A note on the Loewenstein-Prelec theory of intertemporal choice: Corrigendum

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Abstract

In a critique of the Loewenstein and Prelec (1992) theory of intertemporal choice, al-Nowaihi and Dhami (2006) point out four errors. One of the alleged errors was that the elasticity of the value function in prospect theory is decreasing. But it is in fact increasing. We provide a correction and a formal proof. As a corollary, we show that the elasticity of the value function is bounded between zero and one. Nevertheless, all the remaining points in al-Nowaihi and Dhami (2006) remain valid.

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JEL Classification Codes: C60(General: Mathematical methods and programming); D91(Intertemporal consumer choice).

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1. Introduction

In a recent critique of Loewenstein and Prelec's (1992) theory of intertemporal choice published in this journal, the current authors noted four errors. Unfortunately, we made a mistake in relation to one point.[1] This is the alleged Error 1 on pages 105-106 of al-Nowaihi and Dhami (2006). al-Nowaihi and Dhami (2006) claimed that the elasticity of the value function in prospect theory is decreasing. We show here that this is wrong. We show that the elasticity is *increasing* as correctly claimed by Loewenstein and Prelec (1992). In consequence, our correction of the claimed Error 1 was invalid. Nevertheless, all the remaining points in al-Nowaihi and Dhami (2006) remain valid.

2. The Model

Consider a decision maker who, at time t_0, formulates a plan to choose c_i at time t_i, $i = 1, 2, ..., n$, where $t_0 < t_1 < ... < t_n$. Loewenstein and Prelec (henceforth, LP) assume that the utility to the decision maker, at time t_0, is given (LP (9), p579) by :

$$U\left(\left(c_1, t_1\right), \left(c_2, t_2\right), ..., \left(c_n, t_n\right)\right) = \Sigma_{i=1}^{n} v\left(c_i\right) \varphi\left(t_i\right) \tag{2.1}$$

LP adopt the utility function (2.1) taking v to be the value function introduced by Kahneman and Tversky (1979). Thus v satisfies (among other properties)

$$\begin{gathered} v : (-\infty, \infty) \rightarrow (-\infty, \infty) \text{ is continuous, strictly increasing,} \\ v(0) = 0 \text{ and is twice differentiable except at } 0 \end{gathered} \tag{2.2}$$

LP define the *elasticity* of v (LP (15), p583) by:

$$\epsilon(c) = \frac{c}{v}\frac{dv}{dc},\ c \neq 0. \tag{2.3}$$

For $c > 0$ it follows, from (2.2), that $v(c) > 0$. Let $\widetilde{c} = \ln c$, $\widetilde{v}(\widetilde{c}) = \ln v\left(e^{\widetilde{c}}\right)$ and $\widetilde{\epsilon}(\widetilde{c}) = \epsilon\left(e^{\widetilde{c}}\right)$. Then, from (2.3),

$$\widetilde{\epsilon}(\widetilde{c}) = \frac{d\widetilde{v}}{d\widetilde{c}}. \tag{2.4}$$

LP introduce five assumptions, all with good experimental bases (LP, II pp574-578). The two relevant ones here are:

A0 (*Impatience*) $\varphi : [0, \infty) \rightarrow (0, \infty)$ is strictly decreasing[2]. If $0 < x < y$ then $v(x) = v(y)\varphi(t)$ for some $t \in [0, \infty)$.

[1] Professor Kris Kirby recently asked us for a more detailed proof of one of the Propositions. The error came to light when we were drafting a reply. We are very grateful to Professor Kris Kirby for initiating this discovery.

[2] It is sufficient that φ be strictly decreasing in some interval: $(a, a + \delta)$, $a \geq 0, \delta > 0$.

A2 (*The magnitude effect*) If $0 < x < y$, $v(x) = v(y)\varphi(t)$ and $a > 1$, then $v(ax) < v(ay)\varphi(t)$.

3. Correction

Theorem 1 *(Loewenstein and Prelec, 1992): A0 and A2 imply that the value function is more elastic for outcomes of larger absolute magnitude:* $(0 < x < y$ or $y < x < 0) \Rightarrow \epsilon(x) < \epsilon(y)$.

Proof: From assumption A0, LP prove[3] that the value function is *subproportional*: For $0 < x < y$, $a > 1$,

$$\frac{v(ay)}{v(ax)} > \frac{v(y)}{v(x)}. \tag{3.1}$$

Taking logs of (3.1) we get, in succession, $\ln \frac{v(ay)}{v(ax)} > \ln \frac{v(y)}{v(x)}$, $\ln v(ay) - \ln v(ax) > \ln v(y) - \ln v(x)$ and, hence,

$$\ln v(ay) - \ln v(ax) - [\ln v(y) - \ln v(x)] > 0. \tag{3.2}$$

Let $\widetilde{x} = \ln x$, $\widetilde{y} = \ln y$, $\widetilde{a} = \ln a$, $\widetilde{v}(\widetilde{x}) = \ln v\left(e^{\widetilde{x}}\right) = \ln v(x)$ and $\widetilde{v}(\widetilde{y}) = \ln v\left(e^{\widetilde{y}}\right) = \ln v(y)$, then $\widetilde{x} < \widetilde{y}$, $\widetilde{a} > 0$, $\ln v(ay) = \widetilde{v}(\widetilde{y} + \widetilde{a})$, $\ln v(ax) = \widetilde{v}(\widetilde{x} + \widetilde{a})$, $\ln v(y) = \widetilde{v}(\widetilde{y})$ and $\ln v(x) = \widetilde{v}(\widetilde{x})$. Hence, (3.2) becomes:

$$\widetilde{v}(\widetilde{y} + \widetilde{a}) - \widetilde{v}(\widetilde{x} + \widetilde{a}) - [\widetilde{v}(\widetilde{y}) - \widetilde{v}(\widetilde{x})] > 0. \tag{3.3}$$

Take $\delta x > 0$, $\widetilde{a} = \delta x$, $\widetilde{y} = \widetilde{x} + \delta x$, then (3.3) becomes $\widetilde{v}(\widetilde{x} + 2\delta x) - \widetilde{v}(\widetilde{x} + \delta x) - [\widetilde{v}(\widetilde{x} + \delta x) - \widetilde{v}(\widetilde{x})] > 0$ and, hence, $\frac{\widetilde{v}(\widetilde{x}+2\delta x) - \widetilde{v}(\widetilde{x}+\delta x) - [\widetilde{v}(\widetilde{x}+\delta x) - \widetilde{v}(\widetilde{x})]}{(\delta x)^2} > 0$. Thus:

$$\frac{\frac{\widetilde{v}(\widetilde{x}+2\delta x) - \widetilde{v}(\widetilde{x}+\delta x)}{\delta x} - \frac{\widetilde{v}(\widetilde{x}+\delta x) - \widetilde{v}(\widetilde{x})}{\delta x}}{\delta x} > 0.$$

Let $\delta x \to 0$, to get $\widetilde{v}''(\widetilde{x}) \geq 0$. If $\widetilde{v}''(\widetilde{x}) = 0$ in some non-empty open interval, then $v(x) = x^\beta$ on that interval, which would violate (3.1). Hence, $\widetilde{v}''(\widetilde{x}) > 0$ almost everywhere. From (2.4) it follows that $\widetilde{\epsilon}(\widetilde{x})$ is an increasing function of $\widetilde{x}$ and, hence, $\epsilon(x) = \widetilde{\epsilon}(\ln x)$ is an increasing function of x. A similar argument[4] shows that $\epsilon(x)$ is decreasing for $x < 0$. Thus, the value function, v, is more elastic for outcomes that are larger in absolute magnitude, as stated correctly by LP. □

[3] However, inadvertently they write '<' instead of '>'.

[4] For $y < x < 0$, (3.1) still holds. But now we define $\widetilde{x} = \ln(-x)$, $\widetilde{y} = \ln(-y)$ and $\widetilde{v}(\widetilde{x}) = -\ln\left(-v\left(-e^{\widetilde{x}}\right)\right)$. As before, (3.3) holds and $\widetilde{v}''(\widetilde{x}) > 0$ on any non-empty open interval. Thus $\widetilde{\epsilon}(\widetilde{x})$ is increasing in $\widetilde{x}$. Hence $\epsilon(x)$ is decreasing in x.

4. Bounds on the elasticity of the value function

A standard assumption in prospect theory is that the value function is strictly concave for gains and strictly convex for losses. Combining this with LP's theorem, we get:

Corollary 1 : $0 < \epsilon < 1$.

Proof: That $\epsilon > 0$, follows from (2.2) and (2.3). Also from (2.3) we get:

$$v''(x) = \frac{v(x)}{x}\left[\epsilon' - \frac{\epsilon(1-\epsilon)}{x}\right] \tag{4.1}$$

If $x > 0$ then $v(x) > 0$, $v''(x) < 0$, $\epsilon'(x) \geq 0$. From (4.1) it follows that, necessarily, $\epsilon < 1$. If $x < 0$ then $v(x) < 0$, $v''(x) > 0$, $\epsilon'(x) \leq 0$. From (4.1), it follows that, again, $\epsilon < 1$. □

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6. References

References

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