involved in the interaction between a deterrence policy and the optimal tax rate. If, on the other hand, these errors arise from the inherent problems in an endogenous tax measurement technology, but with honest individuals, then there might be important issues between the endogeneity of this technology and the optimal tax rates. Attention to more specific sources of administrative problems can generate a range of useful models that could constitute a more complete theory of "optimal tax systems"; one that combines administrative issues with optimal taxation.

Acknowledgments

I am very grateful to Joel Slemrod, Daniel Seidmann, Jack Mintz, Craig Brett, Abhinay Muthoo and Melvyn Coles for valuable comments and discussions and to three anonymous referees. The usual disclaimers apply.

Notes

1. Textbook treatments of these issue can be found in Atkinson and Stiglitz (1980) and Myles (1995).
2. If Varian's "idiosyncratic uncertainty" is multiplicative then second period income is $S(1+\eta)$; this complicates the model without changing the basic results.

3. Boadway and Wildasin (1996) illustrate measurement problems in 'capital income.' They write: "In principle this should include all forms of returns to assets including interest, dividends, accrued capital gains, capital income from unincorporated business, imputed rent on consumer durables (especially housing) and the imputed return of assets such as transactions balances and insurance. These should all be indexed for inflation and should include an appropriate risk premium. Unfortunately the measurement of these items is difficult or impractical."
4. If a random variable $X \backsim N(\mu, \sigma_X^2)$, then $E[\exp\{-\rho X\}] = \exp\{-\rho(\mu - \frac{\rho}{2}\sigma_X^2)\}$.
5. Savings S are committed to, before the realization of the random shocks. Hence, the consumer implicitly commits to a level of expenditure for second period consumption before the resolution of uncertainty.
6. Cremer and Gavhari (1995) also seem to allude to this role for consumption taxes by distinguishing between consumption goods that are committed to, before and after the resolution to uncertainty.

References

Ahsan, S. M. (1989). "Choice of Tax Base Under Uncertainty." *Journal of Public Economics* 40, 99–134.

Atkinson, A. B. and J. E. Stiglitz. (1980). *Lectures on Public Economics*. New York: McGraw-Hill.

Boadway, R. and D. Wildasin. (1996). "Taxation and Savings." In M. P. Devereux (ed.), *The Economics of Tax Policy*. Oxford: Oxford University Press.

Bradford, D. (1980). "The Case for a Personal Consumption Tax." In J. A. Pechman (ed.), *What Should Be Taxed: Income or Expenditure.* Washington, D.C.: Brookings Institute, 75–113.

Burgess, R. and N. Stern. (1993). "Taxation and Development." *Journal of Economic Literature* 31, 762–830.

Cremer, H. and F. Gavhari. (1995). "Uncertainty and Optimal Taxation: In Defence of Commodity Taxes." *Journal of Public Economics* 56, 291–310.

Devereux, M. P. (1996). "Introduction." In M. P. Devereux (ed.), *The Economics of Tax Policy*. Oxford: Oxford University Press.

Eaton, J. and H. S. Rosen. (1980). "Optimal Redistributive Taxation and Uncertainty." *Quarterly Journal of Economics* 94, 357–364.

Fortin, P. (1995). "An Efficient, Fair and Simple Income Tax for Canada." Invited Presentation to the Magna for Canada Scholarship Fund.

Gale, W. G. and J. Holtzblatt. (2000). "The Role of Administrative Issues in Tax Reform: Simplicity, Compliance and Administration." Forthcoming In G. R. Zodrow and P. Mieszkowski (eds.), *United States Tax Reform in the Twenty-First Century.* Cambridge: Cambridge University Press.

Heady, C. (1996). "Optimal Taxation as a Guide to Tax Policy." In M. P. Devereux (ed.), *The Economics of Tax Policy*. Oxford: Oxford University Press.

Holmstrom, B. (1982). "Moral Hazard in Teams." *Bell Journal of Economics* 13, 324–340.

Holmstrom, B. and P. Milgrom. (1990). "Regulating Trade among Agents." *Journal of Institutional and Theoretical Economics* 146(1), 85–105.

Holmstrom, B. and P. Milgrom. (1991). "Multitask Principal Agent Analysis: Incentive Contracts, Asset Ownership and Job Design." *Journal of Law Economics and Organization* 7(2), 201–228.

Mintz, J. (1997). "The Thorny Problem of Implementing New Consumption Taxes." *National Tax Journal* 49, 461–474.

Mirrlees, J. A. (1990). "Taxing Uncertain Incomes." *Oxford Economic Papers* 42, 34–45.

Myles, G. (1995). *Public Economics.* Cambridge: Cambridge University Press.

Slemrod, J. (1990). "Optimal Taxation and Optimal Tax Systems." *Journal of Economic Perspectives* 4, 157–178.

Skinner, J. (1996). "If Agricultural Land Taxation is so Efficient, Why is it so Rarely Used?" In A. Braverman and J. E. Stiglitz (eds.), *The Economics of Rural Organization: Theory, Practice, and Policy.* Oxford: Oxford University Press.

Stern, N. (1982). "Optimum Taxation with Errors in Administration." *Journal of Public Economics* 17, 181–212.

Varian, H. R. (1980). "Redistributive Taxation as Social Insurance." *Journal of Public Economics* 14, 49–68.

Zodrow, G. R. (1995). "Taxation, Uncertainty and the Choice of a Consumption Tax Base." *Journal of Public Economics* 58, 257–265.