Demographic Transition and Fertility Rebound in Economic Development♣

Asako Ohinata [a]
University of Leicester

Dimitrios Varvarigos [b, *]
University of Leicester

Abstract

Recent evidence on the 'fertility rebound' offers credence to the idea that, from the onset of early industrialisation to the present day, the dynamics of fertility can be represented by an N-shaped curve. An overlapping generations model with parental investment in human capital can account for these observed movements in fertility rates during the different stages of demographic change. A demographic transition with declining fertility emerges at the intermediate stage, when parents engage on a child quantity-quality trade-off. At later stages however, the process of economic growth generates sufficient resources so that households can rear more children while still providing the desirable amount of education investment per child.

Keywords: Demographic transition; Fertility rebound; Human capital

JEL Classification: J11; O41

♣ We would like to thank two anonymous reviewers for their constructive comments and suggestions which helped us improve several aspects of our study. The usual disclaimer applies.

[a] **Asako Ohinata.** School of Business (Economics Division), University of Leicester, Leicester LE1 7RH, UK.

✉ ao150@le.ac.uk
☎ +44 (0) 116 252 2894

[b] **Dimitrios Varvarigos.** School of Business (Economics Division), University of Leicester, Leicester LE1 7RH, UK.

✉ dv33@le.ac.uk
☎ +44 (0) 116 252 2184

[*] **Corresponding author**

1 Introduction

The relation between economic growth and demographic change has been at the forefront of research on the economics of development for at least three decades. Motivated by recent evidence on a phase of demographic change that researchers have termed 'fertility rebound', this study is the first to develop a theory that offers a joint account for three empirically-observed phases of changes in fertility trends, during the various stages of the development process.

Until recently, the conventional wisdom with regard to fertility dynamics seemed to favour the view that, from the onset of early industrialisation, population changes in Northern Europe and the Scandinavian Peninsula can be categorised into two distinct stages. As a means of illustrating this point, in Figure 1 we present trends in the Total Fertility Rate[1] (TFR) in four western regions between 1850 and 2015.[2] Focusing on Northern Europe and the Scandinavian Peninsula, we can observe what has also been pointed out by many researchers, such as Dyson and Murphy (1985), Galor (2005), and Mokyr and Voth (2010) among others: During the first stage (i.e., up to around 1870) fertility rates and population growth increased in these regions[3], whereas the second stage is a prolonged one, during which these regions, together with Southern Europe and the Western 'Offshoots' (see Figure 1), have witnessed a striking decrease in fertility rates.[4]

Given these observations, a number of studies have endeavoured to offer theoretical frameworks that account for both these phases of demographic change (e.g., Tamura

[1] The Total Fertility Rate is defined as the number of children that would be born to a woman if she were to live to the end of her childbearing years and bear children in accordance with current age-specific fertility rates.

[2] Data on the TFR is taken from Gapminder (https://www.gapminder.org/). We plot separate trends for Northern European countries (Austria, Belgium, Germany, Netherlands, Switzerland, United Kingdom, and Ireland), countries of the Scandinavian Peninsula (Denmark, Sweden, Norway, and Finland), Southern European countries (France, Greece, Italy, Portugal, and Spain), and the Western 'Offshoots' (US, Canada and Australia). These figures include both the scatter plots as well as the fitted local polynomials with degree one. The kernel function for each figure is chosen to be alternative Epanechnikov, although the choice of kernel function does not significantly affect the fit to the data. The bandwidth is chosen by the 'Rule-of-Thumb' bandwidth selector. The corresponding bandwidth is stated at the bottom of each figure.

[3] In fact, recent empirical research has shown that, in some countries, a post-Malthusian regime entailing a positive correlation between income and fertility rates emerged prior to the Industrialisation. See Møller and Sharp (2014) for evidence from the United Kingdom, and Klemp and Møller (2016) for evidence from Scandinavian countries.

[4] As it is evident from Figure 1, for many regions the demographic transition was temporarily interrupted by the 'baby boom'. We will elaborate on that point later in our analysis.

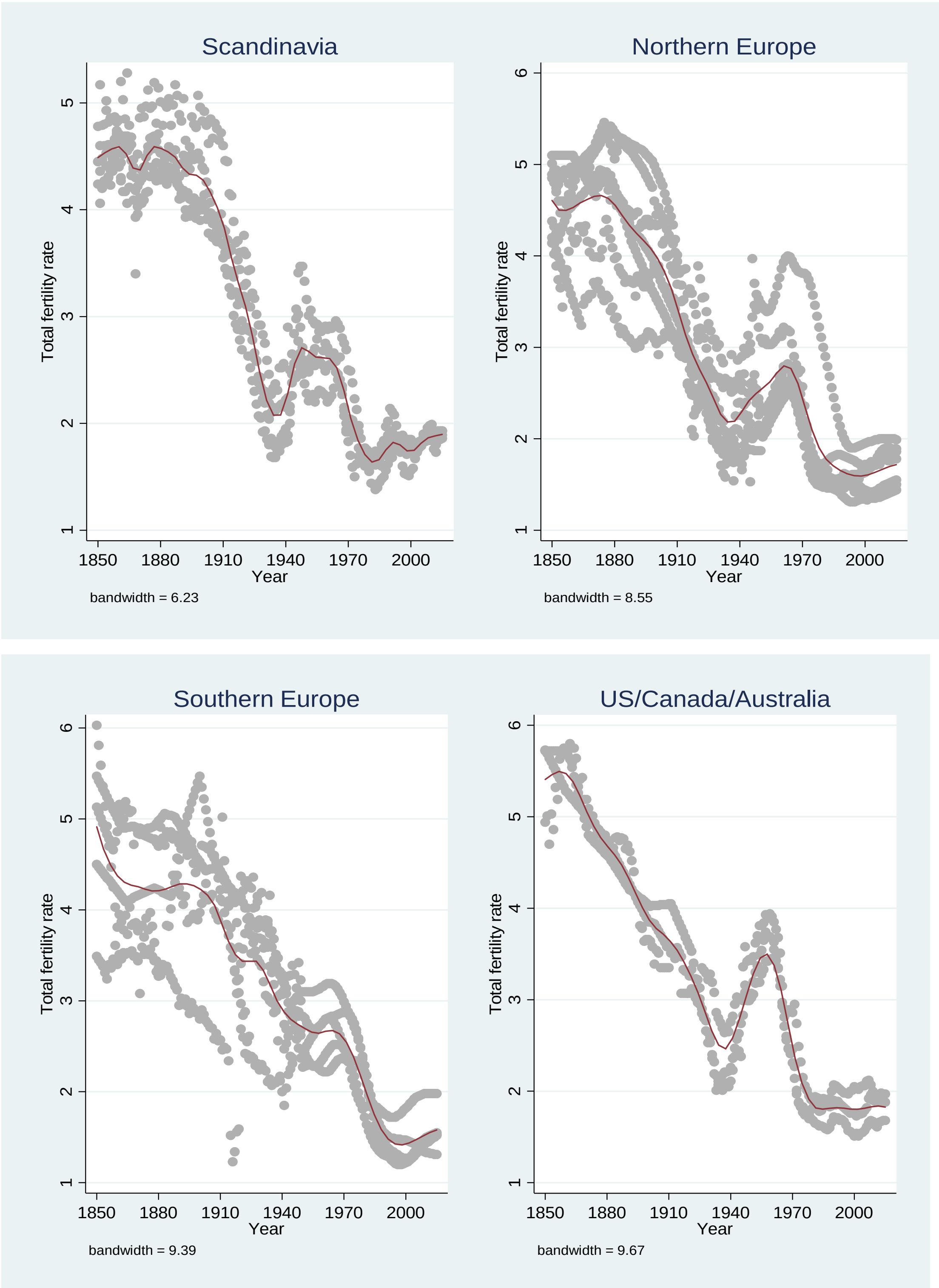

Figure 1. TFR, 1850-2015

1994, 1996; Galor and Weil 2000; Lagerlöf 2003; Tabata 2003; Strulik 2008; Strulik and Weisdorf 2008), while other studies have focused mainly on the second stage of the aforementioned changes (e.g., Becker *et al.* 1990; Galor and Weil 1996; Blackburn and Cipriani 2002; Varvarigos and Zakaria 2013).[5]

More recent data, however, indicates that some developed countries are undergoing a new phase of demographic change. As a means of illustrating and clarifying this point, in Figure 2 we focus on a sample from 1980 to 2015. These scatterplots indicate that the process of fertility reductions may have been reversed in most of the countries in the sample. Rather than pointing out to an outcome that was previously unknown, however, the main message that emanates from Figure 2 is consistent with the message of an emerging literature of empirical studies that have observed a similar outcome, describing it as the 'fertility rebound' in advanced economies. For example, Myrskylä *et al.* (2009), Luci-Greulich and Théveron (2014) and Dominiak *et al.* (2014) employed empirical methods and showed that many developed economies are already going through a phase of fertility rebound.[6]

The aforementioned observations on the fertility rebound, combined with the changes in fertility trends since the beginning of early industrialisation, offer credence to the idea that the dynamics of fertility, along the various stages of economic development, can be traced on an N-shaped curve. In the first stage, fertility rates increase; the second stage witnesses a demographic transition characterised by declining fertility rates; and in the third stage, the trend is once more reversed as fertility rates are increasing. In this study, we develop a theory in which the fertility rebound emerges naturally as the final phase of a 3-stage process of demographic change and economic development, within an overlapping generations model in which households have preferences over the number of children they rear, and the human capital of each of their offspring. The mechanisms that lie behind these outcomes can be summarised as follows: During the early stages of economic development, households find optimal not to invest any of their income towards their children's education; therefore, a rise of

---

[5] Pestieau and Ponthière (2014) construct a model of multiple reproductive periods to argue that optimal fertility can be a source of endogenous cycles. Borck (2011) studies endogenous fertility under different regimes regarding the provision of public education, whereas Dioikitopoulos (2014) employs a model of endogenous fertility to examine the optimal allocation of public expenditure between health and education.
[6] See also Goldstein *et al.* (2009) for a discussion on this issue.

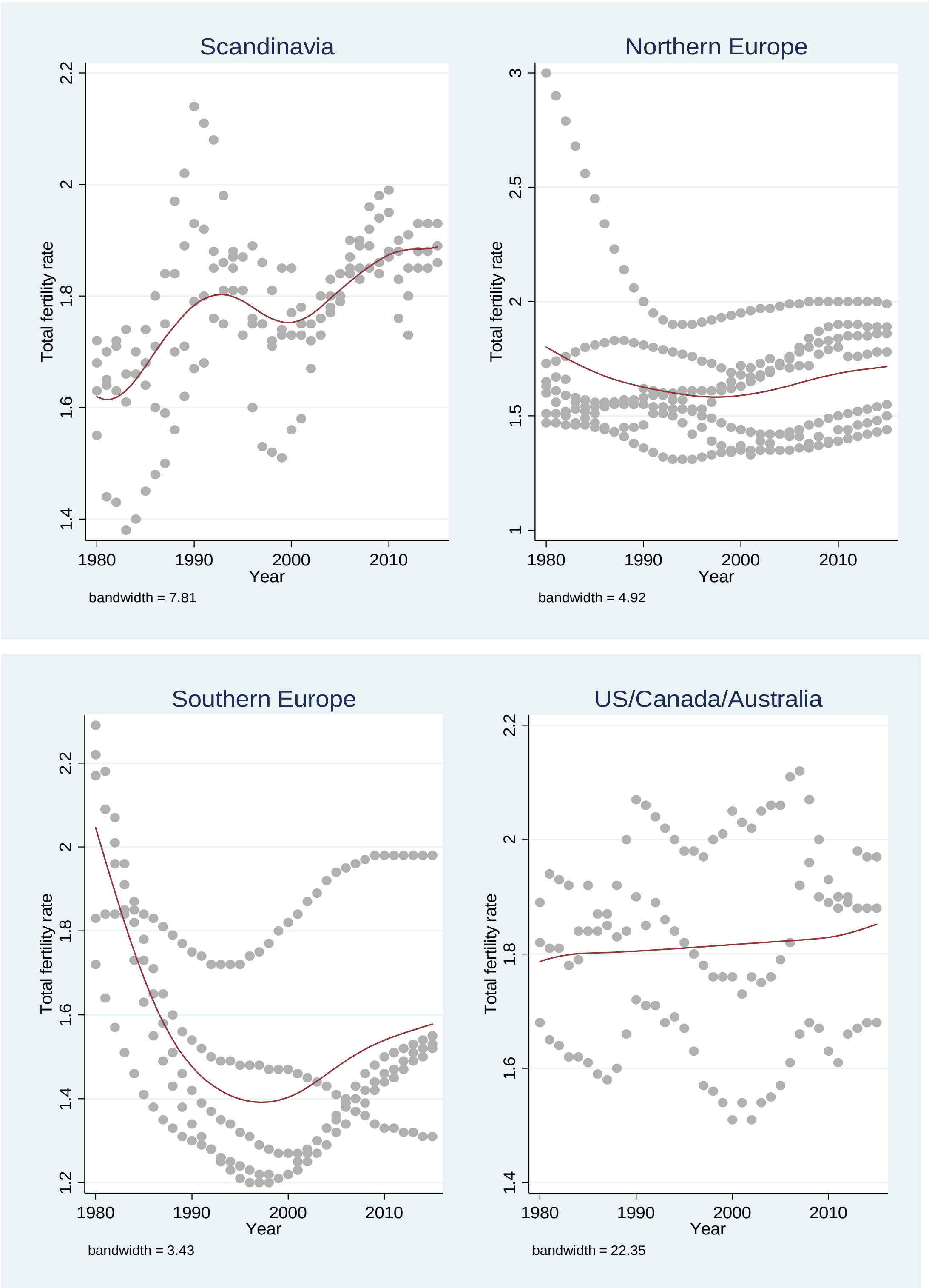

Figure 2. TFR, 1980-2015

disposable income leads to an increase in the number of children raised by each household. As the economy grows, there is a critical stage of development at which parents begin to invest in the education of their children. The high return to education investment, coupled with the relatively limited resources at the disposal of each household, lead to a quantity-quality trade-off. In other words, parental investment in human capital occurs at the expense of family size; thus, fertility rates decline. Nevertheless, as per capita income grows even further, the economy enters a third stage during which families are not constrained by such trade-offs. Instead, disposable income is high enough so that fertility rates increase while parents can still invest the desired resources towards their children's education.

Naturally, the juxtaposition of data (as presented in Figures 1 and 2) with our theoretical framework raises two questions. First, why should we consider the fertility rebound as a change in the dynamics of fertility while neglecting the baby boom that began *circa* 1940 and lasted for roughly 15-20 years? Second, should the more recent fertility rebound merit specific attention, since the increase in fertility rates during this period has been so moderate?

With regard to the first question, the fertility rebound (despite being more moderate in magnitude) has already exceeded, in terms of its length, the baby boom in many countries. Furthermore, for several demographic researchers it is projected as a persistent characteristic in the demographic trends of many developed countries for the foreseeable future (e.g., Alkema *et al.* 2011; Collins and Richards 2013; Schmertmann *et al.* 2014). These are the reasons why, rather than being a temporary (cyclical) change, the fertility rebound reflects a more persistent change in trend, whereas the consensus of the literature on the theory of fertility is that the baby boom was a temporary interruption to the secular decline in fertility rates that began during the late 1800s (e.g., Van Bavel and Reher 2013).[7]

[7] Multiple theories have been proposed to explain the underlying causes of the baby boom. A factor that has been forwarded by several researchers (e.g., Campbell 1974) is the surge in nuptiality following World War II. Greenwood *et al.* (2005) attribute the baby boom to the improved household productivity that was caused by the introduction of electrical household appliances. According to Doepke *et al.* (2015), the baby boom can be attributed to the delayed, post-World War II impact of the increased female labour supply during the wartime mobilisation. Focusing on the Unites States, Tamura and Simon (2017) propose the decline in housing costs, caused by the interstate highway system and the resulting expansion of suburban housing land.

With regard to the second question, it should be emphasised that the countries experiencing the fertility rebound are countries that, during the onset of this change in demographic trends, had very low TFRs, at a level significantly below the replacement one. Coupled with high life expectancy, the low fertility rates can exacerbate population ageing in the future, thus placing significant strain on social security systems, the provision of national health care etc. Therefore, despite the moderate increase in fertility rates, the cumulative impact can be significant, especially as the reversal observed from some countries, such as those in the Scandinavian Peninsula as well as the US, the UK, Australia, Canada and Ireland, is heading towards the replacement TFR (which is 2.1). Even if the increase in TFRs does not reach replacement levels, however, still the cumulative population impact of this process can alleviate the repercussions of population ageing, hence affecting the design and magnitude of policy responses (e.g., social security reform, immigration policy, family policy etc.). In fact, the mechanisms suggested by our paper have an immediate policy implication: Given that the fertility rebound is an inherent part of the development process, heralding a shift to a positive relation between GDP per capita and the fertility rate, growth-promoting policies, in general, can complement more targeted ones in mitigating the problem and the repercussions of population ageing.[8] Finally, it is worth reemphasising that the literature is missing a theoretical foundation for the recent reversal in the TFRs of various Western countries, despite this being documented as a stylised demographic finding by several researchers (e.g. Goldstein *et al.* 2009; Myrskylä *et al.* 2009; Bongaarts and Sobotka 2012; Luci-Greulich and Théveron 2014; Dominiak *et al.* 2014). Our study is a first step towards filling this gap in the literature.

The exposition of the remaining analysis is as follows. In Section 2 we compare the results of our analysis to the existing literature. In Section 3 we build the theoretical model and consider the household's optimal choices regarding child-rearing and education expenditures per child. Section 4 analyses the dynamics of human capital and the dynamics of fertility. In Section 5 we check the robustness of our theoretical results under some extensions of the original model, while Section 6 concludes.

---

[8] In relation to our model's main mechanism, which suggests a shift to a positive relation between per capita income and fertility, it should also be noted that some part of the fertility rebound has coincided with the incidence and the aftermath of the 2007-08 financial crisis. This may have restrained the increase in TFRs across the Westerm world.

2 Related Literature

Given its main focus, this study is related to other analyses that have examined the joint determination of economic and demographic outcomes on the basis of models that introduce choices for fertility and parental investment towards the offspring's education. In addition to the studies that have already been mentioned, other papers in this strand of literature are those by Kalemli-Ozcan (2002); Hazan and Berdugo (2002); de la Croix and Doepke (2004, 2009); Galor and Mountford (2008); and Varvarigos and Arsenis (2015) among others.[9] Under different settings, these studies offer various theoretical arguments for the negative relation between fertility rates and economic development. Another related study is also the one by Hazan and Zoabi (2015). In order to explain evidence showing that the relation between the TFR and the education level of women in the United States is U-shaped, they build a model in which households can purchase marketable services such as education, childcare and housekeeping. They show that the number of children raised is non-monotonic with respect to a household's human capital.[10] Contrary to our analysis, their model is not dynamic; therefore, it does not reproduce the N-shaped fertility dynamics that emerge in this framework.

As we argued in the previous section, our study offers a theoretical foundation behind the fertility rebound, arguing that it is an inherent part of an economy's development process that reaches its advanced stages. Other researchers have suggested some alternative explanations, but through a descriptive approach rather than through the aid of a formal, analytical framework. One explanation that is conventionally used to account for the reversal in fertility trends is based on the tempo effect, i.e., the idea that the timing of childbearing has shifted at later stages of the reproductive age (e.g., Bongaards and Sobotka 2012). Nevertheless, in Myrskylä *et al.* (2009) and Luci-Greulich and Théveron (2014) the significance of the recent switch to a positive relation between fertility and the level of economic development is robust to adjustments in the TFR that are made to account for this tempo effect. Immigration could also be considered as one of the factors behind the fertility rebound, as long as the fertility rate among immigrants

---

[9] For empirical evidence on the quantity-quality trade-off, see Hanushek (1992), Black *et al.* (2005) and Becker *et al.* (2010) among others.

[10] Monstad *et al.* (2008) provide further evidence to support the view that the impact of parents' human capital on fertility is ambiguous.

is, on average, higher than the one of the host country's nationals. This argument, however, does not have enough empirical support. For example, Tromans *et al.* (2009) show that a large part of the recent increase of United Kingdom's TFR is attributed to UK-born women rather than foreign-born ones, thus rejecting the idea that immigration is the only factor behind the fertility rebound.[11]

3 The Model

Time is discrete and indexed by $t = 0,1,2....$ The economy is populated by overlapping generations of households that have a lifespan of two periods – *childhood* and *adulthood*. In childhood, individuals are reared by their parents and receive education that determines the stock of human capital, or *effective* labour, that will be available to them when they become adults. In adulthood, they receive a salary by offering their labour to perfectly competitive firms. These firms produce units of the economy's final good by utilising effective labour under a linear production technology. Such a technology implies that the wage per unit of effective labour (denoted $\omega > 0$) is constant over time. Adult households decide how to allocate their income between consumption, child-rearing, and education expenditures per child.

Consider a household that begins adulthood in period t and suppose that the members of the household wish to bear n_t children. Rearing each child entails a fixed cost of $q > 0$ units of output. Furthermore, parents may wish to spend resources towards the education of each of their offspring. Using x_t to denote the amount of education expenditures per child, it follows that the household's budget constraint is

$$c_t = \omega h_t - n_t(q + x_t), \tag{1}$$

where c_t denotes consumption and h_t is the stock of human capital, i.e., the variable that ultimately determines the amount of effective labour available to each household. The assumption that rearing costs per child are independent of growth-related factors follows Prinz (1990). It is innocuous and adopted purely for expositional clarity. In Section 5.2, we relax this assumption and show circumstances under which our results

[11] Day (2012) argues that the fertility rebound may be attributed to combination of rising female wages and higher productivity of childcare services.

and implications remain unaffected, even when child-rearing costs increase as the economy grows.

As noted earlier, parents can affect each child's human capital by devoting resources towards their education. Particularly, given that each parent devotes x_t units of output per child, human capital will be determined through

$$h_{t+1} = \varphi h_t^{\eta} + \psi h_t^{\mu} x_t , \tag{2}$$

where $\varphi, \psi > 0$ and $\eta, \mu \in (0,1)$. Note that the effect of h_t in Eq. (2) captures intergenerational externalities that generate dynamics in the formation of human capital (e.g., Blackburn and Cipriani 2002; de la Croix and Doepke 2003; Varvarigos and Arsenis 2015). We have assumed constant – rather than diminishing – returns to x_t as this is the only formulation that yields closed-form solutions in our model. Note that the assumption of constant returns to education expenditures is not alien to the existing literature. For example, the same assumption is adopted, among others, by Moav and Neeman (2010, 2012) and Vogl (2016), in models where education investment is in the form of output – like in our current set-up – and by de la Croix and Doepke (2003) and Zhang and Zhang (2005), in models where education investment is in the form of time/effort.

Another issue that should be noted is that the specification of the human capital technology in (2) will be consistent with a stationary solution for the human capital stock, as we shall establish shortly. Nevertheless, this property of the model is not crucial for the determination of our results. Later it will become clear that the dynamics of fertility remain identical even if one sets $\mu = 1$ in (2), thus creating the conditions that allow an ever increasing stock of human capital and, consequently, growth in the long-run.

The lifetime utility of the household is given by

$$u_t = \gamma \ln(c_t) + (1-\gamma)\left[\beta \ln(n_t) + \theta \ln(n_t h_{t+1})\right], \tag{3}$$

where $\gamma \in (0,1)$ and $\beta, \theta > 0$ are preference parameters.[12] In addition to the utility accruing from the consumption of goods, households enjoy utility from the children

[12] Logarithmic utility is the only specification that can yield closed form solutions, hence our decision to adopt it. In fact, the vast majority of existing studies that investigate similar issues of fertility and human capital, use logarithmic utility as well (e.g., Tamura 1994, 1996; Galor and Weil 2000; Kalemli-Ozcan 2002;

they bear and raise over their lifetime. In this context however, children are not only valued *per se* but also in terms of their human capital, i.e., households also enjoy greater felicity by raising more educated children – an (imperfectly) altruistic motive that may capture the idea that parents care about their offspring's human capital because this improves their future prospects – in fact, our results will remain intact of we replace human capital with the offspring's future labour income, wh_{t+1}, in the parental utility function.[13]

Households make their choices so as to maximise their lifetime utility in (3), subject to the constraints in Eq. (1) and (2). In order to solve this problem, we can substitute these constraints in (3) and maximise with respect to n_t and x_t. The respective first order conditions are given by

$$\frac{\gamma(q+x_t)}{\omega h_t - n_t(q+x_t)} \geq \frac{(1-\gamma)(\beta+\theta)}{n_t}, \quad n_t \geq 0, \tag{4}$$

and

$$\frac{\gamma n_t}{\omega h_t - n_t(q+x_t)} \geq \frac{(1-\gamma)\theta\psi h_t^{\mu}}{\varphi h_t^{\eta} + \psi h_t^{\mu} x_t}, \quad x_t \geq 0. \tag{5}$$

The expressions in (4) and (5) offer some familiar conditions according to which the marginal benefit from each activity must be equal to the corresponding marginal cost – both expressed in terms of utility. The marginal utility cost in both cases is associated with the loss of consumption that results from the increase of the resources required to raise and educate the household's offspring. The marginal utility benefit stems from the idea that parents enjoy raising children, as well as supporting their education.

We can express (4) as an equality and solve it to get

$$n_t(q+x_t) = \frac{(1-\gamma)(\beta+\theta)}{\gamma+(1-\gamma)(\beta+\theta)}\omega h_t. \tag{6}$$

Galor and Moav 2002; Hazan and Berdugo 2002; Lagerlöf 2003; de la Croix and Doepke 2003, 2004, 2009; Tabata 2003; Zhang and Zhang 2005; Galor and Mountford 2008; Dioikitopoulos 2014; Varvarigos and Arsenis 2015; Vogl 2016). In this respect, the adoption of similar preferences in our model allows a better comparison of its mechanisms in relation to those in the existing literature.

[13] Rewriting the second part of the utility function as $(1-\gamma)\left[(\beta+\theta)\ln(n_t)+\theta\ln(h_{t+1})\right]$, we can see that the formulation in (3) implies that the utility weight on the number of children each household gives birth to, is higher than the utility weight attached to human capital per child. This assumption is essential for the existence of an equilibrium with an interior solution for n_t. The same technical condition has been used by de la Croix and Doepke (2009) among others.

According to Eq. (6), a household will dedicate a fixed fraction of disposable labour income in order to finance the total costs associated with having children – costs that include both rearing and education. This fraction corresponds to the relative weight attached to the utility that parents enjoy from their offspring.

Next, we can substitute (6) in (5) and express the latter as an equality. Solving this, we get

$$x_t = \frac{(1-\gamma)\theta}{\gamma + (1-\gamma)(\beta+\theta)} \frac{\omega h_t}{n_t} - \frac{\varphi}{\psi} h_t^{\eta-\mu} . \tag{7}$$

This result reveals that the amount of resources that parents spend for the education of each child has two components. With regard to the first component, the fraction of disposable income devoted for education expenditures is associated with the relative weight attached to the utility accruing from the number of children, when these are measured in effective terms (i.e., augmented by each child's human capital). Naturally, the educational resources per child are negatively related to the total number of children raised by the household. As for the second component, it reveals that, on the outset, the interegenerational externality from the current stock of human capital has an ambiguous overall effect on the amount of resources devoted to each child's education. The magnitude of this effect depends on the relative strengths of the relevant coefficients, whereas the ambiguity stems from the fact that, in terms of marginal utility, h_t entails effects that can either substitute or be complementary to x_t: On the one hand, a higher h_t is supportive to a young person's human capital improvements, even without the need for any education investment by her parents; on the other hand, a higher h_t also increases the return (in terms of human capital improvement) of parental investments towards the offspring's education.

The system of equations in (6) and (7) can be solved simultaneously to yield the solutions for private education expenditures per child, and fertility. These solutions are given by

$$x_t = X(h_t) = \max\left\{0, \frac{1}{\beta}\left[\theta q - (\theta+\beta)\frac{\varphi}{\psi} h_t^{\eta-\mu}\right]\right\}, \tag{8}$$

and

$$n_t = N(h_t) = \begin{cases} \dfrac{(1-\gamma)(\beta+\theta)}{\gamma+(1-\gamma)(\beta+\theta)} \dfrac{\omega h_t}{q} & \text{if} \quad x_t = 0 \\ \\ \dfrac{(1-\gamma)\beta}{\gamma+(1-\gamma)(\beta+\theta)} \dfrac{\omega\psi h_t^{1+\mu-\eta}}{q\psi h_t^{\mu-\eta} - \varphi} & \text{if} \quad x_t > 0 \end{cases}, \tag{9}$$

respectively. These two results can facilitate us in analysing and understanding the joint determination of demographics and economic development. The following section will focus on the evolution of human capital and on the dynamics of optimal fertility.

4 Dynamics

4.1 Economic Dynamics

A closer look at the result in (8) reveals that there are circumstances under which parents may find optimal not to invest any resources towards the education of their offspring. The underlying cause for this possibility lies on the fact that, as long as $\varphi > 0$, each child will still be endowed with units of efficient labour, due to the presence of the interegenerational externality, even though parents may not invest any resources towards her education. In order to keep the analysis consistent with the existing literature and the empirical evidence on the matter, it is natural to focus attention on the case where the decision not to invest in the offspring's education materialises at low levels of income. Henceforth, we will be assuming that the condition $\mu > \eta$ holds. Given this, when the stock of human capital is relatively low, the utility cost of foregone consumption outweighs the utility benefit of educating children and increasing their efficiency. Nevertheless, when the stock of human capital is relatively high, its complementary effect becomes strong enough to guarantee that the return to investment in education is sufficiently high to compensate parents for the utility loss due to decreased consumption. The main message from this discussion can be summarised in

Lemma 1. *There exists a threshold* $\tilde{h} \equiv \left[\dfrac{(\theta+\beta)\varphi}{\theta\psi q}\right]^{\frac{1}{\mu-\eta}}$ *such that*

$$x_t = X(h_t) = \begin{cases} 0 & if \quad h_t \le \tilde{h} \\ \dfrac{1}{\beta}\left[\theta q - (\theta+\beta)\dfrac{\varphi}{\psi}h_t^{\eta-\mu}\right] & if \quad h_t > \tilde{h} \end{cases}. \tag{10}$$

Proof. From Eq. (8), we can see that $X(\tilde{h}) = 0$ and $X'(h_t) = \dfrac{(\mu-\eta)(\theta+\beta)\varphi}{\beta\psi}h_t^{\eta-\mu-1} > 0$. Therefore, $x_t > 0$ if and only if $h_t > \tilde{h}$. Furthermore, the non-negativity constraint implies that $x_t = 0 \;\; \forall \;\; h_t \le \tilde{h}$. □

The outcome summarised in Lemma 1 allows us to combine (2) and (10) in order to express human capital accumulation as follows:

$$h_{t+1} = F(h_t) = \begin{cases} \varphi h_t^{\eta} & \text{if} \quad h_t \le \tilde{h} \\ \dfrac{\theta(\psi q h_t^{\mu} - \varphi h_t^{\eta})}{\beta} & \text{if} \quad h_t > \tilde{h} \end{cases}. \tag{11}$$

Using (11), we can characterise the dynamics of human capital through

Proposition 1. *Assume that* $\psi q > \dfrac{\theta+\beta}{\theta}\varphi^{(1-\mu)/(1-\eta)}$ *holds. Then, for any* $h_0 > 0$*, the economy will converge to an asymptotically stable steady state* h^**, such that* $h^* > \tilde{h}$.

Proof. See the Appendix. □

The dynamics of human capital are illustrated on the phase diagram of Figure 3. The return to human capital investment is high enough so that the economy will eventually exceed the threshold that governs households' decisions to devote private resources for the education of their children. This outcome supports the formation of human capital and leads to a (relatively) high steady state equilibrium h^*.

The proof to Proposition 1 (see the Appendix) also reveals the outcome that transpires when the condition $\psi q > \dfrac{\theta+\beta}{\theta}\varphi^{(1-\mu)/(1-\eta)}$ is not satisfied. In the case where $\psi q < \dfrac{\theta+\beta}{\theta}\varphi^{(1-\mu)/(1-\eta)}$ we have $\breve{h} < \tilde{h}$ and $F'(\breve{h}) = \eta \in (0,1)$, where $\breve{h} = \varphi^{1/(1-\eta)}$. Consequently,

$\breve{h}$ will emerge as a stable steady state, at least for some range on the domain of h_t. Given that this equilibrium is associated with $x_t = 0$, whereas our purpose is to examine a scenario for which a transition from $x_t = 0$ to $x_t > 0$ will occur, we rule out this possibility when I analyse the dynamics of fertility in the following section.

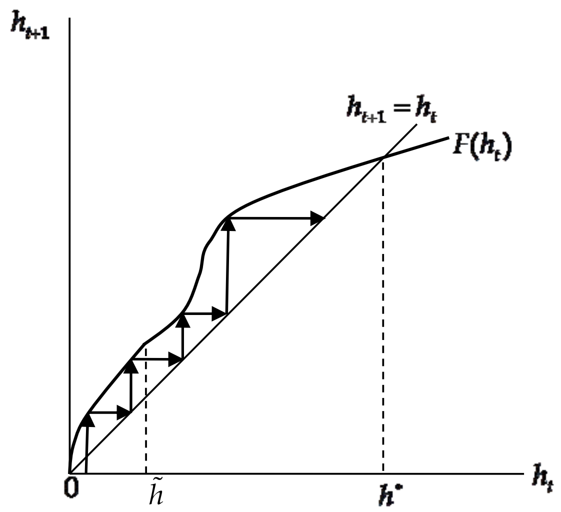

Figure 2. The evolution of human capital

4.2 Fertility Dynamics

The purpose of this section is to trace the dynamics of fertility along the process of economic development. We shall begin the analysis by using the results in (9) in order to examine how fertility varies with the stock of human capital. This analysis leads to

Lemma 2. *Consider* $n_t = N(h_t)$. *It is straightforward to establish that*

i. *When* $x_t = 0$, *then* $N'(h_t) > 0$.

ii. *When* $x_t > 0$, *then there exists* $\hat{h} \equiv \left[\frac{(1+\mu-\eta)\varphi}{\psi q}\right]^{\frac{1}{\mu-\eta}}$ *such that*

$$N'(h_t)\begin{cases} < 0 & for \quad h_t < \hat{h} \\ > 0 & for \quad h_t > \hat{h} \end{cases}.$$

Proof. See the Appendix. □

It will be useful to make a comparison between the threshold values $\tilde{h}$ and $\hat{h}$. This exercise is undertaken in

Lemma 3. *As long as* $\dfrac{(1+\mu-\eta)\theta}{\theta+\beta} > 1$ *holds, it is* $\hat{h} > \tilde{h}$.

Proof. The condition $\hat{h} > \tilde{h}$ implies $\left[\dfrac{(1+\mu-\eta)\varphi}{q\psi}\right]^{\frac{1}{\mu-\eta}} > \left[\dfrac{(\theta+\beta)\varphi}{\theta q\psi}\right]^{\frac{1}{\mu-\eta}}$ which is indeed true for $(1+\mu-\eta)\theta/(\theta+\beta) > 1$. □

In the absence of the condition that underlies Lemma 3, the model's equilibrium would rule out any circumstances under which the relation between the fertility rate and income is negative, hence putting it at odds with the strong evidence regarding the demographic transition. For this reason, Lemma 3 ensures that such a transition is a feature of the economy's dynamic equilibrium.

Now, we can gather the previous results in order to understand the qualitative impact of the human capital stock on fertility. This is summarised in

Proposition 2. *As the stock of human capital grows, the fertility rate increases for* $h_t < \tilde{h}$*; it declines for* $\tilde{h} < h_t < \hat{h}$*; and it increases again for* $h_t > \hat{h}$*. Formally,*

$$N'(h_t)\begin{cases} >0 & \text{for} \quad h_t < \tilde{h} \\ <0 & \text{for} \quad \tilde{h} < h_t < \hat{h} \\ >0 & \text{for} \quad h_t > \hat{h} \end{cases}. \tag{12}$$

Proof. It follows from Lemmas 2 and 3. □

The expression in (12) reveals that the response of optimal fertility to an increasing stock of human capital and, therefore, an increasing level of income per household is non-monotonic. Specifically, the fertility rate increases with h_t at relatively low levels of income; decreases at intermediate levels of income; and increases again at relatively high levels of income, as the stock of human capital and, therefore, fertility converge to their long-run (steady state) values. Comparing with existing theoretical studies on fertility choices and economic growth, we can see that those focusing on declining rates of fertility have identified the outcomes that, in our model, transpire at the second phase of the growth process. Other studies have accounted for the inverted U-shaped relation between fertility rates and output – an outcome that our framework also captures along a two-phase growth process. The novelty of our model is that, contrary to existing studies, it identifies a third, distinct phase on the relation between the rate of fertility and income per household – a relation that is positive. As we argued earlier, this phase is not a mere figment of the theoretical framework; on the contrary, it is empirically observed in many advanced economies.

Naturally, our objective is to analyse an economy that goes through all the stages of possible demographic changes, as it converges to the long-run equilibrium that is characterised by h^*. It follows that the subsequent analysis will focus on a scenario where the steady state equilibrium lies above the two thresholds identified previously. With this in mind, let us consider

Lemma 4. *Assume that* $q\psi > \max\left\{\left(\frac{\beta}{\theta}\right)^{\frac{\mu-\eta}{1-\eta}} \frac{(1+\mu-\eta)\varphi^{(1-\mu)/(1-\eta)}}{(\mu-\eta)^{(\mu-\eta)/(1-\eta)}}, \frac{(\theta+\beta)\varphi^{(1-\mu)/(1-\eta)}}{\theta}\right\}$ *holds. Then* $h^* > \hat{h}$.

Proof. See the Appendix. □

The results in Proposition 2 and Lemmas 3 and 4 allow us to understand the movements in fertility as the economy goes through different stages of the development process towards its convergence to the stationary equilibrium. From these results it follows that, for an initial stock of human capital that satisfies $h_0 < \tilde{h}$, the economy will initially exceed the threshold captured by $\tilde{h}$ and will subsequently exceed the threshold captured by $\hat{h}$ as well. Let us define time periods $\tilde{T}$ and $\hat{T}$ such that

$$h_t \begin{cases} < \tilde{h} & for \quad t = 0, \ldots \tilde{T} \\ \in (\tilde{h}, \hat{h}) & for \quad t = \tilde{T}, \ldots, \hat{T} \\ > \hat{h} & for \quad t = \hat{T}, \ldots \end{cases}, \tag{13}$$

where it should be noted that $\hat{T} > \tilde{T}$ holds by virtue of Proposition 1 and Lemma 3. Given these, a formal characterisation of the dynamics of fertility is possible through

Proposition 3. *There are three different stages of fertility dynamics. Fertility increases from $t = 0$ to $t = \tilde{T}$, it declines from $t = \tilde{T}$ to $t = \hat{T}$, and it increases again from $t = \hat{T}$ onwards.*

Proof. It follows the expressions in (12) and (13). □

The dynamics of fertility are illustrated in Figure 4, where it is clear that they depict an N-shaped graph. The intuition is the following. At the first stage (corresponding to $h_t < \tilde{h}$), the return to the parental investment in education is so low that parents decide to spend the amount of income that they do not consume, entirely for child-rearing purposes. Hence, as disposable income grows, families can afford rearing more children (see Eq. 10 for $x_t = 0$, where child-rearing absorbs a constant fraction of disposable income). Gradually however, the threshold defined by $\tilde{h}$ will be exceeded and the return to private education spending will be high enough to motivate households to dedicate part of their resources towards this purpose. An intuitive explanation of the outcomes that transpire from this point onwards is possible if we use (9) and (10) to write child-rearing and total education expenditures as fractions of disposable income. That is

$$\frac{qn_t}{\omega h_t} = \frac{(1-\gamma)\beta}{\gamma + (1-\gamma)(\beta+\theta)} \frac{q\psi h_t^{\mu-\eta}}{q\psi h_t^{\mu-\eta} - \varphi} = \delta(h_t), \tag{14}$$

and

$$\frac{n_t x_t}{\omega h_t} = \frac{(1-\gamma)(\beta+\theta)}{\gamma + (1-\gamma)(\beta+\theta)} \frac{\left(\frac{\theta}{\beta+\theta} q\psi h_t^{\mu-\eta} - \varphi\right)}{q\psi h_t^{\mu-\eta} - \varphi} = \zeta(h_t), \tag{15}$$

from where we can easily check that $\delta(h_t), \zeta(h_t) \in (0,1)$ for $h_t > \tilde{h}$. From (14) and (15), it follows that $\delta'(h_t) < 0$ and $\zeta'(h_t) > 0$, i.e., as the economy develops, parents devote a decreasing fraction of their income towards child-rearing and an increasing part of their income towards the education of their offspring. In fact, the return to education spending is so high during the second stage (corresponding to $\tilde{h} < h_t < \hat{h}$) that we observe what is effectively a quantity-quality trade-off. In other words, households actually reduce the number of children they rear in order to finance the desired amount of education expenditures per child. Nevertheless, the economy continues to grow and eventually reaches the third stage (corresponding to $h_t > \hat{h}$). Now, disposable income is sufficiently high so that a quantity-quality trade-off is not necessary. In other words, the share of total income on child-rearing may be declining, but the increase in income is so pronounced that the overall amount available for raising children is higher. Households have enough resources to raise more children and still provide the desirable amount of education spending for each of them, as the economy converges to its long-run equilibrium.

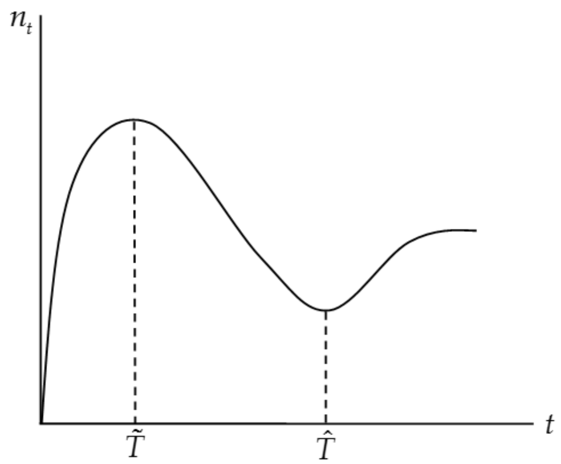

Figure 3. The dynamics of fertility

Given the above, the interplay between optimal fertility and economic development allows our model to generate three historically-observed changes in fertility trends. To the best of our knowledge, this is the first theoretical model to reproduce such outcomes within a single framework, as existing studies have focused either on the phase of fertility reductions, or on the two stages that generate an inverse U-shaped curve on the relation between fertility and income. None of them, however, have reproduced the fertility rebound that is observed over the last two-to-three decades in many advanced economies.

5 Extensions and Different Approaches

5.1 Uncertainty in the Return to Human Capital Investment

So far, we have treated the return to parental investment in education as being deterministic. Nevertheless, as with other forms of investment, its return may incorporate some risk. In this section, our purpose is to examine whether our main results survive under a stochastic environment where the return to education expenditures is risky.

We shall replace the technology in Eq. (2) with

$$h_{t+1} = \varphi h_t^{\eta} + \psi_{t+1} h_t^{\mu} x_t, \tag{16}$$

where ψ_{t+1} is a random variable whose realisation occurs after parents commit resources towards the education of their offspring. We adopt a simple distribution according to which the realised value of ψ_{t+1} will be either $\psi - \sigma$ or $\psi + \sigma$ ($0 < \sigma \leq \psi$), with equal probability $\frac{1}{2}$; in other words, σ represents a mean-preserving spread in the distribution of the return to education. Despite this assumption, the result in Eq. (6) remains the same because the first order condition with respect to n_t remains unaffected. However, the first order condition with respect to x_t changes to

$$\frac{\gamma n_t}{\omega h_t - n_t(q + x_t)} \geq (1-\gamma)\theta h_t^{\mu} \frac{1}{2}\left[\frac{\psi - \sigma}{\varphi h_t^{\eta} + (\psi - \sigma) h_t^{\mu} x_t} + \frac{\psi + \sigma}{\varphi h_t^{\eta} + (\psi + \sigma) h_t^{\mu} x_t}\right], \quad x_t \geq 0. \tag{17}$$

Combining (6), (17) and treating the latter as an equality, yields

$$\frac{\beta + \theta}{q + x_t} = \theta \frac{\psi \varphi h_t^{\eta-\mu} + x_t(\psi^2 - \sigma^2)}{(\varphi h_t^{\eta-\mu} + \psi x_t)^2 - \sigma^2 x_t^2}. \tag{18}$$

Further manipulation of Eq. (18) yields the quadratic equation

$$\frac{\beta}{\theta}(\psi^2-\sigma^2)x_t^2-\left[q(\psi^2-\sigma^2)-\frac{\psi\varphi h_t^{\eta-\mu}(2\beta+\theta)}{\theta}\right]x_t+\psi\varphi h_t^{\eta-\mu}\left[\frac{\psi\varphi h_t^{\eta-\mu}(\beta+\theta)}{\theta}-q\psi\right]=0\,. \quad (19)$$

The expression in (19) has only one acceptable solution for x_t, given by

$$x_t=\frac{q(\psi^2-\sigma^2)-\frac{\psi\varphi h_t^{\eta-\mu}(2\beta+\theta)}{\theta}+\sqrt{[q(\psi^2-\sigma^2)-\psi\varphi h_t^{\eta-\mu}]^2+\beta(\beta+\theta)\left(\frac{2\sigma\varphi h_t^{\eta-\mu}}{\theta}\right)^2}}{2\frac{\beta}{\theta}(\psi^2-\sigma^2)}, \quad (20)$$

whereas the solution for fertility is obtained following the substitution of (20) in (6), as long as $x_t>0$. That is

$$n_t=\frac{(1-\gamma)(\beta+\theta)}{\gamma+(1-\gamma)(\beta+\theta)}\times$$
$$\frac{2\frac{\beta}{\theta}(\psi^2-\sigma^2)\omega h_t}{\frac{2\beta+\theta}{\theta}[q(\psi^2-\sigma^2)-\psi\varphi h_t^{\eta-\mu}]+\sqrt{[q(\psi^2-\sigma^2)-\psi\varphi h_t^{\eta-\mu}]^2+\beta(\beta+\theta)\left(\frac{2\sigma\varphi h_t^{\eta-\mu}}{\theta}\right)^2}}. \quad (21)$$

Obviously, these results are too complicated to investigate analytically. For this reason, the subsequent analysis will be undertaken by means of a numerical example. Particularly, we set the following parameter values: $\eta=0.1$, $\mu=0.8$, $\beta=0.3$, $\theta=0.9$, $\varphi=0.5$, $\psi=1.1$, $\sigma=0.15$, $q=1.1$, $\omega=9$ and $\gamma=0.5$. Subsequently, we examine the effect of h_t on x_t (see Figure 5a). Similarly to the original model, this is a positive effect where, given the non-negativity constraint on human capital investment, we find that $x_t>0$ when $h_t>\tilde{h}=0.426753$. Obviously, for $h_t<0.426753$ the non-negativity constraint implies that $x_t=0$, therefore fertility increases with h_t by virtue of Eq. (6). When $h_t>0.426753$, the dynamics of fertility, as the economy grows, are again identical to the original version of the model. As we can see from Figure 5b, fertility decreases up to $h_t=\hat{h}\approx 0.6$ from which point it increases. All in all, the main implications of our model still remain identical even with presence of uncertainty in the return to education investment.

Of course, the previous analysis does not imply that the introduction of risk is not important for economic outcomes. On the contrary, our numerical examples indicate that (when $h_t>\tilde{h}$) an increase in σ leads to a decline in x_t and an increase in n_t. In terms of intuition, risk-averse parents respond to a mean-preserving spread in the distribution of the return to education, by reducing this type of investment and redirecting more resources towards child-rearing. This effect is, in fact, consistent with evidence on the negative effect of uncertainty on capital accumulation and growth (e.g., Martin and Rogers 2000).

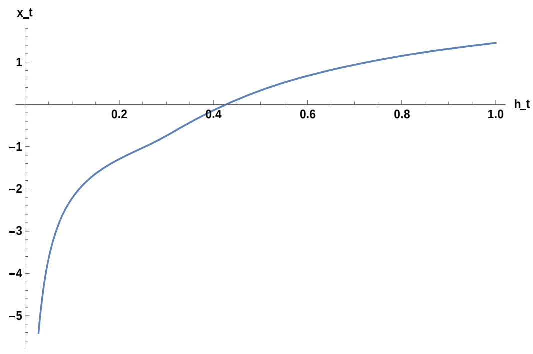

Figure 5a. The effect of h_t on x_t

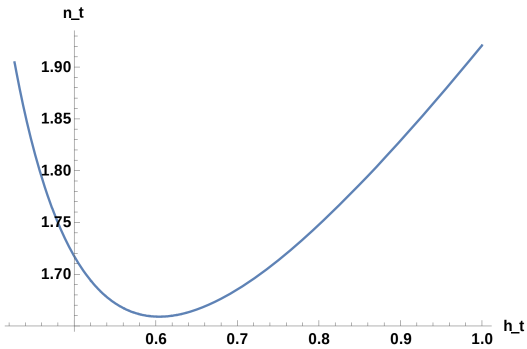

Figure 5b. The dynamics of fertility when $h_t > \tilde{h}$

5.2 Child-Rearing Costs that Vary with Income

The version of the model presented in Sections 3-4 adopted the simplifying assumption that rearing costs per child are fixed at q. In this section, we shall demonstrate that our main results emerge in a version of the model that relaxes this assumption and considers, instead, the case where child-rearing costs increase as the economy grows. Formally, we now assume that each child requires a rearing cost $q(h_t)$, such that $q'(h_t) > 0$. For reasons of analytical convenience, henceforth we adopt the following functional form:

$$q(h_t) = qh_t^{\rho}, \quad q > 0,\ 0 < \rho < 1. \tag{22}$$

In terms of parental investment in education and fertility choices, it can be shown that there exists a threshold $\tilde{h} \equiv \left[\dfrac{(\theta+\beta)\varphi}{\theta\psi q}\right]^{\frac{1}{\rho+\mu-\eta}}$ such that

$$x_t = X(h_t) = \begin{cases} 0 & \text{if} \quad h_t \le \tilde{h} \\ \dfrac{1}{\beta}\left[\theta q h_t^{\rho} - (\theta+\beta)\dfrac{\varphi}{\psi} h_t^{\eta-\mu}\right] & \text{if} \quad h_t > \tilde{h} \end{cases}, \tag{23}$$

and

$$n_t = N(h_t) = \begin{cases} \dfrac{(1-\gamma)(\beta+\theta)}{\gamma+(1-\gamma)(\beta+\theta)} \dfrac{\omega h_t^{1-\rho}}{q} & \text{if} \quad h_t \le \tilde{h} \\ \\ \dfrac{(1-\gamma)\beta}{\gamma+(1-\gamma)(\beta+\theta)} \dfrac{\omega\psi h_t^{1+\mu-\eta}}{q\psi h_t^{\rho+\mu-\eta} - \varphi} & \text{if} \quad h_t > \tilde{h} \end{cases}. \tag{24}$$

As for human capital, it evolves according to

$$h_{t+1} = F(h_t) = \begin{cases} \varphi h_t^{\eta} & \text{if} \quad h_t \le \tilde{h} \\ \dfrac{\theta(\psi q h_t^{\rho+\mu} - \varphi h_t^{\eta})}{\beta} & \text{if} \quad h_t > \tilde{h} \end{cases}. \tag{25}$$

Now, let us examine what these results imply for the model's dynamics. Firstly, check that when $h_t \le \tilde{h}$, $N'(h_t) > 0$ by virtue of (24). When $h_t > \tilde{h}$, on the other hand, we can establish that $X'(h_t) > 0$ (see the expression in 23), whereas (24) can reveal that

$$N'(h_t) \begin{cases} < 0 & \text{if} \quad h_t < \hat{h} \\ \\ > 0 & \text{if} \quad h_t > \hat{h} \end{cases},$$

where $\hat{h} \equiv \left[\dfrac{(1+\mu-\eta)\varphi}{\psi q(1-\rho)}\right]^{\frac{1}{\rho+\mu-\eta}}$. Note that the condition of Lemma 3 now changes to $\dfrac{(1+\mu-\eta)\theta}{(1-\rho)(\theta+\beta)} > 1$, ensuring that $\hat{h} > \tilde{h}$. Furthermore, the condition in Lemma 4 changes to

$$q\psi > \max\left\{\left(\frac{\beta}{\theta}\right)^{\frac{\rho+\mu-\eta}{1-\eta}} \frac{(1+\mu-\eta)[\varphi/(1-\rho)]^{(1-\rho-\mu)/(1-\eta)}}{(\rho+\mu-\eta)^{(\mu-\eta)/(1-\eta)}}, \frac{(\theta+\beta)\varphi^{(1-\rho-\mu)/(1-\eta)}}{\theta}\right\},$$

ensuring that, irrespective of the initial condition $h_0 > 0$, in the long-run human capital will converge to a steady state solution h^*, such that $h^* > \hat{h}$.

The preceding analysis reveals that the main implications of the original version (presented in Sections 3-4) remain intact. To see this, consider an economy for which

$h_0 < \tilde{h}$. Given that h^* is the unique stable solution, and that $h^* > \hat{h} > \tilde{h}$, the process of growth in the transition to the steady state will allow the economy to gradually exceed the threshold levels $\tilde{h}$ and $\hat{h}$. As this happens, the dynamics of fertility are characterised by the process shown in Proposition 3 and Figure 4.

5.3 Opportunity Costs of Child-Rearing

Traditionally, the majority of the existing literature on demographic transition have adopted models in which the cost of child-rearing is purely an opportunity cost of foregone income from the loss of labour time. More recently, however, studies by Hazan and Zoabi (2015) and Vogl (2016), among others, have demonstrated that the alternative approach of direct costs towards child-rearing can generate additional and interesting effects on the relation between fertility and human capital. This is the approach we adopted in our analysis as well. As a means of elucidating the additional effects that this approach brings about in our study, in this section we shall analyse a version of our model where the child-rearing cost takes the form of the opportunity cost of foregone labour income.

Consider Eq. (22) from the previous section and replace it with

$$q(h_t) = \omega q h_t . \tag{26}$$

Substituting (26) in (1) yields

$$c_t = \omega h_t (1 - q n_t) - n_t x_t . \tag{27}$$

Evidently, we have modified our set-up into one where parents forego labour income in order to rear their offspring. Solving the model under the constraint in (27), the equilibrium is characterised by a threshold $\tilde{h} \equiv \left[\frac{(\theta+\beta)\varphi}{\theta\psi q\omega} \right]^{\frac{1}{1+\mu-\eta}}$ such that

$$x_t = X(h_t) = \begin{cases} 0 & \text{if} \quad h_t \le \tilde{h} \\ \dfrac{1}{\beta}\left[\theta q \omega h_t - (\theta+\beta)\dfrac{\varphi}{\psi} h_t^{\eta-\mu} \right] & \text{if} \quad h_t > \tilde{h} \end{cases}, \tag{28}$$

where $X'(h_t) \ge 0$, and

$$n_t = N(h_t) = \begin{cases} \dfrac{(1-\gamma)(\beta+\theta)}{[\gamma+(1-\gamma)(\beta+\theta)]q} & \text{if} \quad h_t \le \tilde{h} \\ \\ \dfrac{(1-\gamma)\beta}{\gamma+(1-\gamma)(\beta+\theta)} \dfrac{\omega\psi h_t^{1+\mu-\eta}}{q\omega\psi h_t^{1+\mu-\eta}-\varphi} & \text{if} \quad h_t > \tilde{h} \end{cases}, \tag{29}$$

from which it is straightforward to establish that

$$N'(h_t) \begin{cases} = 0 & \text{if} \quad h_t \le \tilde{h} \\ \\ < 0 & \text{if} \quad h_t > \tilde{h} \end{cases}. \tag{30}$$

These results reveal that the replacement of direct child-rearing costs with opportunity costs in our model, has eliminated the fertility rebound at advanced stages of economic development (in this version, the fertility rate keeps declining), as well as the fertility increase in the initial stages of economic development (in this version, the fertility rate is constant). The reason is because, under (26)-(27), the impact of human capital on the cost of raising children is high enough to eliminate the effect whereby an increase in income is supportive to child-rearing – the effect that underlies the two stages of rising fertility rates in the original version of the model.

From an empirical point of view, both the opportunity and the direct costs of raising children are significant. For instance, Dotti Sani and Treas (2016) estimate that a couple's time spent in child-care could have increased their labour supply by 25%, whereas Lino *et al.* (2017) report estimates according to which the direct expenditures per child on items such as shelter, food, clothing, health-care, child-care, education, entertainment and other personal items account for a significant fraction (roughly ranging from 15%-30%) of a household's income. Our modelling choice to focus exclusively on the direct costs of child-rearing is by no means a deliberate attempt to relegate the importance of the relevant opportunity costs, such as foregone labour income. However, while the aforementioned statistics are a testament to the fact that the direct costs are perhaps equally significant, the vast majority of existing studies have employed models that focus exclusively on the time costs of raising – and, sometimes, educating – children. As our results demonstrate, the differences are not trivial. Indeed, the results of this section, when compared to the results of the original model in Sections 3-4, reveal that ignoring the direct costs of child-rearing obscures mechanisms that are important in shaping the dynamics of fertility. Furthermore, our main results, in comparison to those in the existing literature, reveal that the inclusion of such direct costs into a model of fertility and growth can generate a stage of fertility dynamics that, despite the evidence favouring its relevance, has eluded the attention of most of the existing work on the economics of fertility.

As a final note, the comparison of the results in Sections 5.2 and 5.3 should not lead us into attributing the lack of a positive relation between fertility and human capital to the fact that output production is linear in human capital. What is in fact critical for the different implications between the fertility dynamics in Sections 4.2-5.2 and those implied from (30) is that, in the former cases, the elasticity of child-rearing costs with respect to human capital is lower compared to the elasticity of output with respect to human capital.

6 Conclusion

Recent empirical evidence suggests that many of the countries that witnessed marked reductions in their fertility since the onset of demographic transition, now appear to experience a fertility rebound with rising fertility rates. Among the various explanations for this reversal, existing empirical evidence suggests that economic development – in the past, the main engine behind the demographic transition – is now one of the driving forces behind this new phase of demographic change. Furthermore, such evidence indicates that, from the beginning of early industrialisation to the present day, the fertility trends can be traced along an N-shaped curve.

In this paper we presented a theory that accounts for this dynamic behaviour. The main theme of the analysis is that the child quantity-quality trade-off – one of the main explanations for the demographic transition in the economics literature – materialises in an intermediate stage where the joint effects of the high return to human capital investment and the income constraint faced by households, implies that parents can increase the investment towards their children's education, but only at the expense of the number of children they bear over their reproductive age. As income grows even further though, there is a new stage where parents can provide the desirable expenditures towards the education of their offspring, without necessarily reducing the number of children they give birth to.

As it became evident from the main part of the paper, the model that underlines our theory is qualitative rather than quantitative. Furthermore, it was constructed with the purpose of offering analytical solutions that pinpoint the main mechanisms that characterise the equilibrium outcomes, without blurring their intuition. For instance, specifications such as logarithmic utility functions and constant returns to education expenditures were adopted because they are exactly those specifications that keep the analysis tractable. Naturally, a more general framework can be a fruitful avenue for future research. Nevertheless, even in this simple form, our model is able to draw attention to an outcome that, although it is strongly supported by evidence, it has so far evaded the attention of the literature on the economic growth-demographic change nexus.

At this point, we want to elaborate a bit further on one of the issues which we did not address in our study. It relates to the idea that, in addition to expenditures on education, parents can increase their offspring's resources by means of direct income transfers. Our choice not to consider those in our analysis was certainly not an attempt to downgrade their relevance and importance; it was rather a choice to make our analysis comparable to other studies that have analysed the characteristics of demographic transition. Indeed, the vast majority of the studies that examine demographic change along the process of economic development have not incorporated direct income transfers; instead, they have adopted the mechanism of parental investment towards the children's human capital (e.g., Becker *et al.* 1990; Tamura 1994, 1996; Galor and Weil 2000; Kalemli-Ozcan 2002; Galor and Moav 2002; Hazan and Berdugo 2002; Lagerlöf 2003; de la Croix and Doepke 2003, 2004, 2009; Tabata 2003; Zhang and Zhang 2005; Galor and Mountford 2008; Borck 2011; Dioikitopoulos 2014; Varvarigos and Arsenis 2015; Vogl 2016). The empirical justification of this approach is twofold: Firstly, from a comparative development perspective, evidence suggests that human capital has been one of the most important and persistent contributors to the historical growth process of currently developed economies (e.g., Glaeser *et al.* 2004). Secondly, there is evidence showing that the characteristics of human capital accumulation are critical in explaining the different stages of demographic transition historically (e.g., Murtin 2013). Generally speaking, direct income transfers (such as bequests) seem to be more pertinent for issues relating to income distribution, thus they have been incorporated in studies that focus on the dynamics and persistence of income inequality (e.g., Galor and Zeira 1993; Aghion and Bolton 1997). In this respect, one exception in terms of the fertility-growth nexus is the study of Dahan and Tsiddon (1998) who incorporate bequests to analyse the joint evolution of fertility and income inequality.[14] Issues of income inequality, however, go beyond the objective of our paper; that is why we have decided to keep our analysis tightly focused on the demographic implications of parental investment to education.

As a final note, we should emphasise that our theory formalises just one of a variety of possible explanations behind the fertility rebound in developed economies – the outcome that is ultimately responsible for the emergence of N-shaped fertility dynamics. This should not be viewed as a stance against other legitimate explanations for this phenomenon (e.g., the 'tempo' effect; immigration etc.). These may offer important accounts behind the reversal of fertility trends in developed countries; therefore, their exploration certainly merits formal analysis through future research work. In any case however, existing evidence (cited in the Introduction) reveals that the tempo effect and immigration cannot fully account for the change in fertility trends in developed countries. Therefore, offering an additional or complementary explanation was an endeavour certainly worth undertaking.

Appendix

Proof of Proposition 1

Consider the case where $h_t \leq \tilde{h}$. According to (11), we have $F(h_t) = \varphi h_t^{\eta}$ where $F'(h_t) = \eta\varphi h_t^{\eta-1} > 0$, $F'(0) = \infty$ and $F''(h_t) = (\eta-1)\eta\varphi h_t^{\eta-2} < 0$. Furthermore, note that $\breve{h} = \varphi\breve{h}^{\eta} \Rightarrow \breve{h} = \varphi^{\frac{1}{1-\eta}}$. However, it is true that $\breve{h} > \tilde{h}$ given that $\psi q > \frac{\theta+\beta}{\theta}\varphi^{(1-\mu)/(1-\eta)}$ holds by

[14] Another study that includes bequests in a model of fertility is by Baland and Robinson (2000). It is different in scope to the issues analysed in our paper, as it is not concerned with the process of demographic transition at different stages of economic development. Instead, it shows that whether the parental bequest motive is operative or not has implications for the incidence and efficiency of child labour.

assumption. Consequently $F(h_t) > h_t \;\; \forall \;\; h_t \leq \tilde{h}$, meaning that the economy will not reach the steady state $\breve{h}$ because the dynamic behaviour of human capital will change when its stock exceeds $\tilde{h}$. According to (11), the transition equation becomes $F(h_t) = \dfrac{\theta}{\beta}(\psi q h_t^{\mu} - \varphi h_t^{\eta})$ thereafter. Note that, by the definition of $\tilde{h}$

$$\tilde{h} = \left[\frac{(\theta+\beta)\varphi}{\theta\psi q}\right]^{\frac{1}{\mu-\eta}} \Rightarrow$$

$$\theta\psi q \tilde{h}^{\mu-\eta} = (\theta+\beta)\varphi \Rightarrow$$

$$\frac{\theta}{\beta}\psi q \tilde{h}^{\mu} = \frac{\theta+\beta}{\beta}\varphi \tilde{h}^{\eta} \Rightarrow$$

$$\frac{\theta}{\beta}\psi q \tilde{h}^{\mu} + \left(1 - \frac{\theta+\beta}{\beta}\right)\varphi \tilde{h}^{\eta} = \varphi \tilde{h}^{\eta} \Rightarrow$$

$$\frac{\theta}{\beta}(\psi q \tilde{h}^{\mu} - \varphi \tilde{h}^{\eta}) = \varphi \tilde{h}^{\eta} \Rightarrow$$

$$\lim_{h_t \to \tilde{h}^+} F(h_t) = \lim_{h_t \to \tilde{h}^-} F(h_t) .$$

It is,

$$F'(h_t) = \frac{\theta}{\beta}(\mu\psi q h_t^{\mu-1} - \eta\varphi h_t^{\eta-1}) , \tag{A1}$$

so that $F'(h_t) > 0$ as long as

$$\mu\psi q h_t^{\mu-1} > \eta\varphi h_t^{\eta-1} \Rightarrow$$

$$h_t^{\mu-\eta} > \frac{\eta\varphi}{\mu\psi q} \Rightarrow$$

$$h_t > \left(\frac{\eta\varphi}{\mu\psi q}\right)^{\frac{1}{\mu-\eta}} \equiv \underset{\sim}{h} ,$$

which is true because $\tilde{h} > \underset{\sim}{h}$ for $\mu > \eta$. Furthermore, given $\eta, \mu \in (0,1)$, Equation (A1) reveals that $F'(\infty) = 0$. Therefore, for $h_t > \tilde{h}$, the transition graph will cross the 45^0 line at a point h^*, such that $h^* = F(h^*)$ and

$$F(h_t)\begin{cases} > h_t & \text{for} \quad \tilde{h} < h_t < h^* \\ \\ < h_t & \text{for} \quad h_t > h^* \end{cases}.$$

It follows that $F'(h^*) \in (0,1)$, allowing us to conclude that the fixed point h^* corresponds to a stable equilibrium. □

Proof of Lemma 2

The first part of Lemma 2 is easily proven after using (9) for $x_t = 0$, and showing that $N'(h_t) = \dfrac{(1-\gamma)(\beta+\theta)}{\gamma+(1-\gamma)(\beta+\theta)} \dfrac{\omega}{q} > 0$. Next, we can consider the expression for fertility that corresponds to $x_t > 0$. First of all, notice that the denominator $q\psi h_t^{\mu-\eta} - \varphi$ is positive for $h_t > \tilde{h}$. Calculating the derivative, we get

$$N'(h_t) = \frac{(1-\gamma)\beta}{\gamma+(1-\gamma)(\beta+\theta)} \omega\psi \left[\frac{(1+\mu-\eta)h_t^{\mu-\eta}(q\psi h_t^{\mu-\eta} - \varphi) - h_t^{1+\mu-\eta}(\mu-\eta)q\psi h_t^{\mu-\eta-1}}{(q\psi h_t^{\mu-\eta} - \varphi)^2} \right].$$

Obviously, the sign of the derivative will depend on the sign of the expression inside squared brackets. In particular, it will be $N'(h_t) > 0$ as long as

$$(1+\mu-\eta)h_t^{\mu-\eta}(q\psi h_t^{\mu-\eta} - \varphi) - h_t^{1+\mu-\eta}(\mu-\eta)q\psi h_t^{\mu-\eta-1} > 0 \Rightarrow$$

$$(1+\mu-\eta)h_t^{2(\mu-\eta)}q\psi - (1+\mu-\eta)\varphi h_t^{\mu-\eta} - (\mu-\eta)h_t^{2(\mu-\eta)}q\psi > 0 \Rightarrow$$

$$h_t^{2(\mu-\eta)}q\psi - (1+\mu-\eta)\varphi h_t^{\mu-\eta} > 0 \Rightarrow$$

$$h_t^{\mu-\eta}[h_t^{\mu-\eta}q\psi - (1+\mu-\eta)\varphi] > 0 \Rightarrow$$

$$h_t > \left[\frac{(1+\mu-\eta)\varphi}{q\psi} \right]^{\frac{1}{\mu-\eta}} \equiv \hat{h}.$$

Therefore, for $h_t < \hat{h}$ it is $N'(h_t) < 0$. □

Proof of Lemma 4

Given $F(h^*) = h^*$ and $F(h_t) < h_t$ for $h_t > h^*$, it is sufficient to show that $F(\hat{h}) > \hat{h}$. This condition corresponds to

$$\frac{\theta}{\beta}(\psi q \hat{h}^{\mu} - \varphi \hat{h}^{\eta}) > \hat{h} \Rightarrow$$

$$\frac{\theta}{\beta}(\psi q\hat{h}^{\mu-\eta}-\varphi)>\hat{h}^{1-\eta}\Rightarrow$$

$$\psi q\hat{h}^{\mu-\eta}-\varphi>\frac{\beta}{\theta}\hat{h}^{1-\eta}\Rightarrow$$

$$\psi q\left[\frac{(1+\mu-\eta)\varphi}{q\psi}\right]-\varphi>\frac{\beta}{\theta}\left[\frac{(1+\mu-\eta)\varphi}{q\psi}\right]^{\frac{1-\eta}{\mu-\eta}}\Rightarrow$$

$$(\mu-\eta)\varphi>\frac{\beta}{\theta}\left[\frac{(1+\mu-\eta)\varphi}{q\psi}\right]^{\frac{1-\eta}{\mu-\eta}}\Rightarrow$$

$$q\psi>\left(\frac{\beta}{\theta}\right)^{\frac{\mu-\eta}{1-\eta}}\frac{(1+\mu-\eta)\varphi^{(1-\mu)/(1-\eta)}}{(\mu-\eta)^{(\mu-\eta)/(1-\eta)}}.$$

Together with the fact that $q\psi>\frac{(\theta+\beta)\varphi^{(1-\mu)/(1-\eta)}}{\theta}$ holds by virtue of Proposition 1, then $q\psi>\max\left\{\left(\frac{\beta}{\theta}\right)^{\frac{\mu-\eta}{1-\eta}}\frac{(1+\mu-\eta)\varphi^{(1-\mu)/(1-\eta)}}{(\mu-\eta)^{(\mu-\eta)/(1-\eta)}},\frac{(\theta+\beta)\varphi^{(1-\mu)/(1-\eta)}}{\theta}\right\}$ is sufficient to establish that $h^*>\hat{h}$. □

References

1. Aghion, P., and Bolton, P. 1997. "A theory of trickle-down growth and development," *Review of Economic Studies*, 64, 151-172.
2. Alkema, L., Raftery, A.E., Gerland, P., Clark, S.J., Pelletier, F., Buettner, T., and Heiling, G.K. 2011. "Probabilistic projections of the Total Fertility Rate for all countries," *Demography*, 48, 815-839.
3. Baland, J.M., and Robinson, J.A. 2000. "Is child labor inefficient?" *Journal of Political Economy*, 108, 663-679.
4. Becker, G.S., Murphy, K.M., and Tamura, R. 1990. "Human capital, fertility, and economic growth," *Journal of Political Economy*, 98, S12-S37.
5. Becker, S., Cinnirella, F., and Woessmann, L. 2010. "The trade-off between fertility and education: Evidence from before the demographic transition," *Journal of Economic Growth*, 15, 177-204.

6. Black, S.E., Deveraux, P.J., and Salvanes, K.G. 2005. "The more the merrier? The effect of family composition on children's education," *Quarterly Journal of Economics,* 120, 669-700.
7. Blackburn, K., and Cipriani, G.P. 2002. "A model of longevity, fertility, and growth," *Journal of Economic Dynamics and Control*, 26, 187-204.
8. Bongaards, J., and Sobotka, T. 2012. "A demographic explanation for the recent rise in European fertility," *Population and Development Review*, 38, 83-120.
9. Borck, R. 2011. "Federalism, fertility, and growth," *Scandinavian Journal of Economics*, 113, 30–54.
10. Campbell, A.A. 1974. "Beyond the demographic transition," *Demography*, 11, 549-561.
11. Collins, J., and Richards, O. 2013. "Evolution, fertility and the ageing population," Department of Economics, discussion paper no 13-02, University of Western Australia.
12. Dahan, M., and Tsiddon, D. 1998. "Demographic transition, income distribution, and economic growth," *Journal of Economic Growth*, 3, 29-52.
13. Day, C. 2012. "'Economic growth, gender wage gap and fertility rebound," *Economic Record*, 88, 88-99.
14. de la Croix, D., and Doepke, M. 2003. "Inequality and growth: Why differential fertility matters," *American Economic Review*, 93, 1091-1113.
15. de la Croix, D., and Doepke, M. 2004. "Public versus private education when differential fertility matters," *Journal of Development Economics*, 73, 607-629.
16. de la Croix, D., and Doepke, M. 2009. "To segregate or to integrate: Educational policies and democracy," *Review of Economic Studies*, 76, 597-628.
17. Dioikitopoulos, E. 2014. "Aging, growth and the allocation of public expenditures on health and education," *Canadian Journal of Economics*, 47, 1173-1194.
18. Doepke, M., Hazan, M., and Maoz, Y. D. 2015. "The baby boom and World War II: A macroeconomic analysis," *Review of Economic Studies*, 82, 1031-1073.

19. Dominiak, P., Lechman, E., and Okonowicz, A. 2014. "The fertility rebound and economic growth: New evidence for 18 countries over the period 1970-2011," Working Paper no.05/2014, GUT Faculty of Management and Economics.
20. Dotti Sani, G.M., and Treas, J. 2016. "Educational gradients in parents' child-care time across countries, 1965–2012," *Journal of Marriage and Family*, 78, 1083-1096.
21. Dyson, T., and Murphy, M. 1985. "The onset of fertility transition," *Population and Development Review*, 11, 399-440.
22. Galor, O. 2005. "The demographic transition and the emergence of sustained economic growth," *Journal of the European Economic Association*, 3, 494-504.
23. Galor, O., and Moav, O. 2002. "Natural selection and the origin of economic growth," *Quarterly Journal of Economics*, 117, 1133-1191.
24. Galor, O., and Mountford, A. 2008. "Trading population for productivity," *Review of Economic Studies*, 75, 1143-1179.
25. Galor, O., and Weil, D.N. 1996. "The gender gap, fertility, and growth," *American Economic Review*, 86, 374-387.
26. Galor, O., and Weil, D.N. 2000. "Population, technology, and growth: From Malthusian stagnation to demographic transition and beyond," *American Economic Review*, 90, 806-828.
27. Galor, O., and Zeira, J. 1993. "Income distribution and macroeconomics," *Review of Economic Studies*, 60, 35-52.
28. Glaeser, E.L., La Porta, R., Lopez-de-Silanes, F., and Shleifer, A. 2004. "Do institutions cause growth?" *Journal of Economic Growth*, 9, 271-303.
29. Goldstein, J.R., Sobotka, T., and Jasilioniene, A. 2009. "The end of 'lowest-low' fertility?" *Population and Development Review*, 15, 663-699.
30. Greenwood, J., Seshadri, A., and Vandenbroucke, G. 2005. "The baby boom and baby bust," *American Economic Review*, 95, 183-207.
31. Hanushek, E.A. 1992. "The trade-off between child quantity and quality," *Journal of Political Economy*, 100, 84-117.
32. Hazan, M., and Berdugo, B. 2002. "Child labour, fertility, and economic growth," *Economic Journal*, 112, 810-828.

33. Hazan, M., and Zoabi, H. 2015. "Do highly educated women choose smaller families?" *Economic Journal*, 125, 1191–1226.

34. Kalemli-Ozcan, S. 2002. "Does the mortality decline promote economic growth?" *Journal of Economic Growth*, 7, 411-439.

35. Klemp, M., and Møller, N.F. 2016. "Post-Malthusian dynamics in pre-industrial Scandinavia," *Scandinavian Journal of Economics*, 118, 841-867.

36. Lagerlöf, N.P. 2003. "From Malthus to modern growth: Can epidemics explain the three regimes?" *International Economic Review*, 44, 755-777.

37. Lino, M., Kuczynski, K., Rodriguez, N., and Schap, T. 2017. "Expenditures on children by families, 2015," Miscellaneous publication no. 1528-2015, U.S. Department of Agriculture, Center for Nutrition Policy and Promotion.

38. Luci-Greulich A., and Thévenon O. 2014. "Does economic advancement 'cause' a re-increase in fertility? An empirical analysis for OECD countries (1960–2007)," *European Journal of Population*, 30, 187-221.

39. Martin, P., and Rogers, C.A. 2000. "Long-term growth and short-term economic instability," *European Economic Review*, 44, 359-381.

40. Moav, O., and Neeman, Z. 2010. "Status and poverty," *Journal of the European Economic Association*, 8, 413-420.

41. Moav, O., and Neeman, Z. 2012. "Saving rates and poverty: The role of conspicuous consumption and human capital," *Economic Journal*, 122, 933-956.

42. Mokyr, J., and Voth, H.J. 2010. "Understanding growth in Europe, 1700-1870: Theory and evidence," in Broadberry, S., and O'Rourke, K.H. (Eds): *The Cambridge Economic History of Modern Europe, Volume 1: 1700-1870*, Cambridge University Press, Cambridge.

43. Møller, N.F., and Sharp, P. 2014. "Malthus in cointegration space: Evidence of a post-Malthusian pre-industrial England," *Journal of Economic Growth*, 29, 105-140.

44. Monstad, K., Propper, C., and Salvanes, K.G. 2008. "Education and fertility: Evidence from a natural experiment," *Scandinavian Journal of Economics*, 110, 827–852.

45. Murtin, F. 2013. "Long-term determinants of the demographic transition, 1870-2000," *Review of Economics and Statistics*, 95, 617-631.

46. Myrskylä, M., Kohler, H.P., and Billari, F. 2009. "Advances in development reverse fertility declines," *Nature*, 460, 741-743.
47. Pestieau, P., and Ponthière, G. 2014. "Optimal fertility along the life cycle," *Economic Theory*, 55, 185-224.
48. Prinz, A., 1990. "Endogenous fertility, altruistic behavior across generations, and social security systems," *Journal of Population Economics*, 3, 179-192.
49. Schmertmann, C., Zagheni, E., Goldstein, J.R., and Myrskylä, M. 2014. "Bayesian forecasting of cohort fertility," *Journal of the American Statistical Association*, 109, 500-513.
50. Strulik, H. 2008. "Geography, health, and the pace of demo-economic development," *Journal of Development Economics*, 86, 61-75.
51. Strulik, H., and Weisdorf, J.L. 2008. "Population, food, and knowledge: A simple unified growth theory," *Journal of Economic Growth*, 13, 195-216.
52. Tabata, K. 2003. "Inverted U-shaped fertility dynamics, the poverty trap and growth," *Economics Letters*, 81, 241-248.
53. Tamura, R. 1994. "Fertility, human capital and the wealth of nations," *Economic Theory*, 4, 593-603.
54. Tamura, R. 1996. "From decay to growth: A demographic transition to economic growth," *Journal of Economic Dynamics and Control*, 20, 1237-1261.
55. Tamura, R., and Simon, C. 2017. "Secular fertility declines, baby booms, and economic growth: International evidence," *Macroeconomic Dynamics*, 21, 1601-1672.
56. Tromans, N., Natamba, E., and Jefferies, J. 2009. "Have the women born outside the UK driven the rise in UK births since 2001?" *Population Trends*, 136, 28-42.
57. Van Bavel, J., and Reher, D.S. 2013. "The baby boom and its causes: What we know and what we need to know," *Population and Development Review*, 39, 257-288.
58. Varvarigos, D., and Arsenis, P. 2015. "Corruption, fertility, and human capital," *Journal of Economic Behavior and Organization*, 109, 145-162.

59. Varvarigos, D., and Zakaria, I.Z. 2013. "Endogenous fertility in a growth model with public and private health expenditures," *Journal of Population Economics*, 26, 67-85.
60. Vogl, T.S. 2016. "Differential fertility, human capital, and development," *Review of Economic Studies*, 83, 365-401.
61. Zhang, J., and Zhang, J. 2005. "The effect of life expectancy on fertility, saving, schooling and economic growth: theory and evidence," *Scandinavian Journal of Economics*, 107, 45-66.