Risky Loans and Collateralized Fund Obligations

Dilip B. Madan
Joint work with Ernst Eberlein
and Helyette Geman

June 19 2009
Spectral and Cubature Methods in
Financial Econometrics
University of Leicester

Motivation

- We recognize that ratings of risky loan are broadly related to the spreads over the risk free rate on such loans.
- Though we have no direct contribution to make on ratings, we hope to better understand the structure of spreads as the latter is a more direct exercise in pricing and valuation.
- It is now recognized that jumps in the underlying value processes are important especially for the shorter maturities as analysed in Kou and Wang (2003, 2004) and Lipton (2002). This is the class of jump diffusion processes.
- For the more general class of Lévy processes we have access to closed forms for certain Laplace transforms and then to the spreads by Laplace inversion when the underlying value process is spectrally negative or has no positive jumps.

- Furthermore, Eberlein and Madan (2008) have investigated the relavance of such processes for the option surface at the longer maturities noting that the absence of short positions coupled with call overwriting strategies mitigates the need for such jumps in the implied long maturity risk neutral distributions.
- Here we present an analysis of loan spreads when the underlying value process is a spectrally negative Lévy process.

Two Risky Loan Contracts

- We shall analyse two stylized risky loan contracts.
- One that only takes a loss at maturity when the underlying asset values are insufficient.
- The second is a Collateralized Fund Obligation (CFO) that takes losses through time as and when asset values drop below prespecified thresholds.
- We refer to the first as classic loan obligation (CLO) while the second is a CFO.

The CLO

- Let the loan amount be L and suppose there are lower priority loans in the amount B and an initial equity of H.
- The initial asset value is A_0.
- This loan takes a loss of principal if the final asset value A is below $A_0 - (B + H)$.
- The loss is the smaller of L and $A_0 - (B + H) - A$.
- The principal returned is
$$\left(L - (A_0 - (B + H) - A)^+\right)^+ .$$
- Let c be the continuously compounded coupon rate on the loan with a single payment at maturity on the outstanding balance.

The Coupon Formula

- Let $f(A)$ be the risk neutral distribution of asset value at maturity.
- The loan pricing relation equates the expected risk and risk free returns to maturity T.

$$e^{cT}\int_0^\infty \left(L-(A_0-(B+H)-A)^+\right)^+ f(A)dA = e^{rT}L$$

- With $K = B + H$ one solves for c from the price of a call spread on the asset value of maturity T.

$$e^{-cT} = \frac{1}{L}\left[Call_A(A_0-K-L) - Call_A(A_0-K)\right]$$

- The strikes involved here are typically so far in the money that the usual Fourier methods for option pricing in Lévy models of Carr and Madan (1999) break down.
- We were led to develop saddlepoint methods Carr and Madan (2009, Journal of Computational Finance forthcoming) to complete the computations of this paper.

The CFO

- For the CFO one introduces the process for the infimum of the asset value to date deflated by the advance rate η
$$X(t) = \frac{1}{\eta} \inf_{0 \leq s \leq t} A(s).$$
- The coupon payment at time $u < t$ is on the outstanding balaunce given by
$$c \left(L - \left(X(0) - (B + H) - X(u) \right)^{+} \right)^{+}.$$
- The return of principal at T is
$$\left(L - \left(X(0) - (B + H) - X(T) \right)^{+} \right)^{+}$$

The CFO coupon formula

- We may derive the coupon in terms of the final and integrated call spreads on the $X(t)$ as

$$c = \frac{1 - \frac{1}{L}\left[C_{X,T}(X(0)-K-L) - C_{X,T}(X(0)-K)\right]}{\frac{1}{L}\int_0^T \left[C_{X,u}(X(0)-K-L) - C_{X,u}(X(0)-K)\right] du}$$

- For computation we employ the law of the infimum of a spectrally negative process via the Wiener-Hopf decomposition along with the technique of Rogers (2000) for changing the contour of integration to avoid having to solve complex valued equations for the exponential parameter of the law of the supremum.

The Lévy Model employed

- The spectrally negative Lévy model we employ is $CGMY$ with M set to infinity to get CGY with an added diffusion.
- The diffusion volatility is σ and the Lévy measure is
$$k(x) = C\frac{e^{-G|x|}}{|x|^{1+Y}}\mathbf{1}_{x<0}.$$
- We work with $Y < 1$ as the method of Rogers (2000) breaks down for $Y > 1$. I have had this investigated for $Y > 1$ using the Gavier Stehfest algorithm and the results are qualititatively. Here we report $Y < 1$.

The total asset volatility v for the asset value process satisfies
$$v^2 = \sigma^2 + \frac{C}{\Gamma(2-Y)G^{2-Y}}.$$

Stylized Spread Investigation

- We employ 3 levels for the proportion of total volatility due to diffusion
$$.25, .5, .75.$$
- There are 3 levels for the aggregate volatility of 25%, 50%, and 75% and we solve for C given Y, G.
- There are three levels for G and Y respectively.
- The parameter choices give us 81 cases.
- The loan specific variables are maturity, and priority while the market specific variable is the level of the risk free rate.
- Using 3 settings for each we have a total of $2187 = 81 * 27$ cases.
- For each of these cases we computed the CLO and CFO spread.

Design of Fixed Effect Regression

- The relationships between inputs and spreads are possibly nonlinear and to summarize the effects we conducted a fixed effect regression of coupon spreads on input levels.
- There are seven inputs

$$
\begin{aligned}
&vol, G, Y,\\
&diffusion\; proportion,\\
&maturity,\\
&lower\; priority\; capital,\\
&level\; of\; rates
\end{aligned}
$$

- Two levels for each beyond the base case.
- This gives 15 explanatory variables including the constant term for the regression of two dependent spreads.

Input Settings

- We present the input settings in a Table.

TABLE 1			
Input Settings			
Variable	Levels		
volatility	.25	.5	.75
G	1	5	10
Y	.25	.5	.75
diffusion proportion	.25	.5	.75
Lower Capital	70	80	90
Maturity	1	3	5
Interest Rate	.025	.05	.1

Results of Fixed Effects Regression

- The Table presents the results.
- The average CFO coupon exceeds the classic coupon suggesting that more risk is taken in the CFO structure.
- The diffusion component has a positive effect on spreads suggesting that spreads are responsive to the level of small activity.
- The total volatility has a high, positive and nonlinear effect that is more pronounced for the CFO structure.

- Interestingly, the effect of raising G which increases the relative size of the small activity has a positive effect that is relatively linear. This also suggests that the cumulated effects of small jumps are important.
- The effect of increasing Y are positive. This again suggests that raising the level of small activity raises spreads.

- The effect of higher priority is negative as expected, nonlinear and more pronounced for the CFO structure.
- The effects of maturity are positive, slightly nonlinear, and more pronounced for the CFO structures.
- Interestingly, lower interest rate environments necessitate larger spreads.

TABLE 2

Regressions of Classic and CFO coupons in basis points

	Classic Coupon	CFO coupon
Variable	Coefficient	Coefficient
Constant	99.0039	147.0567
vol2	139.9759	237.9077
vol3	580.6227	959.2871
G2	30.1862	61.7368
G3	31.6579	69.0987
Y2	2.1139	5.1053
Y3	4.2783	9.6203
dp2	0.1807	12.5320
dp3	4.6589	29.4034
LC2	-194.6241	-365.3774
LC3	-345.89	-622.57
T2	136.2704	234.4470
T3	176.64	301.15
R2	-26.7642	-36.4063
R3	-72.73	-99.9435
RSQUARE	0.7668	0.7599

Rates and Spreads

- We present a Tables of average spreads by rates and maturities for both the CLO and CFO structures.

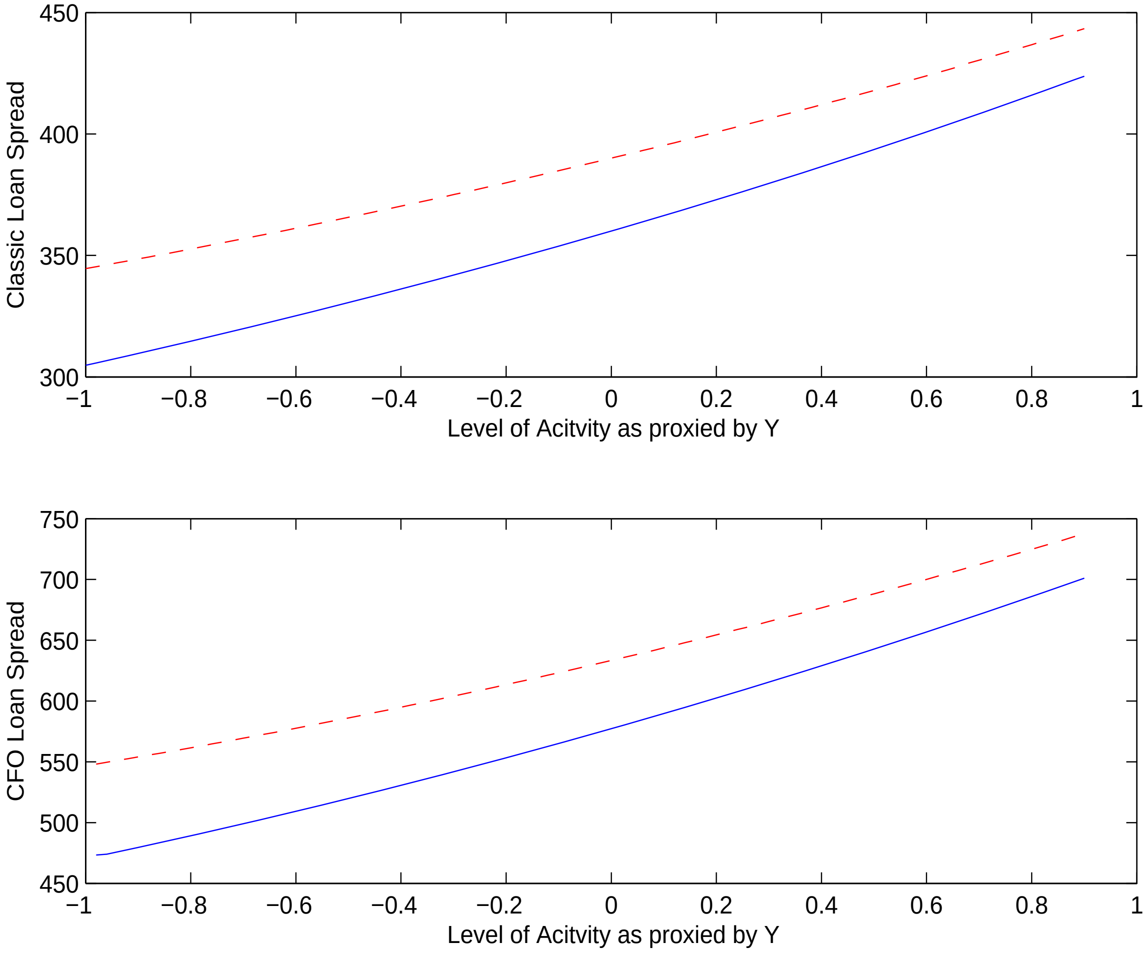
TABLE 3			
Classic Coupons by Rate and Maturity			
	Rate1	Rate2	Rate3
Maturity1	160.38	152.92	138.09
Maturity2	323.88	293.95	241.65
Maturity3	378.81	336.64	265.14

TABLE 4			
CFO Coupons by Rate and Maturity			
	Rate1	Rate2	Rate3
Maturity1	250.86	237.55	213.15
Maturity2	519.94	478.64	406.33
Maturity3	602.71	548.09	454.19

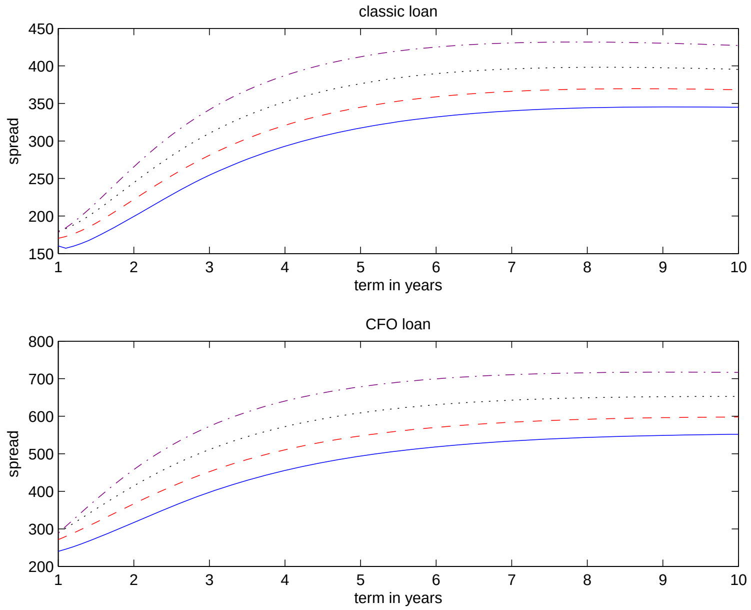
Activity Rates and Loan Spreads

- Our observations on the effects of increasing G and Y raising spreads leads to the conjecture that a high level of small activity raises spreads for spectrally negative Lévy processes as opposed to the presence of a few sizable jumps in the underlying value process.
- We know that we transition from finite to infinite activity as Y gets positive.
- We go to infinite variation as Y rises above unity.
- We present graphically the effects of Y for volatility at 50% and $G = 1$ with a diffusion proportion of 50% in blue and 60% in red.

- We also present the term structure effects of Y using four settings and graphing against maturity. The Y settings are $-0.75, -0.25, 0.25, 0.75$.


Classic Loan Spread
450
400
350
300
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
Level of Acitvity as proxied by Y
CFO Loan Spread
750
700
650
600
550
500
450
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
Level of Acitvity as proxied by Y

1.


classic loan
450
400
350
300
250
200
150
spread
1
2
3
4
5
6
7
8
9
10
term in years
CFO loan
800
700
600
500
400
300
200
spread
1
2
3
4
5
6
7
8
9
10
term in years

2.

Calibrating Merton Compound Option Model

- We now suppose the underlying asset value process is a spectrally negative process with dynamics $CGYSN$ and evolution

$$\begin{aligned} dA(t) &= (r-q)\,A(t\_)dt + \sigma A(t\_)dW \\ &\quad + A(t\_)\int_{-\infty}^{0} \left(e^{x}-1\right)\left(\mu(dx,dt) - k(x)dxdt\right) \end{aligned}$$

- The characteristic function for the log price is

$$\begin{aligned} E\left[e^{iuA(t)}\right] &= \exp\left(t\psi(u)\right) \\ \psi(u) &= -\frac{\sigma^2 u^2}{2} \\ &\quad + C\Gamma(-Y)\left(\left(G+iu\right)^{Y} - G^{Y}\right) \\ &\quad + iu\omega \\ \omega &= \ln(A(0)) + \\ &\quad \left(\begin{array}{c} r-q- \\ \frac{\sigma^2}{2} - C\Gamma(-Y)\left(\left(G+1\right)^{Y} - G^{Y}\right) \end{array}\right) \end{aligned}$$

- We take the debt level of the company as the strike and the debt maturity as the maturity and solve for asset value using
  $$S_0 = C(A_0, D, M; \sigma, C, G, Y)$$
- where S_0 is the market stock price. Hence the stock is a call option on the asset value.
- We calibrate the parameters σ, C, G, Y to fit the equity option surface. We simulate the asset value process from a prospective set of parameters and transform to stock prices using
  $$S(t) = C(A(t), D, M; \sigma, C, G, Y)$$
- We then price options on the path space and match these with traded market option prices.
- The estimated model is then used to price loan spreads of both types for the said company on the stated date.

Illustration on GM 2003

- The Debt level for 2003 was $191,133$ millions of dollars.
- The outstanding equity was $25,268$ millions of dollars.
- The stock price averaged over December 2003 was 49.44.
- The implied number of shares was 511.1076 million shares.
- We set the asset value strike at the debt level per share of 373.9584.
- The maturity was set at the period duration of 12 years.

- The average option surface for December 2003 was synthesized by $VGSSD$ Sato process parameters of

$$\sigma = .2803,\ \nu = .9027,\ \theta = -.1131,\ \gamma = .5314.$$

- From these parameter values and initial stock price of 49.44 we build a target option price surface using 36 prices with 9 strikes for each of four maturities.

- We calibrate the asset value process to these option prices using our simulated compound option pricing model.
- The parameter values were

$$\sigma = .0.0189,\ C = 0.2297,\ G = 1.0991,\ Y = 0.3604$$

- We also did a standard equity option calibration of the $CGYSN$ model to equity option prices to get parameters

$$\sigma = 0.2064,\ C = 0.0956,\ G = 1.1818,\ Y = 0.4953$$

- We then computed loan spreads for 70% lower priority capital for the 5 year CLO and CFO.

CLO and CFO Loan Spreads on GM 2003

- The resulting loan spreads using both the direct equity calibration and the compound option model are as follows.

TABLE 5

GM Loan Spreads 2003

Loan	Equity Calibration	Asset Calibration
Classic	110.55	224.94
CFO	192.02	336.52

- The CDS at this time was at 174.50.

Conclusion

- Classic loan spreads for spectrally negative Lévy processes are implemented by call spread calculations using new saddlepoint methods of Carr and Madan (2009).
- CFO loan spread computations are done using Rogers (2000).
- It is observed that the level of small jump activity strongly influences the level of spreads in addition to other factors.
- The Merton compound option model is used to calibrate asset value parameters to an equity option surface to infer asset value parameters
- Loan spread computations are then illustrated for both loan types using calibarted parameters.