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A note on the Loewenstein–Prelec theory of intertemporal choice

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Abstract

In one of the major contributions to behavioral economics, Loewenstein and Prelec [Loewenstein, G., Prelec, D., 1992. Anomalies in intertemporal choice: evidence and an interpretation. The Quarterly Journal of Economics 107, 573–597] set the foundations for the behavioral approach to decision making over time. We correct a number of errors in Loewenstein and Prelec (1992). Furthermore, we provide a correct, more direct and simpler derivation of their generalized hyperbolic discounting formula that has formed the basis of much recent work on temporal choice.

Keywords: Anomalies of the DU model; Intertemporal choice; Generalized hyperbolic discounting; Prospect theory

JEL classification: C60 (General: Mathematical methods and programming); D91 (Intertemporal consumer choice)

1. Introduction

It is well known that the discounted utility model of intertemporal choice (henceforth, DU) is contradicted by a relatively large body of empirical and experimental evidence; see, for instance, Thaler (1981). Furthermore, it appears that these anomalies are not simply mistakes; see, for instance, Frederick et al. (2002). If we wish to develop models that better explain economic behavior, then we have no choice but to take account of these anomalies. Furthermore, certain types of behavior, and several institutional features, can be explained by decision makers

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attempting to deal with time-inconsistency problems that arise from non-exponential discounting.[2]

Loewenstein and Prelec (1992) (henceforth, LP) give a formal statement of the known anomalies of the DU model. Of the anomalies mentioned in LP, the subsequent literature has focussed largely on the evidence for and implications of declining discount rates (DU in contrast assumes constant discount rates). The importance of LP's contribution is that it remains, as far as we know, the only theoretical contribution that provides an explanation of the other anomalies of the DU model, in particular, the magnitude effect and the gain-loss asymmetry; we give definitions below. Furthermore, LP give the first statement and axiomatic derivation of the generalized hyperbolic discounting formula which has been the main, but not the only, alternative to the exponential discounting model.[3]

The purpose of our note is two-fold. First, we correct a number of errors in LP (1992) which have potentially serious implications.[4] Second, we provide a correct, more direct and simpler derivation of their generalized hyperbolic discounting formula.

2. Loewenstein–Prelec theory of intertemporal choice

Consider a decision maker who, at time t_0, formulates a plan to choose c_i at time t_i, $i=1, 2, \ldots, n$, where $t_0 < t_1 < \ldots < t_n$. LP assume that the utility to the decision maker, at time t_0, is given (LP(9), p. 579) by

$$U((c_1,t_1),(c_2,t_2),\ldots,(c_n,t_n)) = \sum_{i=1}^{n} v(c_i)\varphi(t_i) \tag{2.1}$$

We get the standard DU model for the special case of exponential discounting:

$$\varphi(t_i) = e^{-\beta t_i}, \beta > 0 \tag{2.2}$$

Aside from its tractability, the main attraction of DU is that it leads to time-consistent choices (at least, in non-game-theoretic situations). If the plan (c_1,t_1), (c_2,t_2), …, (c_n,t_n) is optimal at time t_0, then at time t_k the plan (c_{k+1}, t_{k+1}), (c_{k+2}, t_{k+2}), …, (c_n,t_n) is also optimal. But this may no longer be true for more general specifications of the discount function φ.

LP adopt the utility function (2.1) taking v to be the value function introduced by Kahneman and Tversky (1979). Thus, v satisfies (among other properties)

$$\begin{aligned} &v:(-\infty,\infty)\rightarrow(-\infty,\infty) \text{ is continuous, strictly increasing,}\\ &v(0)=0 \text{ and is differentiable except at } 0 \end{aligned} \tag{2.3}$$

[2]Time inconsistency problems can lead individuals to make suboptimal decisions about, for instance, savings, pensions, retirement, etc. The existences of mandatory pension plans, retirement age, compulsory insurance of several sorts, etc. are possible institutional responses to these time inconsistency problems; see, for instance, Frederick et al. (2002).

[3]The simpler quasi-hyperbolic formulation, due to Phelps and Pollak (1968) and later popularized by Laibson (1997) often tends to be used in applied theoretical work on account of its tractability. However, LP's formulation is the most general form of the hyperbolic discounting function.

[4]We have checked a more recent reprint of the LP article in Kahneman and Tversky (2000) but the same errors remain.

They define the elasticity of v (LP (16), p. 583) by

$$\epsilon_v(c) = \frac{c}{v}\frac{\mathrm{d}v}{\mathrm{d}c}, \quad c \neq 0 \tag{2.4}$$

LP introduce five assumptions, all with good experimental bases (LP, II p. 574–578). The first four of these are

- A0 (*impatience*). φ: $[0, \infty) \rightarrow (0, \infty)$ is strictly decreasing.[5] If $0<x<y$ then $v(x)=v(y)\varphi(t)$ for some $t \in [0, \infty)$.
- A1 (*gain-loss asymmetry*). If $0<x<y$ and $v(x)=v(y)\varphi(t)$, then $v(-x)>v(-y)\varphi(t)$.
- A2 (*the magnitude effect*). If $0<x<y$, $v(x)=v(y)\varphi(t)$ and $a>1$, then $v(ax)<v(ay)\varphi(t)$.
- A3 (*common difference effect*). If $0<x<y$, $v(x)=v(y)\varphi(t)$ and $s>0$, then $v(x)\varphi(s)<v(y)\varphi(s+t)$.

  A0 is only implicit in LP; however, it is essential for Theorems 1–3 (below). In this note, we make no use of their fifth assumption: delay–speedup asymmetry (LP, p. 578).

  To derive the LP formula for generalized hyperbolic discounting (LP (15), p. 580), a stronger form of A3 is needed. We adopt:
- A3a (*common difference effect with quadratic delay*). If $0<x<y$, $v(x)=v(y)\varphi(t)$ and $s>0$, then $v(x)\varphi(s)=v(y)\varphi(s+t+\alpha st)$, $\alpha>0$.

Note that A3a $\Rightarrow$ A3 and that $\alpha=0$ gives exponential discounting.[6]

Three theorems follow.

Theorem 1. *A0 and A1 imply that the value function is more elastic for losses than for gains:* $x>0 \Rightarrow \epsilon_v(-x)>\epsilon_v(x)$.

Theorem 2. *A0 and A2 imply that the value function is less elastic for outcomes of larger absolute magnitude:*

$$(0 < x < y \text{ or } y < x < 0) \Rightarrow \epsilon_v(x) > \epsilon_v(y).$$

Theorem 3. *A0 and A3a imply that the discount factor is a generalized hyperbola:* $\varphi(t)=(1+\alpha t)^{-\frac{\beta}{\alpha}}, \beta > 0, \ t \geq 0$ *(α is as in A3a).*

Corollary 1. *A0 and A3a imply that* $-\frac{\dot{\varphi}}{\varphi} = \frac{\beta}{1+\alpha t}$. *Hence, the discount rate is positive and declining.*

We discuss the errors in LP more fully in Appendix A. For the moment, we indicate the main errors. Theorem 2 is stated incorrectly in LP and their proof contains an error (LP p. 583–584, V3 p. 584). The generalized hyperbola given in Theorem 3 does not follow from the assumptions made in LP. Our A3a is a corrected version of the one that appears in LP (LP (11), p. 579) and enables us to derive the required generalized hyperbola. There are further errors in an example of intertemporal consumption–savings choice. We correct and sharpen the LP

[5] It is sufficient that φ be strictly decreasing in some interval: $(a, a+\delta)$, $a \geq 0$, $\delta>0$.

[6] As pointed out by the referee, A1 and A2 (in the presence of A0) are equivalent to

$$\frac{v(ax)}{v(ay)} < \frac{v(x)}{v(y)} \text{ for } 0 < x < y, a > 1 \text{ or } a = -1.$$

results. We also give a more direct, simpler and correct[7] proof of Theorem 3. In Section 3, we give the correct proofs.

3. Proofs

Proof of Theorem 1. See LP, V2 p. 583. □

Proof of Theorem 2. Let

$$0 < x < y \tag{3.1}$$

By A0, there is a time, t, such that the consumer is indifferent between receiving the increment x now and receiving the increment y, t-periods from now. Then, letting v be the value function and φ the discount function, we get

$$v(x) = v(y)\varphi(t) \tag{3.2}$$

Let

$$a > 1 \tag{3.3}$$

then the magnitude effect, A2, predicts that

$$v(ax) < v(ay)\varphi(t) \tag{3.4}$$

(3.2) gives:

$$\frac{v(x)}{v(y)} = \varphi(t) \tag{3.5}$$

Since y, a are positive, it follows that ay, and hence, $v(ay)$ are also positive. Hence, (3.4) gives

$$\frac{v(ax)}{v(ay)} < \varphi(t) \tag{3.6}$$

(3.5) and (3.6) give

$$\frac{v(x)}{v(y)} > \frac{v(ax)}{v(ay)}, \quad 0 < x < y; \quad a > 1 \tag{3.7}$$

It follows from (3.7) that the value function, v, is subproportional.[8] It also follows that ln(v) is a concave function of ln(x) (for $x>0$) and that the derivative of ln(v) with respect to ln(x) is decreasing. It then follows that the value function is less elastic for outcomes that are larger in absolute magnitude:

$$(0 < x < y \text{ or } y < x < 0) \Rightarrow \epsilon_v(x) > \epsilon_v(y). \quad \square$$

[7] In Appendix A, we show that LP's generalized hyperbolic discounting formula contradicts their specialized form of A3. Hence, it cannot follow from it.

[8] See Kahneman and Tversky (1979, p. 282) for the definition of subproportionality. Note that our, and LP's, $a>1$ corresponds to their $0<r<1$.

Proof of Theorem 3. Let

$$0 < x < y \tag{3.8}$$

By A0, there is a time, t, such that the consumer is indifferent between receiving the increment x now and receiving the increment y, t-periods from now. Then, letting v be the value function and φ the discount function, we get

$$v(x) = v(y)\varphi(t) \tag{3.9}$$

Multiply (3.9) by $\varphi(s)$,where $s>0$, to get

$$v(x)\varphi(s) = v(y)\varphi(s)\varphi(t) \tag{3.10}$$

A3a, (3.8) and (3.9) give

$$v(x)\varphi(s) = v(y)\varphi(s + t + \alpha st), \quad \alpha > 0 \tag{3.11}$$

(3.10) and (3.11) give

$$\varphi(s + t + \alpha st) = \varphi(s)\varphi(t) \tag{3.12}$$

Let

$$X = 1 + \alpha s, Y = 1 + \alpha t \tag{3.13}$$

Hence,

$$s = \frac{X-1}{\alpha}, \quad t = \frac{Y-1}{\alpha}, \quad s + t + \alpha st = \frac{XY-1}{\alpha} \tag{3.14}$$

Define the function G: $[1, \infty) \to (0, \infty)$ by

$$G(X) = \varphi\left(\frac{X-1}{\alpha}\right) \tag{3.15}$$

Hence,

$$G(Y) = \varphi\left(\frac{Y-1}{\alpha}\right), G(XY) = \varphi\left(\frac{XY-1}{\alpha}\right) \tag{3.16}$$

From (3.12), (3.14) (3.15) (3.16)

$$\begin{aligned} G(XY) &= \varphi\left(\frac{XY-1}{\alpha}\right) = \varphi(s + t + \alpha st) = \varphi(s)\varphi(t) = \varphi\left(\frac{X-1}{\alpha}\right)\varphi\left(\frac{Y-1}{\alpha}\right) \\ &= G(X)G(Y) \end{aligned} \tag{3.17}$$

Define the function h: $[0, \infty) \to (0, \infty)$ by

$$h(x) = G(e^x), \quad x \geq 0 \tag{3.18}$$

Hence, and in the light of A0, h satisfies

$$h : [0, \infty) \to (0, \infty) \text{ is strictly decreasing and } h(x + y) = h(x)h(y) \tag{3.19}$$

As is well known, see, for example, Corollary 1.4.11 in Eichhorn (1978), the unique solution of (3.19) is the exponential function[9]

$$h(x) = e^{cx}, \quad x \geq 0, \quad c < 0 \tag{3.20}$$

(3.13), (3.15), (3.18), and (3.20) give

$$\varphi(t) = (1 + \alpha t)^{c} \tag{3.21}$$

Let

$$\beta = -\alpha c \tag{3.22}$$

(3.21) and (3.22) give

$$\varphi(t) = (1 + \alpha t)^{-\frac{\beta}{\alpha}}, \quad \alpha, \beta > 0, \quad t \geq 0 \tag{3.23}$$

where $\beta > 0$ because $\alpha > 0$ and $c < 0$. □

4. Optimal consumption plans

LP provide a rich set of applications. However, one of them needs correction (their application 5 p. 591–593). The corrected version, given in this section, has sharper conclusions.

Consider a consumer with an exogenously given stream of real income whose present value at time 0 is I. Let the real interest rate, r, be positive and constant. Let the consumer's reference real consumption, $\bar{c}$, be a non-negative constant. At time 0, the consumer chooses her consumption plan $c(t) = c^*(t)$ so as to maximize $\int_{t=0}^{T} v(c(t) - \bar{c})\varphi(t)\mathrm{d}t$ subject $\int_{t=0}^{T} c(t)e^{-rt}\mathrm{d}t \leq I$. Suppose that the consumer is able to commit to this (generally time-inconsistent) plan. Following Kahneman and Tversky (1979), assume that the consumer's value function, v, is strictly concave for gains but strictly convex for losses, i.e., for $x > 0$, $v''(x) < 0$, $v''(-x) > 0$.

4.1. The consumer is always in the domain of gains: $c(t) > \bar{c}$

Here the value function is strictly concave. The problem can be transformed into a standard control problem as follows. Let $y(t)$ be the exogenously given stream of real income and $s(t)$ the stock of saving at time t. Then $c^*(t)$ is the solution to the following problem

$$\text{Maximize} \int_{t=0}^{T} v(c(t) - \bar{c})\varphi(t)\mathrm{d}t$$
$$\text{subject to } \dot{s} = y(t) + rs - c; s(0) = s(T) = 0$$

where the dot represents the time derivative. The Hamiltonian is

$$H(c, s, \lambda, t) = v(c - \bar{c})\varphi(t) + \lambda(y(t) + rs - c)$$

where c is the control variable, s is the state variable and λ is the costate variable. $v\varphi(t)$ is strictly concave in c (hence, jointly in c and s, since s does not occur in $v\varphi(t)$) and $\dot{s}$ is linear in s and c. Hence, Hamilton's equations are both necessary and sufficient for a global optimum (Kamien and

[9]It is sufficient that h be strictly decreasing in some interval: $(a, a+\delta)$, $a \geq 0$, $\delta > 0$.

Schwartz, 1981, II.3). Solving Hamilton's equations: $\frac{\partial H}{\partial c}=0, \dot{\lambda}=-\frac{\partial H}{\partial s}, \dot{s}=y(t)+rs-c$, gives

$$\dot{c}^*=\left[r-\left(-\frac{\dot{\varphi}}{\varphi}\right)\right]\frac{v'}{(-v'')} \tag{4.1}$$

(4.1) gives

(i) If $r\geq\left(-\frac{\dot{\varphi}}{\varphi}\right)_{t=0}$ then, since $-\frac{\dot{\varphi}}{\varphi}$ is declining (by Corollary 1), we get $r>-\frac{\dot{\varphi}}{\varphi}$ for all $t>0$. Hence, $\dot{c}^*>0$ for all $t>0$.

(ii) Suppose $r<\left(-\frac{\dot{\varphi}}{\varphi}\right)_{t=0}$. Here we have two sub-cases:
(iia) $r<-\frac{\dot{\varphi}}{\varphi}$ for all $t<T$. Then $\dot{c}^*<0$ for all $t<T$.
(iib) $r(t_0)=\left(-\frac{\dot{\varphi}}{\varphi}\right)_{t=t_0}$ for some t_0, $0<t_0<T$. Then $\dot{c}^*<0$ for $t<t_0$ and $\dot{c}^*>0$ for $t>t_0$.

4.2. The consumer is always in the domain of losses: $c(t)<\bar{c}$

Here the value function is strictly convex. Hence, it is optimal for the consumer to consume all her income at one point in time. This point has to be at the upper boundary, $t=T$. There are two reasons for this. First, the value of real lifetime income at time t is Ie^{rt}, which is increasing with time. Hence, postponing consumption as long as possible pushes consumption closer to reference consumption and, hence, reduces loss. Second, because of discounting, postponing consumption reduces this loss even further.

Here is a formal derivation. If the consumer consumes all her income at time t, its present value in utility terms will be $u(t)=v(Ie^{rt}-\bar{c})\varphi(t)$. The consumer will postpone consumption as long as $\dot{u}\equiv\dot{\varphi}v+\varphi rIe^{rt}v'>0$. But this quantity is always positive, since $\dot{\varphi}<0$, $v<0$, $\varphi>0$, $r>0$, $I>0$, $e^{rt}>0$, $v'>0$. It follows that the consumer will consume all her income at the last possible moment, i.e., at $t=T$.

5. Conclusions

Loewenstein and Prelec (1992) is a foundational paper in economics. To the best of our knowledge, it provides the first and only available theoretical framework to explain several important anomalies to the DU model. Furthermore, it provides an axiomatic derivation of the generalized hyperbolic discounting formula that forms the basis of much recent research in temporal choice.

We correct several errors in the paper, some with potentially serious implications. We also provide direct and simple proofs of some their most important results and sharpen others.

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Appendix A. Errors in Loewenstein and Prelec (1992)

A.1. Error 1

LP state Theorem 2 incorrectly. What they state (LP, V3 p. 584) is equivalent to:

Theorem 2. *A2 implies that the value function is more elastic for outcomes of larger absolute magnitude: $(0<x<y$ or $y<x<0)\rightarrow\epsilon_v(x)<\epsilon_v(y)$. ('more' should be 'less' and the last '<' should be '>').*

In the course of their proof, LP derive, incorrectly, the formula (LP (18) p. 583):

$$\frac{v(x)}{v(y)}<\frac{v(ax)}{v(ay)}, 0<x<y; a>1 (\text{the first '<' should be '>'}) \tag{6.1}$$

They then conclude that the value function is subproportional. This is correct, but it follows from (3.7), not (6.1). They follow this with the two incorrect statements that ln(v) is a convex function of ln(x) (for $x>0$) and that the derivative of ln(v) with respect to ln(x) is increasing. Of course, what they should have said is that ln(v) is a concave function of ln(x) (for $x>0$) and that the derivative of ln(v) with respect to ln(x) is decreasing.

A.2. Error 2

To derive the formula for generalized hyperbolic discounting, a stronger form of A3 is needed. What LP adopt (LP, (11) p. 579) is equivalent to:

A3b (*common difference effect with linear delay*). If $0<x<y$, $v(x)=v(y)\varphi(t)$ and $s>0$, then $v(x)\varphi(s)=v(y)\varphi(ks+t)$, where $k=k(x,y)$ is a function of x and y but not of s or t. (Note that A3b $\Rightarrow$ A3).

We will now show that generalized hyperbolic discounting implies that $k=1+\alpha t$. Hence, k depends on t but is independent of x and y.

Let

$$0<x<y \tag{6.2}$$

Assume that a consumer is indifferent between receiving the increment x now and receiving the increment y, t-periods from now. Then, letting v be the value function and φ the discount function, we get equation (10) of LP:

$$v(x)=v(y)\varphi(t) \tag{6.3}$$

The common difference effect, A3, then implies that the consumer, now, strictly prefers y, $(s+t)$-periods ahead to x, s-periods ahead. LP say (p. 579 just above (11)) "We now derive a … general functional form, by postulating that the delay that compensates for the larger outcome [y] is a linear function [$t+ks$] of the time [s] to the smaller, earlier outcome [x] (holding fixed the two outcomes x and y)". Hence, their equation (11):

$$v(x)=v(y)\varphi(t)\Rightarrow v(x)\varphi(s)=v(y)\varphi(t+ks) \tag{6.4}$$

LP continue: "for some constant k, which, of course depends on x and y".

From the above equation they prove that the discount function φ must be given by their Eq. (15), reproduced immediately below.

$$\varphi(t)=(1+\alpha t)^{-\frac{\beta}{\alpha}}, \alpha,\beta>0 \tag{6.5}$$

We shall show that if the discount function, φ, is given by their Eq. (15) ((6.5), above), then

$$k=1+\alpha t \tag{6.6}$$

(From (6.6) it is clear that k is a function of t but not of x or y.) Multiplying (6.3) by $\varphi(s)$ gives

$$v(x)\varphi(s) = v(y)\varphi(s)\varphi(t) \tag{6.7}$$

Comparing (6.4) and (6.7), we see that

$$\varphi(t + ks) = \varphi(s)\varphi(t) \tag{6.8}$$

From (6.5) and (6.8), we get

$$(1 + \alpha(t + ks))^{-\frac{\beta}{\alpha}} = (1 + \alpha s)^{-\frac{\beta}{\alpha}}(1 + \alpha t)^{-\frac{\beta}{\alpha}} \tag{6.9}$$

Successive simplifications of (6.9) give

$$(1 + \alpha t + \alpha ks)^{-\frac{\beta}{\alpha}} = [(1 + \alpha s)(1 + \alpha t)]^{-\frac{\beta}{\alpha}} \tag{6.10}$$

$$(1 + \alpha t + \alpha ks)^{-\frac{\beta}{\alpha}} = \left[1 + \alpha s + \alpha t + \alpha^2 st\right]^{-\frac{\beta}{\alpha}} \tag{6.11}$$

$$1 + \alpha t + \alpha ks = 1 + \alpha s + \alpha t + \alpha^2 st \tag{6.12}$$

$$\alpha ks = \alpha s + \alpha^2 st \tag{6.13}$$

$$k = 1 + \alpha t \tag{6.14}$$

It follows that, to get their Eq. (15) ((6.5) above) k must be given by (6.14). Hence, their Eq. (11) ((6.4) above) has to be written as

$$v(x) = v(y)\varphi(t) \Rightarrow v(x)\varphi(s) = v(y)\varphi(s + t + \alpha st). \tag{6.15}$$

A.3. Error 3

(The optimal consumption plan when the consumer is always in the domain of gains) LP's equation (23) p. 592 is incorrect. What they state is equivalent to

$$\dot{c}^* = r - \left(-\frac{\dot{\varphi}}{\varphi}\right)\frac{v'}{(-v'')} \tag{6.16}$$

The correct equation is given by (4.1) above. However, their conclusions are correct but follow from (4.1), not (6.16).

A.4. Error 4

LP derive the incorrect equation (LP (24) p592):

$$\text{The consumer will postpone consumption as long as } r < \frac{-\dot{\varphi}/\varphi}{\epsilon_v(Ie^{rt})} \tag{6.17}$$

Furthermore, they claim, incorrectly, that $\epsilon_v(Ie^{rt})$ is increasing. In fact, $\epsilon_v(Ie^{rt})$ is decreasing (see Theorem 2). They merely conclude that, in the domain of losses, consumption will be concentrated at a single point in time, which could be anywhere in the interval [0,T]. The correct form of (6.17) is

$$\text{The consumer will postpone consumption as long as } r > \frac{-\dot{\varphi}/\varphi}{\frac{I e^{rt}}{I e^{rt} - \bar{c}} \epsilon_v (I e^{rt} - \bar{c})} \tag{6.18}$$

Since the right hand side of (6.18) is negative, the inequality in (6.18) always holds. Hence, we get the sharper result that the consumer will consume all her income at the last possible moment, i.e., at $t=T$. However, we gave a simpler proof of this result in Section 4.2, above.

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