Efficient Estimation of Copula-based Semiparametric Markov Models

Xiaohong Chen (Yale), Wei Biao Wu (Chicago), Yanping Yi (NYU)

Finance and Econometrics Workshop, June 19, 2009

Outline

- Partial review of copula-based time series models
- Ergodic properties
- Problems with existing estimators
- Our new estimator and its large sample properties
- Monte Carlo study: comparison of different estimators
- Future work

Review of copulas

- A d-dimensional *Copula* is a d-dimensional multivariate joint prob. distribution on $[0, 1]^d$ with uniform marginal cdfs.

Review of copulas

- A d-dimensional *Copula* is a d-dimensional multivariate joint prob. distribution on $[0,1]^d$ with uniform marginal cdfs.
- Why copula?
- Sklar's theorem (1959): (1) Any d-dimensional multivariate joint prob. dist. H with marginal cdfs $G_j, j = 1, ..., d$ can be represented as $H(x_1, ..., x_d) = C(G_1(x_1), ..., G_d(x_d))$, where C is a d-dimensional copula; (2) the representation is unique when $G_j, j = 1, ..., d$ is absolutely continuous wrt Leb.
- Copula approach allows for separately modelling marginal behavior and dependence structure.
- Copula measure of dependence is invariant to monotone transformation.

Partial review of copula-based multivariate time series models

- **Class 1** (copula modeling of *concurrent dependence of multivariate raw time series*): Embrechts et al. (01), Embrechts (08, survey), Patton (02), Patton (07, survey), Kluppelberg and her co-authors,......

Partial review of copula-based multivariate time series models

- **Class 1** (copula modeling of *concurrent dependence of multivariate raw time series*): Embrechts et al. (01), Embrechts (08, survey), Patton (02), Patton (07, survey), Kluppelberg and her co-authors,......
- **Class 2** (copula modeling of *concurrent dependence of multivariate time series fitted residuals*): Chen-Fan (06b), Chen-Fan-Patton (03), Chan et al. (07), Patton (07, survey), Tawn and his co-authors,......

Partial review of copula-based multivariate time series models

- **Class 1** (copula modeling of *concurrent dependence of multivariate raw time series*): Embrechts et al. (01), Embrechts (08, survey), Patton (02), Patton (07, survey), Kluppelberg and her co-authors,......
- **Class 2** (copula modeling of *concurrent dependence of multivariate time series fitted residuals*): Chen-Fan (06b), Chen-Fan-Patton (03), Chan et al. (07), Patton (07, survey), Tawn and his co-authors,......
- **Class 3** (copula modeling of *temporal dependence of univariate time series*): Darsow et al. (92), Joe (97), Chen-Fan (04), Chen-Fan (06a), Beare (08), Ibragimov (07), Ibragimov-Lentzas (08), Patton (07, survey), Chen et al. (08a), Bouye-Salmon (08), Atlason (08)......

Review of copula-based Markov models of order 1

- Any strictly stationary 1-order Markov process $\{Y_t\}$ is completely determined by its bivariate joint dist. of Y_{t-1} and Y_t: $H(y_1, y_2) \equiv C_0(G_0(y_1), G_0(y_2))$; $C_0(\cdot)$ is the true copula linking Y_{t-1} to Y_t; $G_0(\cdot)$ is the true marginal cdf.

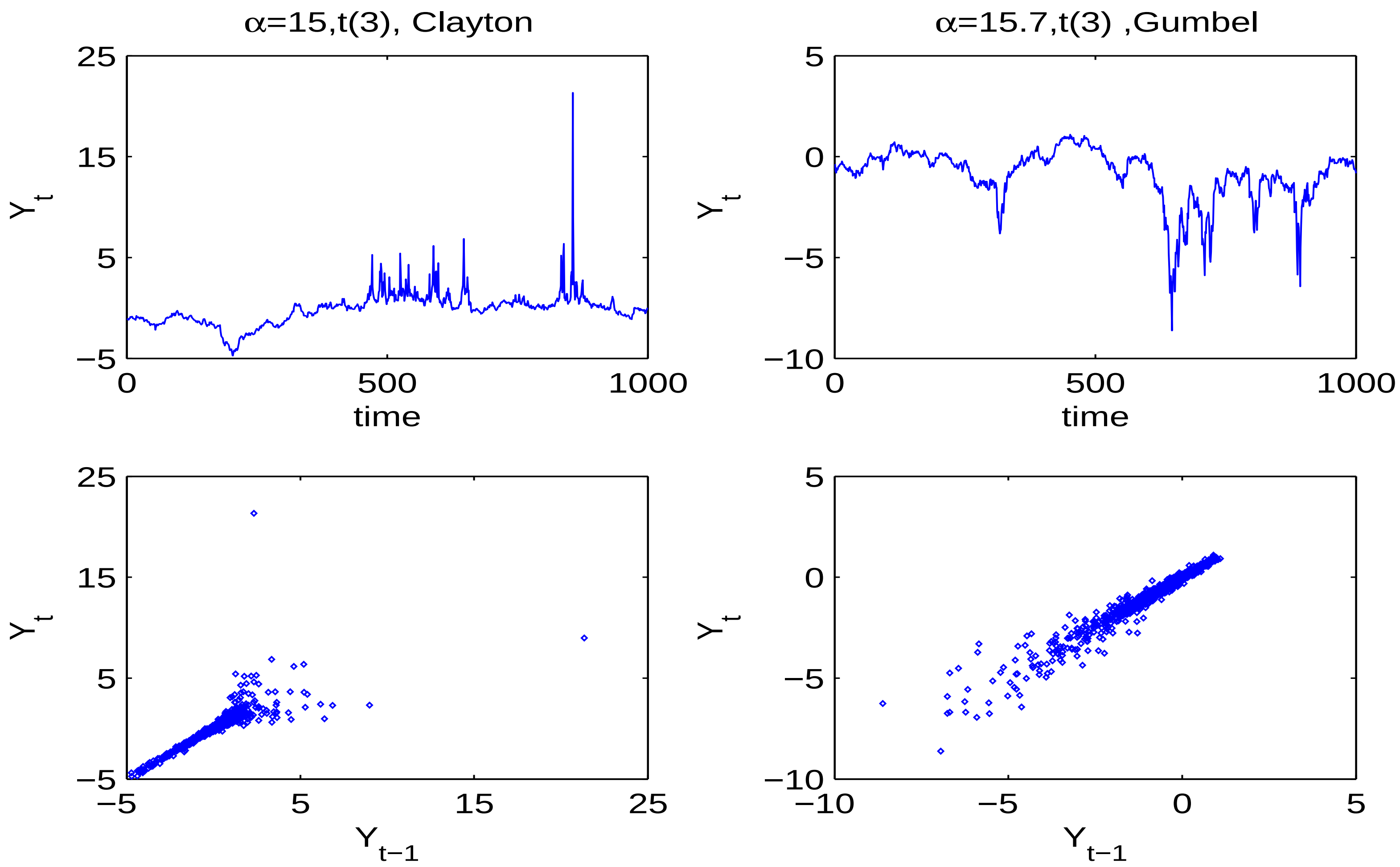
Review of copula-based Markov models of order 1

- Any strictly stationary 1-order Markov process $\{Y_t\}$ is completely determined by its bivariate joint dist. of Y_{t-1} and Y_t: $H(y_1, y_2) \equiv C_0(G_0(y_1), G_0(y_2))$; $C_0(\cdot)$ is the true copula linking Y_{t-1} to Y_t; $G_0(\cdot)$ is the true marginal cdf.
- **DGP**: (1) $\{Y_t\}_{t=1}^{n}$ is a sample of a strictly stationary 1-order Markov process generated from $(G_0(\cdot), C(\cdot, \cdot; \alpha_0))$, where $G_0(\cdot)$ is the true invariant dist. that is absolutely continuous wrt Leb. on $\mathcal{R}$; $C(\cdot, \cdot; \alpha_0)$ is the true parametric copula for (Y_{t-1}, Y_t) up to unknown value α_0, is absolutely continuous wrt Leb. on $[0, 1]^2$. (2) the true marginal density $g_0(\cdot)$ of $G_0(\cdot)$ is positive on its support; and the true copula density $c(\cdot, \cdot; \alpha_0)$ of $C(\cdot, \cdot; \alpha_0)$ is positive on $(0, 1)^2$.

Review of copula-based Markov models of order 1

- separately modeling all the scale-free temporal and tail dependence (via copulas), and heavy-tails (via fat-tailed marginals);
- generating persistent clustering of extreme values via tailed dependent copulas + fat-tailed marginals;
- implied (nonlinear) conditional quantile is automatically monotonic across quantiles;
- generating behaviors like asymmetric dependence, Garch, SV, near-unit root, long-memory, structural break, Markov switching, ...

Time series plots



Review of copula-based Markov models of order 1

- true cond. prob. density of Y_t given $Y^{t-1} \equiv (Y_{t-1}, ..., Y_1)$ is: $h_0(\cdot|Y_{t-1}) = g_0(\cdot)c(G_0(Y_{t-1}), G_0(\cdot); \alpha_0)$.

Review of copula-based Markov models of order 1

- true cond. prob. density of Y_t given $Y^{t-1} \equiv (Y_{t-1}, ..., Y_1)$ is: $h_0(\cdot|Y_{t-1}) = g_0(\cdot)c(G_0(Y_{t-1}), G_0(\cdot); \alpha_0)$.
- $q-$th, $q \in (0, 1)$, cond. quantile of Y_t given Y^{t-1} is: $Q_q^Y(y) = G_0^{-1}\left(C_{2|1}^{-1}\left[q|G_0(y); \alpha_0\right]\right)$.
- $C_{2|1}[\cdot|u; \alpha_0] \equiv \frac{\partial}{\partial u}C(u, \cdot; \alpha_0) \equiv C_1(u, \cdot; \alpha_0)$ is the cond. dist. of $U_t \equiv G_0(Y_t)$ given $U_{t-1} = u$; $C_{2|1}^{-1}\left[q|u; \alpha_0\right]$ is the $q-$th cond. quantile of U_t given $U_{t-1} = u$.

Review of copula-based Markov models of order 1

- true cond. prob. density of Y_t given $Y^{t-1} \equiv (Y_{t-1}, ..., Y_1)$ is: $h_0(\cdot|Y_{t-1}) = g_0(\cdot)c(G_0(Y_{t-1}), G_0(\cdot); \alpha_0)$.
- $q-$th, $q \in (0,1)$, cond. quantile of Y_t given Y^{t-1} is: $Q_q^Y(y) = G_0^{-1}\left(C_{2|1}^{-1}\left[q|G_0(y); \alpha_0\right]\right)$.
- $C_{2|1}[\cdot|u; \alpha_0] \equiv \frac{\partial}{\partial u} C(u, \cdot; \alpha_0) \equiv C_1(u, \cdot; \alpha_0)$ is the cond. dist. of $U_t \equiv G_0(Y_t)$ given $U_{t-1} = u$; $C_{2|1}^{-1}\left[q|u; \alpha_0\right]$ is the $q-$th cond. quantile of U_t given $U_{t-1} = u$.
- Kendall's tau is: $\tau = 4 \int\int_{[0,1]^2} C(u_1, u_2) dC(u_1, u_2) - 1$;
- lower tail dependence coeff. is: $\lambda_L = \lim_{u \to 0^+} \frac{C(u,u)}{u}$,
- upper tail dependence coeff. is: $\lambda_U = \lim_{u \to 1^-} \frac{1-2u+C(u,u)}{1-u}$.

Review of copula-based Markov models of order 1

- Semiparametric Regression Transformation Models:

$$\Lambda_{1,\theta_1}(G_0(Y_t)) = \Lambda_{2,\theta_2}(G_0(Y_{t-1})) + \sigma_{\theta_3}(G_0(Y_{t-1}))e_t,$$

where $\Lambda_{1,\theta_1}(\cdot)$ is a parametric increasing function, $\Lambda_{2,\theta_2}(\cdot)$ and $\sigma_{\theta_3}(\cdot) > 0$ are parametric functions, e_t is independent of Y_{t-1}, and $\{e_t\}$ is i.i.d. with a parametric pdf $f_e(\cdot;\theta_4)$ satisfying mean 0 and variance 1. Then $\{Y_t\}$ is a copula-based Markov process with the parametric copula density:

$$c(u_0, u_1; \alpha_0) = f_e(\frac{\Lambda_{1,\theta_1}(u_1) - \Lambda_{2,\theta_2}(u_0)}{\sigma_{\theta_3}(u_0)}; \theta_4) \times \frac{d\Lambda_{1,\theta_1}(u_1)}{du_1},$$

where α_0 consists of the distinct elements of $\theta_1, \theta_2, \theta_3, \theta_4$.

Review of copula-based Markov models of order 1

- Generalized semiparametric regression transformation:

$$\Lambda_{1,\theta_1}(G_0(Y_t)) = \Lambda_{2,\theta_2}(G_0(Y_{t-1})) + \varepsilon_t, \quad E\{\varepsilon_t|Y_{t-1}\} = 0,$$

where $G_0(\cdot)$ is the unknown cdf of Y_t, $\Lambda_{1,\theta_1}(\cdot)$ is a parametric increasing function,
$\Lambda_{2,\theta_2}(u_0) \equiv E\{\Lambda_{1,\theta_1}(G_0(Y_t))|G_0(Y_{t-1}) = u_0\}$, and the conditional density of ε_t given $G_0(Y_{t-1}) = u_0$ is:
$f_{\varepsilon_t|G_0(Y_{t-1})-u_0}(\varepsilon) =$

$$c(u_0, \Lambda^{-1}_{1,\theta_1}(\varepsilon + \Lambda_{2,\theta_2}(u_0)); \alpha_0) \div \frac{d\Lambda_{1,\theta_1}(\varepsilon + \Lambda_{2,\theta_2}(u_0))}{d\varepsilon}$$

Review of copula-based Markov models of order 1

- If $C(\cdot, \cdot; \alpha)$ is a Gaussian copula, then $\{\Phi^{-1}(G_0(Y_t))\}$ is

$$\Phi^{-1}(G_0(Y_t)) = \alpha \Phi^{-1}(G_0(Y_{t-1})) + \varepsilon_t,$$

  where $\varepsilon_t \sim N(0, 1 - \alpha^2)$, and is independent of Y_{t-1}. Data $\{Y_t\}$ is linear AR(1) iff $G_0 = \Phi$. If $G_0 \neq \Phi$, then $\{Y_t\}$ is a stationary nonlinear Markov process generated by the Gaussian copula with non-Gaussian marginals.
- Gaussian copula Markov process does not have any clustering, nor tail dependence (even with heavy tailed G_0).
- Tail-dependent copulas such as Clayton, t, Gumbel will generate clustering and tail dependence of Y_t.

typical tail dependent copulas

- Clayton: $C(u_1, u_2) = \left[u_1^{-\alpha} + u_2^{-\alpha} - 1\right]^{-1/\alpha}$, $\alpha > 0$. Kendall's tau $\tau = \frac{\alpha}{2+\alpha}$, lower tail dependence $\lambda_L = 2^{-1/\alpha}$.

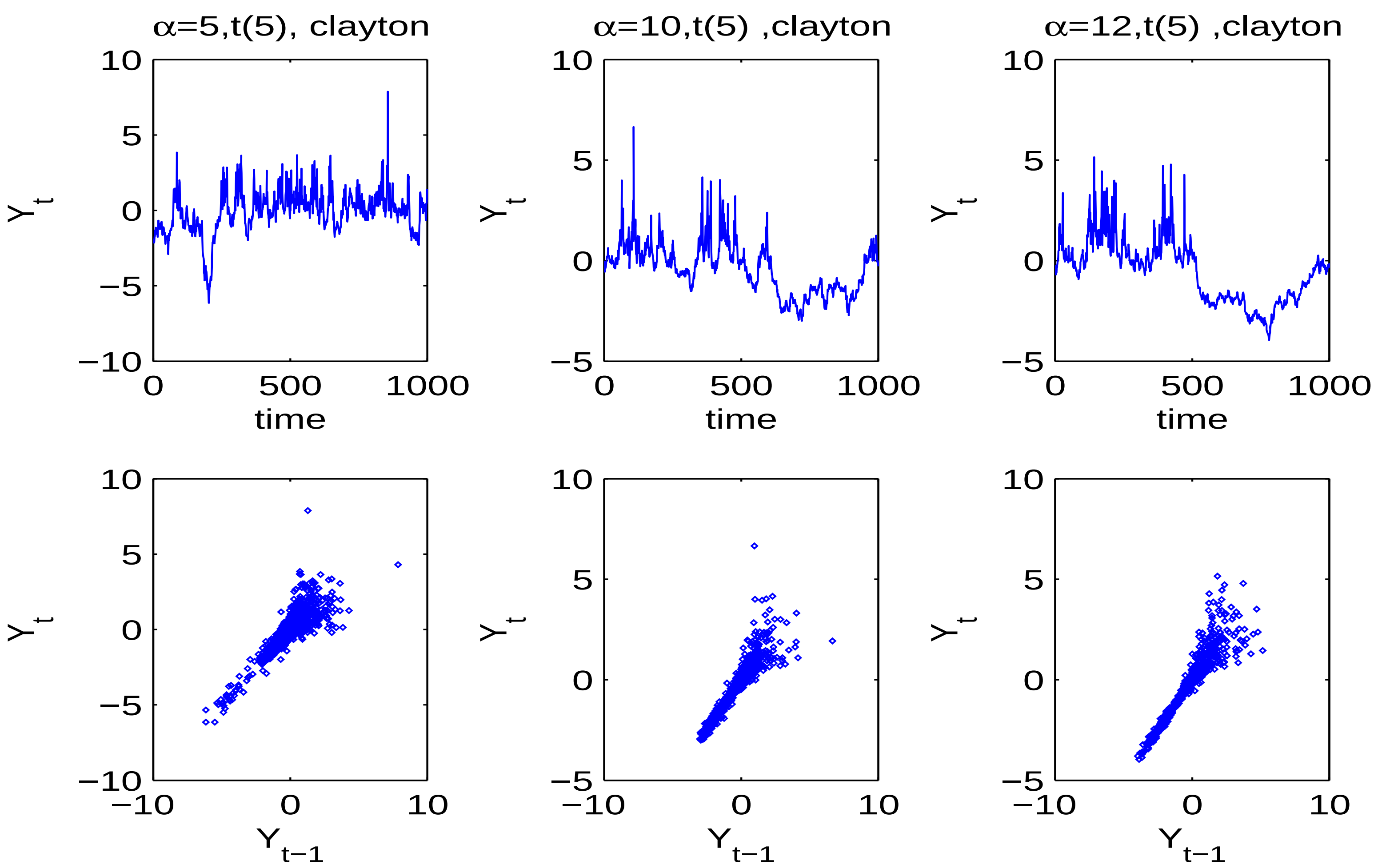
typical tail dependent copulas

- Clayton: $C(u_1, u_2) = \left[u_1^{-\alpha} + u_2^{-\alpha} - 1\right]^{-1/\alpha}$, $\alpha > 0$. Kendall's tau $\tau = \frac{\alpha}{2+\alpha}$, lower tail dependence $\lambda_L = 2^{-1/\alpha}$.
- Gumbel: $C(u_1, u_2) = \exp(-[(-\ln u_1)^{\alpha} + (-\ln u_2)^{\alpha}]^{1/\alpha})$, $\alpha > 1$. Kendall's tau $\tau = 1 - \frac{1}{\alpha}$, upper tail dependence $\lambda_U = 2 - 2^{1/\alpha}$.

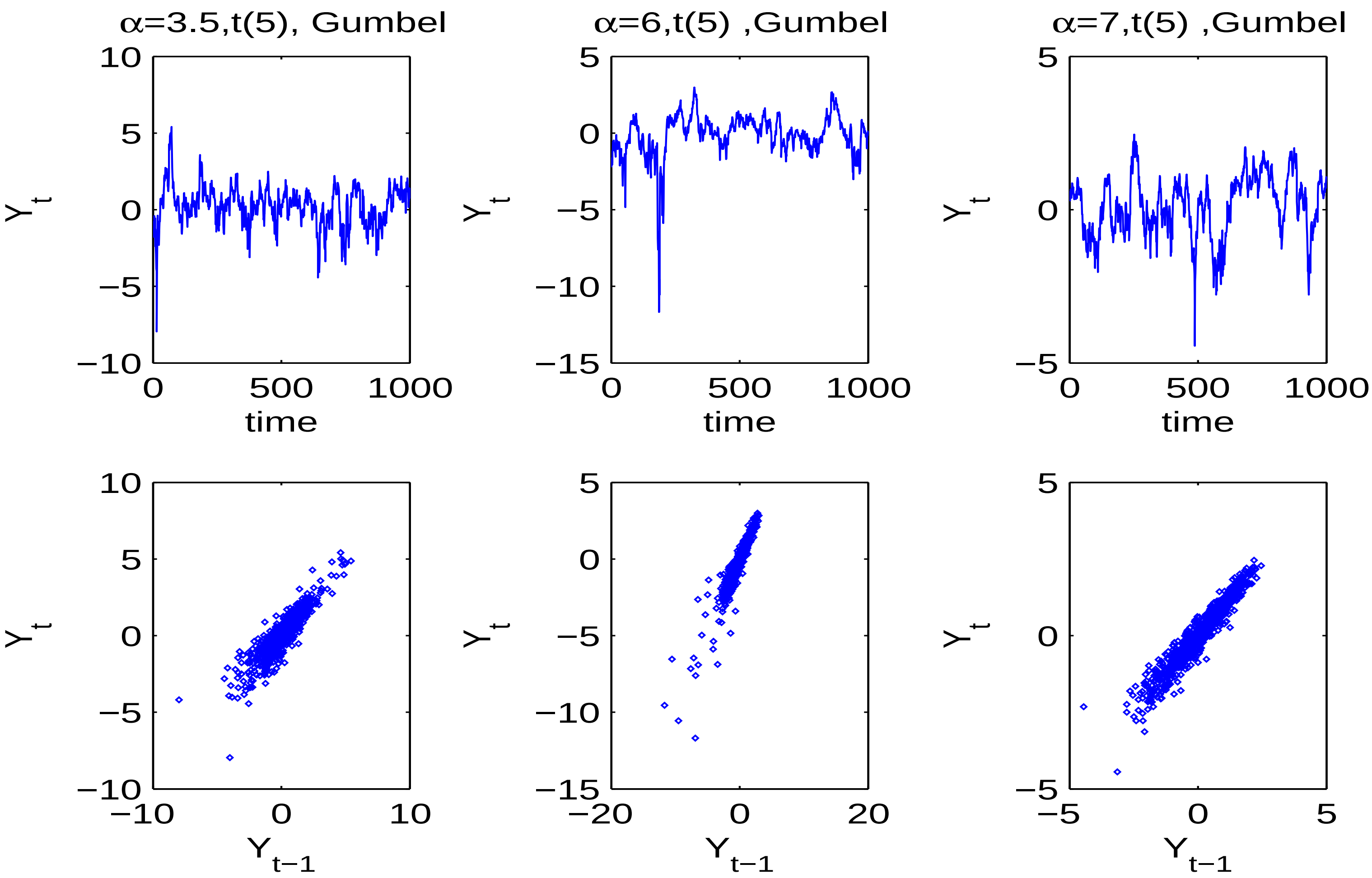
typical tail dependent copulas

- Clayton: $C(u_1, u_2) = \left[u_1^{-\alpha} + u_2^{-\alpha} - 1\right]^{-1/\alpha}$, $\alpha > 0$. Kendall's tau $\tau = \frac{\alpha}{2+\alpha}$, lower tail dependence $\lambda_L = 2^{-1/\alpha}$.
- Gumbel: $C(u_1, u_2) = \exp(-[(-\ln u_1)^\alpha + (-\ln u_2)^\alpha]^{1/\alpha})$, $\alpha > 1$. Kendall's tau $\tau = 1 - \frac{1}{\alpha}$, upper tail dependence $\lambda_U = 2 - 2^{1/\alpha}$.
- Student: $C(u_1, u_2) = \mathbf{t}_{\nu,\rho}(t_\nu^{-1}(u_1), t_\nu^{-1}(u_2))$, $\alpha = (\nu, \rho)$, $|\rho| < 1$. Kendall's tau $\tau = \frac{2}{\pi} \arcsin \rho$, symmetric tail dependence:
  $\lambda_L = \lambda_U = 2t_{\nu+1}(-\sqrt{(\nu+1)(1-\rho)/(1+\rho)})$

Clayton copula ($\alpha = 5, 10, 12$); t(5) marginal



Gumbel copula ($\alpha = 3.5, 6, 7$); t(5) marginal



Clayton-copula stationary Markov vs breaks vs MSR

- **True Model** is a Clayton copula strictly stationary Markov process of order one, that is, $\{Y_t\}_{t=1}^{t=n}$ is completely determined by a bi-variate Clayton copula $C(G(Y_t), G(Y_{t+1}))$ and a marginal cdf $G(Y_t)$:
  1. Generate n independent $U(0,1)$ r.v. $(X_t)_{t=1}^{n}$.
  2. Let $U_1 = X_1$ and $U_t = C_{2|1}^{-1}(X_t|U_{t-1};\alpha)$.
  3. Let $Y_t \triangleq G^{-1}(U_t)$.
- In this simulation, $n = 1000$, Clayton(15) copula + t(3) marginal. Thus

$$C_{2|1}^{-1}(X_t|U_{t-1};\alpha) = [(X_t^{-\alpha/(1+\alpha)} - 1)U_{t-1}^{-\alpha} + 1]^{-1/\alpha}$$

with $\alpha = 15$ and G is the cdf of t(3).

Clayton-copula stationary Markov vs breaks vs MSR

- Compare a break model vs a Markov switching regression model, while the true model is clayton-copula stationary 1st order Markov.
- Davis et al. (05) models a class of non-stationary time series using piecewise AR processes (PAR). The unknown parameters are:
  1. The number (m) and location ($\tau_j, \forall j = 1, ..., m$) of the piecewise AR processes.
  2. Orders of the AR processes ($p_j, \forall j = 1, ..., m$).
  3. Coefficients of the AR processes ($\psi_j \triangleq (\gamma_j, \phi_{1j}, ..., \phi_{jp_j}, \sigma_j^2), \forall j = 1, ..., m$).
- Davis et al. (05) propose LS estimation, MDL model selection, Genetic Algorithm

Copula Markov vs breaks vs MSR

- We consider 2 nested MS regression models:
  1. $MSR(0)$:
     $Y_t = \mu_0(1 - s_t) + \mu_1 s_t + (\sigma_0(1 - s_t) + \sigma_1 s_t)\varepsilon_t$.
  2. $MSR(1)$:
     $Y_t = \mu_0(1 - s_t) + \mu_1 s_t + \rho Y_{t-1} + (\sigma_0(1 - s_t) + \sigma_1 s_t)\varepsilon_t$.

  with $\varepsilon_t \sim i.i.N(0, 1)$.

Figure: PAR Breaks (Black) and MSR(0) Probs

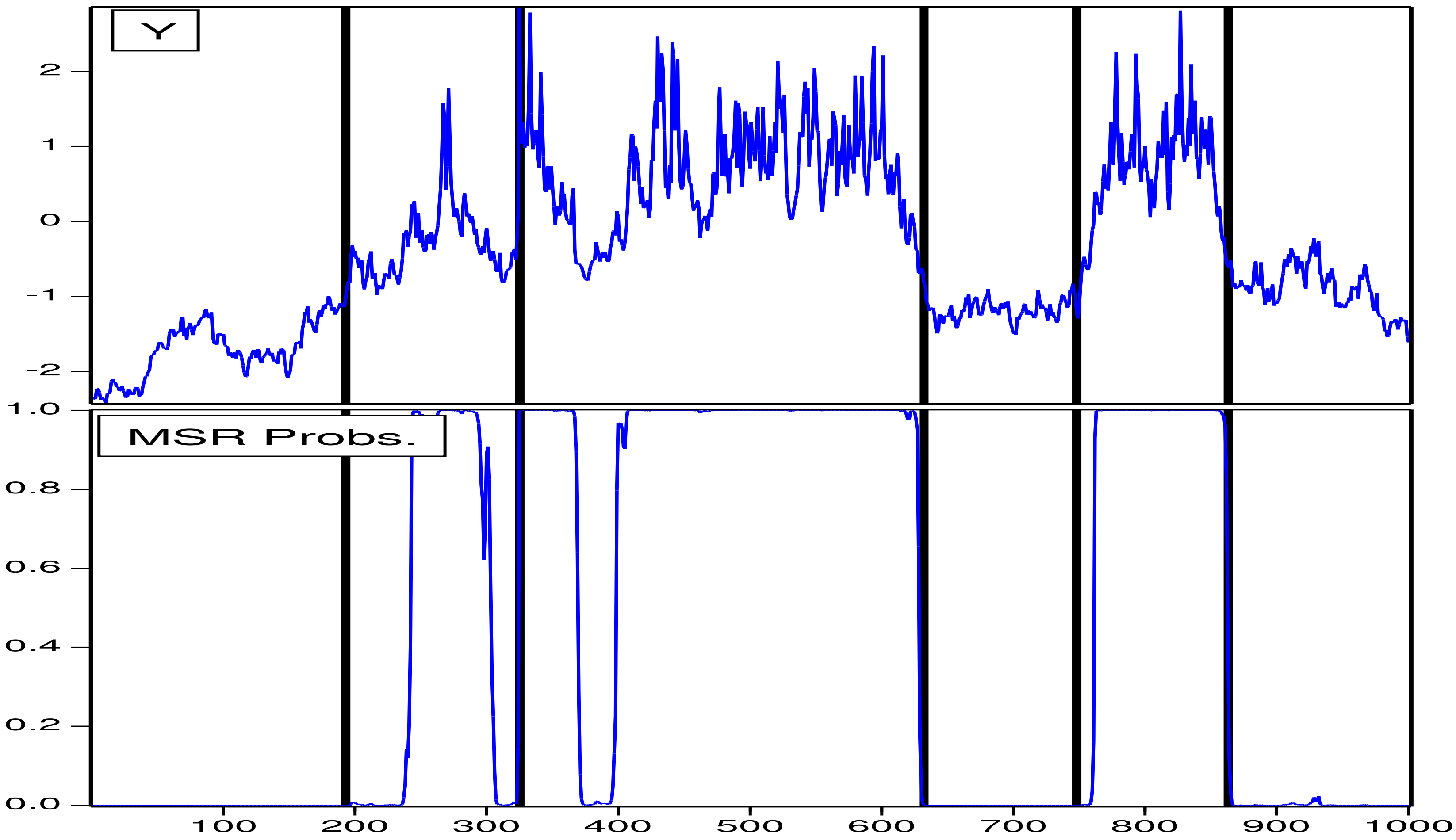
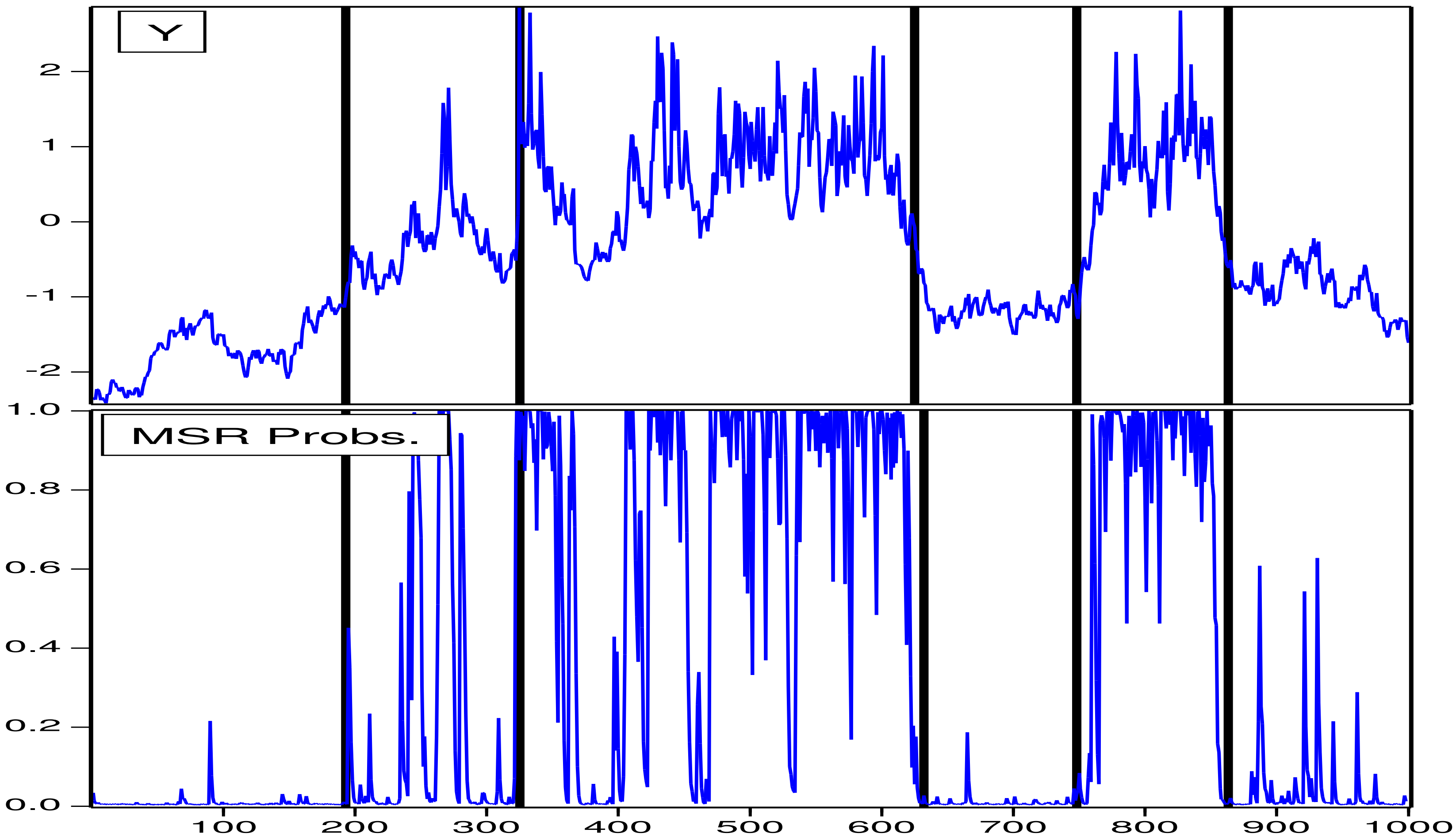


Figure: PAR Breaks (Black) and MSR(1) Probs



Conclusions drawn from this simulation

- **Misspecification.** The “true model” doesn’t have any breaks. But both MSR and PAR models capture breaks.
- **Break location.** MSR(0) and PAR models almost coincide in the location of the breaks.
- **Forecast.** For forecasting, the Copula model and MSR are easy to implement, whereas PAR doesn’t.

Semiparametric stationary 1st order Markov model

- strictly stationary 1st order Markov model: joint cdf of (Y_t, Y_{t+1}) is: $C_0(G_0(Y_t), G_0(Y_{t+1}))$.
- assume true marginal $G_0()$ is unknown, but true copula C_0 has a known parametric form: $C_0() = C(\cdot, \cdot; \alpha_0)$ up to unknown copula parameter α_0.
- **Aim**: estimation of $G_0()$ and α_0.
- Log-likelihood of $\{Y_t\}$ (in terms of $G(\cdot)$ and α):

$$L_n(G, \alpha) = \frac{1}{n} \sum_{t=1}^{n} \log g(Y_t) + \frac{1}{n-1} \sum_{t=2}^{n} \log c(G(Y_{t-1}), G(Y_t); \alpha)$$

- $c(\cdot, \cdot; \alpha)$: copula density, $g(\cdot)$: marginal density.

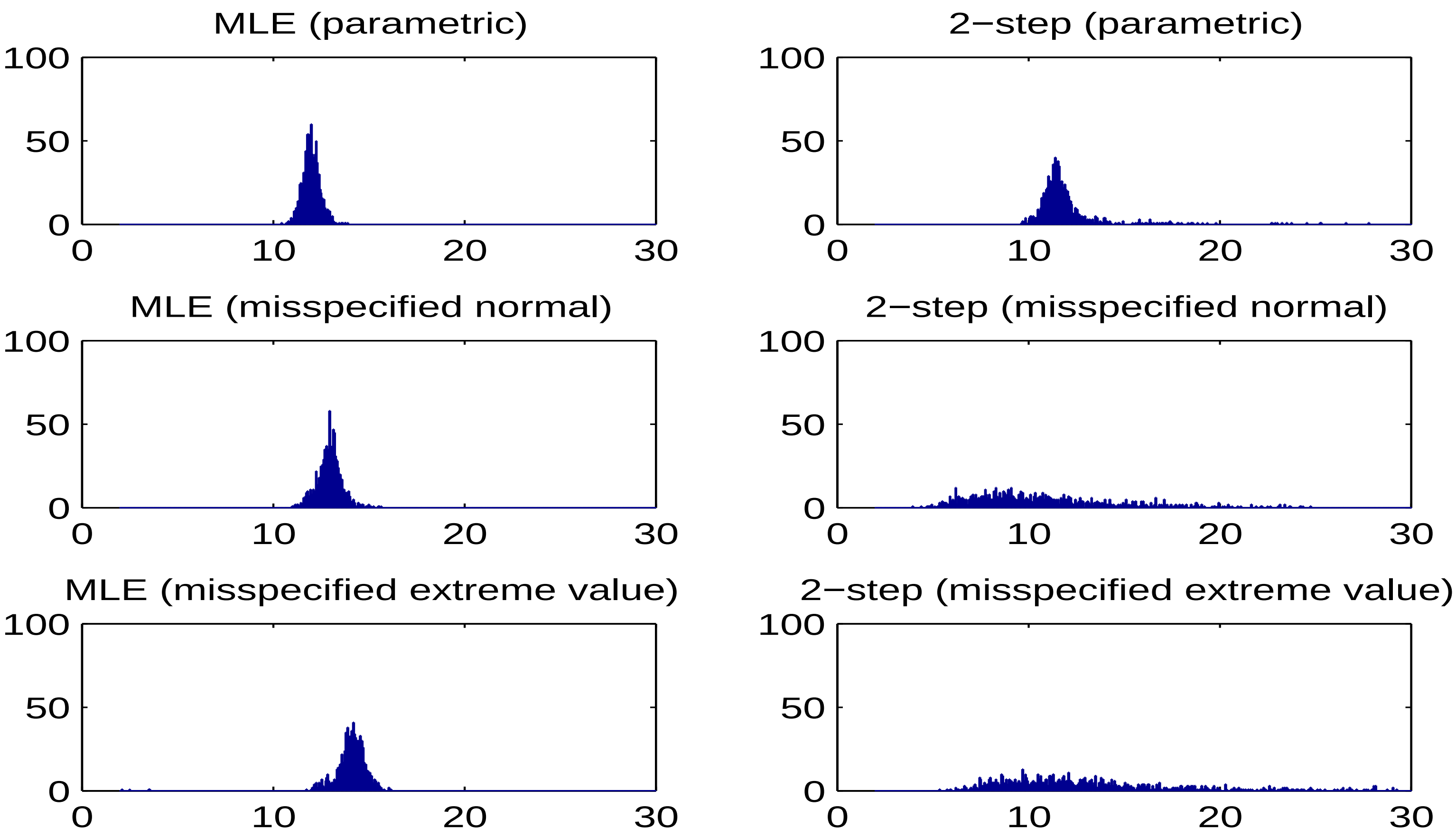
Existing estimators and problems

- MLE-based method: jointly estimate copula parameter and marginal parameter by maximizing the log-likelihood of the Markov model: (1) ideal (or infeasible) MLE: marginal is completely known; (2) parametric MLE: true marginal is known up to unknown parameters; (3) misspecified parametric MLE: marginal is parametrically misspecified.

Existing estimators and problems

- MLE-based method: jointly estimate copula parameter and marginal parameter by maximizing the log-likelihood of the Markov model: (1) ideal (or infeasible) MLE: marginal is completely known; (2) parametric MLE: true marginal is known up to unknown parameters; (3) misspecified parametric MLE: marginal is parametrically misspecified.
- 2-step method: Step 1 estimate marginal cdf by ignoring copula dependence information; Step 2 plug in the estimated marginal cdfs into the copula likelihood and maximize wrt to copula parameter. (1) 2-step parametric estimator; (2) 2-step estimator of Chen-Fan (06a): step 1 estimates unknown G_0 by its rescaled empirical cdf G_n.

Clayton copula ($\alpha = 12$); t(5) marginal



New contributions of this paper

- show that several tail-dependent copula based Markov models are geometric ergodic (hence geometric beta-mixing).
- propose a sieve MLE for the copula-based time series model.
- establish $\sqrt{n}$-normality and semiparametric efficiency of the sieve MLE.
- show that the profiled sieve MLE is asymptotically chi-square distributed.
- propose a Markov semiparametric bootstrap procedure and show its higher-order improvements over the first order asymptotic normal approximation.
- Monte Carlo study.

Geometric ergodicity

- Chen-Fan (2006): all positive copula density generated strictly stationary 1st order Markov model is ergodic and beta-mixing.
- Beare (2008): all symmetric, square integrable, positive copula density generated stationary 1st order Markov is geometric beta-mixing; but square integrable copula density rules out all copulas with tail dependence.
- Ibragimov-Lentzas (2008): Clayton-copula generated strictly stationary 1st order Markov looks like long-memory via simulation study.

Geometric ergodicity

- Chen-Fan (2006): all positive copula density generated strictly stationary 1st order Markov model is ergodic and beta-mixing.
- Beare (2008): all symmetric, square integrable, positive copula density generated stationary 1st order Markov is geometric beta-mixing; but square integrable copula density rules out all copulas with tail dependence.
- Ibragimov-Lentzas (2008): Clayton-copula generated strictly stationary 1st order Markov looks like long-memory via simulation study.
- Chen-Wu-Yi (2008): we show that Clayton, Gumbel, Student t' copula, survival Clayton, survivial Gumbel copula generated strictly stationary 1st order Markov models are all geometric ergodic (hence geometric beta mixing).

Sieve MLE

- $(\hat{g}_n, \hat{\alpha}_n)$ solve $\max_{g_{K_n},\alpha} L_n(g_{K_n};\alpha)$, where

$$L_n(g_{K_n};\alpha) = \frac{1}{n}\sum_{t=1}^{n} \log g_{K_n}(Y_t)$$
$$+\frac{1}{n}\sum_{t=2}^{n} \log c(\int 1(y \le Y_{t-1}) g_{K_n}(y)dy, \int 1(y \le Y_t) g_{K_n}(y)dy;$$

- $g_{K_n}(x) = \exp\left(\sum_{k=1}^{K_n} a_k A_k(x)\right)$, $\int g_{K_n}(x)dx = 1$, $K_n \to \infty$, $\frac{K_n}{n} \to 0$; $\{A_k(\cdot) : k \ge 1\}$ are known basis functions, and $\{a_k : k \ge 1\}$ are unknown sieve coefficients.
- $\hat{g}_n(x) = \exp\left(\sum_{k=1}^{K_n} \hat{a}_k A_k(x)\right)$, where $\{\hat{a}_k : k = 1, \cdots, K_n\}$ and $\hat{\alpha}_n$ are jointly chosen to maximize the log likelihood $L_n(g_{K_n};\alpha)$ of the Markov process.

Monte Carlo simulation study

- $\{Y_t\}$ is a strictly stationary Markov process generated from Clayton copula with $\alpha = 5, 10, 12$ respectively, with true marginal t(5).
- Clayton copula with para. α: $C(u_1, u_2, \alpha) = \left[u_1^{-\alpha} + u_2^{-\alpha} - 1\right]^{-1/\alpha}$. Its copula density is:

$$c(u_1, u_2, \alpha) = (1+\alpha) u_1^{-(1+\alpha)} u_2^{-(1+\alpha)} \left[u_1^{-\alpha} + u_2^{-\alpha} - 1\right]^{-(\alpha^{-1}+2)}$$

- Kendall's $\tau = \frac{\alpha}{2+\alpha}$.
- lower tail dependence index $\lambda = \lim_{u \to 0} \frac{C(u,u)}{u} = 2^{-1/\alpha}$
- no upper tail dependence for Clayton copula
- stronger overall dependence and lower tail dependence increase in α.

Monte Carlo study(cont.)

- Log likelihood for the Clayton copula-based Markov model is:

$$
\begin{aligned}
L_n(G,\alpha) &= \frac{1}{n}\sum_{t=1}^{n} \log g(Y_t) + \frac{n-1}{n}\log(1+\alpha) \\
- &\quad \frac{1}{n}\sum_{t=2}^{n} \left[(1+\alpha)\log G(Y_{t-1}) + (1+\alpha)\log G(Y_t)\right] \\
- &\quad \frac{1}{n}\sum_{t=2}^{n} \left((\frac{1}{\alpha}+2)\log\left[G(Y_{t-1})^{-\alpha} + G(Y_t)^{-\alpha} - 1\right] \right)
\end{aligned}
$$

- We show that for ideal MLE, the asymptotic variance for $\hat{\alpha}$ is bounded above by α^2. The root-n normality and efficiency of the ideal MLE can be derived without any mixing rate condition.

Why Clayton copula?

- All bivariate archimedian copula's limiting tail behavior could be approximated by Clayton copula and survival Clayton copula.
- Clayton copula can produce time series with asymmetric dependence structure (lower tail dependence, no upper tail dependence, positive overall dependence)
- Degree of asymmetry becomes stronger as α increases
- Coupled with fat-tailed marginal cdfs, Clayton copula with larger α produces clusters of small values—persistent in small values.
- conditional quantiles (VaR) can be written in closed form.

Brief summary of MC results

- Sieve MLE always perform better than 2 step estimator in terms of bias and MSE.
- For strong tail dependence case, all the 2step based estimators perform poorly, having big bias.
- For strong tail dependence case, empirical cdf estimator of marginals perform poorly.
- extreme conditional quantiles estimated via sieve MLE is much more precise than those estimated via 2step estimator.

B Tables and Figures

Results are all based on 1000 MC replications of estimates using $n = 1000$ time series simulation, except that $\mathcal{X}^2$ inverted confidence intervals are based on 500 MC replications. τ = Kendall's τ, λ = lower tail dependence index. $Bias^2_{10^3}$, Var_{10^3} and MSE_{10^3} are the true values of $Bias^2$, Var and MSE multiplied by 1000 respectively.
Different two-step estimators: 2step-sieve = 2step procedure while estimating marginal by sieves in 1st step; 2step-empirical = Chen-Fan = 2step; 2step-para = 2step procedure while estimating marginal correctly assuming parametric Student's t_ν distribution in 1st step; 2step-misN = 2step procedure while estimating marginal assuming parametric normal distribution in 1st step; 2step-misEV = 2step procedure while estimating marginal assuming parametric extreme value distribution in 1st step.
Different one-step estimators: Sieve = Sieve MLE; Ideal = Ideal MLE; Para = correctly specified parametric MLE; Mis-N = parametric MLE using misspecified normal distribution as marginal; Mis-EV = parametric MLE using misspecified extreme value distribution as marginal.

Table 1: Clayton copula with true $\alpha = 12$, true marginal $G = t_5$: 2-step estimates of α

	2step-sieve	2step-empirical	2step-para	2step-misN	2step-misEV
Mean	11.370	7.896	12.098	10.709	13.185
Bias	-0.631	-4.104	0.098	-1.291	1.185
Var	3.584	5.656	6.801	14.469	23.827
MSE	3.982	22.5	6.811	16.135	25.231
$\alpha^{MC}_{(2.5,97.5)}$	(8.91,16.52)	(4.35,13.6)	(10.18, 18.42)	(5.65, 20.33)	(7.19, 26.81)

Table 2: Clayton copula with true $\alpha = 12$, true marginal $G = t_5$: 2-step estimates of G

	2step-sieve		2step-empirical		2step-para		2step-misN		2step-misEV	
	$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$
Mean	0.326	0.664	0.331	0.665	0.332	0.668	0.329	0.642	0.340	0.607
$Bias^2_{10^3}$	0.014	0.039	0.001	0.023	0.003	0.003	0.000	0.786	0.104	4.012
Var_{10^3}	2.151	1.196	28.83	12.08	0.039	0.039	25.763	15.154	16.729	15.208
MSE_{10^3}	2.165	1.235	28.83	12.10	0.041	0.041	25.764	15.941	16.833	19.220

Table 3: Clayton copula, true marginal $G = t_5$: estimation of α

		Sieve	Ideal	2step	Para	Mis-N	Mis-EV
$\alpha = 2$	Mean	2.001	2.005	1.920	2.001	2.111	2.907
τ	Bias	0.001	0.005	-0.080	0.001	0.111	0.907
(0.500)	Var	0.020	0.008	0.102	0.012	0.015	0.019
λ	MSE	0.020	0.008	0.109	0.012	0.027	0.841
(0.707)	$\alpha^{MC}_{(2.5,97.5)}$	(1.74,2.28)	(1.84,2.18)	(1.40, 2.63)	(1.78, 2.23)	(1.92,2.37)	(2.67,3.16)
$\alpha = 5$	Mean	4.970	5.006	4.400	5.002	5.379	6.026
	Bias	-0.030	0.006	-0.600	0.002	0.379	1.026
τ	Var	0.139	0.027	1.257	0.044	0.054	0.186
(0.714)	MSE	0.140	0.027	1.617	0.044	0.198	1.239
λ	$\alpha^{MC}_{(2.5,97.5)}$	(4.40, 5.77)	(4.69, 5.33)	(2.71,6.93)	(4.60 , 5.43)	(4.96,5.83)	(5.47,6.50)
(0.871)	$\alpha^{\mathcal{X}^2}_{(0,95)}$	(4.41, 5.45)					
$\alpha = 10$	Mean	9.889	10.01	7.169	10.01	10.77	11.75
	Bias	-0.111	0.01	-2.831	0.01	0.77	1.75
τ	Var	0.483	0.086	4.620	0.143	0.247	0.568
(0.833)	MSE	0.495	0.086	12.63	0.143	0.841	3.637
λ	$\alpha^{MC}_{(2.5,97.5)}$	(8.83 ,11.25)	(9.44,10.6)	(4.02,12.5)	(9.29,10.8)	(9.78,11.7)	(10.4,12.8)
(0.933)	$\alpha^{\mathcal{X}^2}_{(0,95)}$	(8.96, 10.8)					
$\alpha = 12$	Mean	11.85	12.01	7.896	12.00	12.94	14.04
	Bias	-0.149	0.01	-4.104	0.00	0.94	2.04
τ	Var	1.623	0.119	5.656	0.206	0.405	0.960
(0.857)	MSE	1.646	0.120	22.5	0.207	1.285	5.112
λ	$\alpha^{MC}_{(2.5,97.5)}$	(10.6,13.6)	(11.3, 12.7)	(4.35,13.6)	(11.2 , 13.0)	(11.7 , 14.2)	(12.4, 15.3)
(0.944)	$\alpha^{\mathcal{X}^2}_{(0,95)}$	(10.8, 12.9)					

Table 4: Clayton copula, true marginal $G = t_3$: estimation of α

		Sieve	Ideal	2step	Para	Mis-N	Mis-EV
$\alpha = 2$	Mean	1.969	2.002	1.912	1.989	2.400	2.957
τ	Bias	-0.031	0.002	-0.088	-0.011	0.400	0.957
(0.500)	Var	0.019	0.007	0.101	0.012	0.103	0.056
λ	MSE	0.020	0.007	0.109	0.012	0.264	0.971
(0.707)	$\alpha^{MC}_{(2.5,97.5)}$	(1.70, 2.25)	(1.83, 2.17)	(1.36, 2.60)	(1.76,2.19)	(1.99,3.28)	(2.57, 3.36)
$\alpha = 5$	Mean	4.849	5.003	4.359	4.979	5.859	5.923
τ	Bias	-0.151	0.003	-0.642	-0.021	0.859	0.923
(0.714)	Var	0.093	0.026	1.247	0.041	0.189	0.338
λ	MSE	0.116	0.026	1.658	0.042	0.927	1.190
(0.871)	$\alpha^{MC}_{(2.5,97.5)}$	(4.25, 5.48)	(4.69, 5.32)	(2.67,7.12)	(4.58, 5.35)	(5.36, 6.95)	(4.89, 6.62)
$\alpha = 10$	Mean	9.687	10.00	7.115	9.967	11.42	11.57
τ	Bias	-0.313	0.004	-2.886	-0.033	1.425	1.570
(0.833)	Var	0.351	0.085	4.852	0.129	0.577	1.194
λ	MSE	0.449	0.085	13.18	0.130	2.607	3.659
(0.933)	$\alpha^{MC}_{(2.5,97.5)}$	(8.68, 10.87)	(9.43, 10.6)	(3.87, 12.5)	(9.26,10.6)	(10.33,12.9)	(9.68, 12.9)
$\alpha = 12$	Mean	11.62	12.01	7.896	11.98	13.67	13.82
τ	Bias	-0.382	0.012	-4.104	-0.016	1.668	1.816
(0.857)	Var	0.541	0.119	5.656	0.222	0.770	1.917
λ	MSE	0.687	0.120	22.50	0.222	3.552	5.214
(0.944)	$\alpha^{MC}_{(2.5,97.5)}$	(10.5, 13.3)	(11.3,12.7)	(4.35, 13.6)	(11.0, 12.9)	(12.3, 15.7)	(11.4, 15.4)

Table 5: Gumbel copula, true marginal $G = t_5$: estimation of α

		Sieve	Ideal	2step	Para	Mis-N	Mis-EV
$\alpha = 2$	Mean	2.003	1.999	1.982	1.996	2.110	1.991
	Bias	0.003	-0.001	-0.018	-0.004	0.110	-0.009
τ	Var	0.007	0.002	0.013	0.004	0.020	0.030
(0.5)	MSE	0.007	0.002	0.014	0.004	0.032	0.030
	$\alpha^{MC}_{(2.5,97.5)}$	(1.85, 2.17)	(1.91,2.10)	(1.78, 2.23)	(1.87, 2.13)	(1.94, 2.55)	(1.69,2.35)
$\alpha = 3.5$	Mean	3.477	3.498	3.352	3.491	3.672	4.028
	Bias	-0.023	-0.002	-0.148	-0.009	0.172	0.528
τ	Var	0.066	0.008	0.130	0.018	0.050	0.245
(0.714)	MSE	0.066	0.008	0.152	0.018	0.0794	0.524
	$\alpha^{MC}_{(2.5,97.5)}$	(3.03, 4.06)	(3.34, 3.68)	(2.76, 4.20)	(3.25, 3.77)	(3.35, 4.26)	(3.06, 4.91)
$\alpha = 6$	Mean	5.778	5.998	5.253	5.994	6.220	7.439
	Bias	-0.222	-0.002	-0.747	-0.006	0.220	1.439
τ	Var	0.315	0.023	0.676	0.062	0.107	1.230
(0.833)	MSE	0.365	0.023	1.235	0.062	0.155	3.302
	$\alpha^{MC}_{(2.5,97.5)}$	(4.72, 6.96)	(5.72, 6.31)	(3.92,7.17)	(5.54, 6.51)	(5.55,6.79)	(5.03, 9.46)
$\alpha = 7$	Mean	6.622	6.997	5.873	6.993	7.250	8.775
	Bias	-0.378	-0.003	-1.127	-0.007	0.250	1.775
τ	Var	0.457	0.032	0.968	0.086	0.142	1.833
(0.857)	MSE	0.600	0.032	2.238	0.086	0.204	4.983
	$\alpha^{MC}_{(2.5,97.5)}$	(5.31, 8.04)	(6.67,7.37)	(4.23, 8.20)	(6.47, 7.59)	(6.50, 7.94)	(5.75, 11.3)

Table 6: Gumbel copula, true marginal $G = t_3$: estimation of α

		Sieve	Ideal	2step	Para	Mis-N	Mis-EV
$\alpha = 2$	Mean	2.002	1.999	1.982	1.992	2.377	1.864
	Bias	0.002	-0.001	-0.018	-0.008	0.377	-0.136
τ	Var	0.007	0.002	0.013	0.005	0.153	0.026
(0.5)	MSE	0.007	0.002	0.014	0.005	0.295	0.045
	$\alpha^{MC}_{(2.5,97.5)}$	(1.85, 2.18)	(1.91, 2.10)	(1.78,2.23)	(1.85, 2.14)	(1.99, 3.55)	(1.60, 2.22)
$\alpha = 3.5$	Mean	3.486	3.498	3.352	3.481	3.906	3.629
	Bias	-0.014	-0.002	-0.148	-0.019	0.406	0.129
τ	Var	0.064	0.008	0.130	0.021	0.269	0.315
(0.714)	MSE	0.064	0.008	0.152	0.021	0.434	0.331
	$\alpha^{MC}_{(2.5,97.5)}$	(3.06, 4.07)	(3.34, 3.68)	(2.76, 4.20)	(3.21, 3.87)	(3.21, 5.38)	(2.73, 4.83)
$\alpha = 6$	Mean	5.797	5.998	5.253	5.971	6.359	6.8805
	Bias	-0.203	-0.002	-0.747	-0.029	0.359	0.881
τ	Var	0.320	0.023	0.676	0.071	0.396	2.328
(0.833)	MSE	0.362	0.023	1.235	0.072	0.525	3.103
	$\alpha^{MC}_{(2.5,97.5)}$	(4.67, 6.95)	(5.72,6.31)	(3.92,7.17)	(5.47, 6.67)	(5.20, 7.48)	(4.32, 9.78)
$\alpha = 7$	Mean	6.667	6.997	5.873	6.971	7.357	8.257
	Bias	-0.333	-0.003	-1.127	-0.029	0.357	1.257
τ	Var	0.456	0.032	0.968	0.106	0.506	3.859
(0.857)	MSE	0.566	0.032	2.238	0.107	0.633	5.438
	$\alpha^{MC}_{(2.5,97.5)}$	(5.34, 8.12)	(6.67,7.37)	(4.23, 8.20)	(6.34, 7.79)	(6.01, 8.58)	(4.96, 12.24)

Table 10: Clayton copula, true marginal $G = t_5$: estimation of G. Reported $Bias^2$, Var and MSE are the true ones multiplied by 1000.

		Sieve		2step		Para		Mis-N		Mis-EV	
		$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$
$\alpha = 2$	Mean	0.327	0.671	0.334	0.667	0.333	0.667	0.349	0.619	0.346	0.595
	$Bias^2_{10^3}$	0.007	0.002	0.015	0.008	0.011	0.011	0.357	2.558	0.258	5.703
$\tau(0.500)$	Var_{10^3}	0.061	0.059	1.282	0.719	0.002	0.002	0.678	1.865	0.503	0.824
$\lambda(0.707)$	MSE_{10^3}	0.067	0.061	1.297	0.727	0.012	0.012	1.035	4.423	0.761	6.527
$\alpha = 5$	Mean	0.326	0.670	0.333	0.667	0.333	0.667	0.337	0.600	0.334	0.590
	$Bias^2_{10^3}$	0.017	0.000	0.012	0.009	0.011	0.011	0.046	4.874	0.019	6.421
$\tau(0.714)$	Var_{10^3}	0.101	0.105	6.018	2.686	0.002	0.002	1.093	3.734	1.293	3.134
$\lambda(0.871)$	MSE_{10^3}	0.117	0.105	6.030	2.695	0.013	0.013	1.139	8.608	1.312	9.554
$\alpha = 10$	Mean	0.323	0.663	0.331	0.666	0.333	0.667	0.356	0.627	0.362	0.633
	$Bias^2_{10^3}$	0.046	0.054	0.002	0.014	0.011	0.011	0.657	1.857	1.014	1.404
$\tau(0.833)$	Var_{10^3}	0.142	0.123	20.93	8.944	0.002	0.002	0.690	2.364	1.345	1.810
$\lambda(0.933)$	MSE_{10^3}	0.188	0.177	20.93	8.958	0.013	0.013	1.347	4.221	2.359	3.214
$\alpha = 12$	Mean	0.322	0.660	0.331	0.665	0.333	0.667	0.363	0.638	0.367	0.642
	$Bias^2_{10^3}$	0.069	0.102	0.001	0.023	0.011	0.011	1.116	1.038	1.389	0.810
$\tau(0.857)$	Var_{10^3}	0.243	0.140	28.83	12.08	0.002	0.002	1.158	2.149	1.632	2.473
$\lambda(0.944)$	MSE_{10^3}	0.312	0.243	28.83	12.10	0.013	0.013	2.274	3.188	3.022	3.283

Table 11: Clayton copula, true marginal $G = t_3$: estimation of G. Reported $Bias^2$, Var and MSE are the true ones multiplied by 1000.

		Sieve		2step		Para		Mis-N		Mis-EV	
		$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$
$\alpha = 2$	Mean	0.325	0.673	0.333	0.666	0.333	0.667	0.347	0.557	0.382	0.614
	$Bias^2_{10^3}$	0.026	0.007	0.011	0.013	0.009	0.009	0.282	12.84	2.710	3.145
$\tau(0.500)$	Var_{10^3}	0.054	0.049	1.430	0.801	0.002	0.002	1.921	5.651	0.755	0.947
$\lambda(0.707)$	MSE_{10^3}	0.080	0.056	1.441	0.814	0.011	0.011	2.203	18.49	3.465	4.092
$\alpha = 5$	Mean	0.322	0.671	0.332	0.667	0.333	0.667	0.331	0.537	0.342	0.579
	$Bias^2_{10^3}$	0.072	0.002	0.003	0.011	0.009	0.009	0.001	17.65	0.134	8.276
$\tau(0.714)$	Var_{10^3}	0.081	0.085	6.474	2.969	0.002	0.002	1.401	5.697	2.234	5.346
$\lambda(0.871)$	MSE_{10^3}	0.153	0.087	6.478	2.980	0.011	0.011	1.403	23.35	2.369	13.62
$\alpha = 10$	Mean	0.319	0.664	0.331	0.666	0.333	0.667	0.364	0.584	0.371	0.624
	$Bias^2_{10^3}$	0.128	0.042	0.001	0.013	0.009	0.009	1.132	7.452	1.642	2.123
$\tau(0.833)$	Var_{10^3}	0.109	0.137	22.28	9.800	0.003	0.003	0.711	3.410	2.103	4.192
$\lambda(0.933)$	MSE_{10^3}	0.236	0.178	22.29	9.813	0.012	0.012	1.843	10.86	3.744	6.315
$\alpha = 12$	Mean	0.318	0.661	0.331	0.665	0.333	0.667	0.374	0.598	0.375	0.633
	$Bias^2_{10^3}$	0.154	0.079	0.001	0.023	0.010	0.010	1.903	5.242	2.052	1.351
$\tau(0.857)$	Var_{10^3}	0.127	0.141	28.83	12.08	0.003	0.003	0.950	2.662	2.494	4.934
$\lambda(0.944)$	MSE_{10^3}	0.281	0.220	28.83	12.10	0.013	0.013	2.853	7.904	4.547	6.286

Table 12: Gumbel copula, true marginal $G = t_5$: estimation of G. Reported $Bias^2$, Var and MSE are the true ones multiplied by 1000.

		Sieve		2step		Para		Mis-N		Mis-EV	
		$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$
$\alpha = 2$	Mean	0.329	0.672	0.333	0.666	0.333	0.667	0.363	0.633	0.402	0.650
	$Bias^2_{10^3}$	0.002	0.005	0.007	0.018	0.010	0.010	1.055	1.376	5.236	0.384
$\tau(0.500)$	Var_{10^3}	0.053	0.057	0.755	1.025	0.002	0.002	1.059	1.414	3.459	4.357
	MSE_{10^3}	0.055	0.062	0.762	1.043	0.012	0.012	2.114	2.790	8.694	4.742
$\alpha = 3.5$	Mean	0.328	0.674	0.332	0.665	0.333	0.667	0.407	0.670	0.429	0.648
	$Bias^2_{10^3}$	0.005	0.017	0.005	0.030	0.010	0.010	5.964	0.000	9.694	0.487
$\tau(0.714)$	Var_{10^3}	0.134	0.140	2.353	3.482	0.003	0.003	8.112	4.451	14.32	11.69
	MSE_{10^3}	0.139	0.158	2.358	3.511	0.013	0.013	14.08	4.451	24.01	12.18
$\alpha = 6$	Mean	0.324	0.680	0.330	0.664	0.333	0.667	0.391	0.651	0.394	0.606
	$Bias^2_{10^3}$	0.034	0.100	0.000	0.036	0.011	0.011	3.761	0.375	4.042	4.139
$\tau(0.833)$	Var_{10^3}	0.241	0.239	6.840	10.37	0.003	0.003	24.82	13.31	22.64	18.28
	MSE_{10^3}	0.275	0.339	6.840	10.41	0.014	0.014	28.58	13.69	26.68	22.42
$\alpha = 7$	Mean	0.322	0.683	0.329	0.665	0.333	0.667	0.370	0.630	0.378	0.591
	$Bias^2_{10^3}$	0.066	0.177	0.000	0.029	0.011	0.011	1.593	1.576	2.341	6.219
$\tau(0.857)$	Var_{10^3}	0.285	0.272	9.362	13.79	0.004	0.004	28.87	16.86	24.44	20.39
	MSE_{10^3}	0.352	0.449	9.362	13.82	0.014	0.014	30.46	18.43	26.78	26.61

Table 13: Gumbel copula, true marginal $G = t_3$: estimation of G. Reported $Bias^2$, Var and MSE are the true ones multiplied by 1000.

		Sieve		2step		Para		Mis-N		Mis-EV	
		$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$	$Q_{1/3}$	$Q_{2/3}$
$\alpha = 2$	Mean	0.328	0.673	0.333	0.666	0.333	0.667	0.401	0.613	0.519	0.737
	$Bias^2_{10^3}$	0.004	0.011	0.007	0.018	0.009	0.009	5.069	3.239	35.53	4.456
$\tau(0.500)$	Var_{10^3}	0.059	0.063	0.755	1.025	0.003	0.003	2.389	3.111	10.44	7.202
	MSE_{10^3}	0.063	0.074	0.762	1.043	0.012	0.012	7.457	6.350	45.98	11.66
$\alpha = 3.5$	Mean	0.328	0.675	0.332	0.665	0.333	0.667	0.524	0.719	0.565	0.746
	$Bias^2_{10^3}$	0.004	0.025	0.005	0.030	0.009	0.009	37.55	2.386	55.42	5.762
$\tau(0.714)$	Var_{10^3}	0.139	0.147	2.353	3.482	0.004	0.004	18.71	9.238	28.40	18.12
	MSE_{10^3}	0.143	0.171	2.358	3.511	0.013	0.013	56.26	11.62	83.82	23.88
$\alpha = 6$	Mean	0.325	0.681	0.330	0.664	0.333	0.667	0.501	0.700	0.497	0.676
	$Bias^2_{10^3}$	0.025	0.120	0.000	0.036	0.009	0.009	29.17	0.899	27.97	0.037
$\tau(0.833)$	Var_{10^3}	0.273	0.255	6.840	10.37	0.005	0.005	40.49	20.60	40.98	29.81
	MSE_{10^3}	0.298	0.375	6.840	10.41	0.014	0.014	69.66	21.50	68.96	29.84
$\alpha = 7$	Mean	0.324	0.684	0.329	0.665	0.333	0.667	0.477	0.679	0.476	0.655
	$Bias^2_{10^3}$	0.041	0.182	0.000	0.029	0.009	0.009	21.46	0.076	21.35	0.227
$\tau(0.857)$	Var_{10^3}	0.314	0.275	9.362	13.79	0.006	0.006	49.51	26.89	45.82	33.93
	MSE_{10^3}	0.355	0.457	9.362	13.82	0.016	0.016	70.97	26.96	67.16	34.16

Table 18: Clayton copula, true marginal $G = t_3$: estimation of 0.01 conditional quantile

		Sieve	Ideal	2step	Para	Mis-N	Mis-EV
$\alpha = 5$	Int$Bias^2_{10^3}$	36.26	0.000	150.0	0.172	900.7	704.8
$\tau(0.714)$	IntVar_{10^3}	32.15	5.450	985.3	10.18	463.7	313.4
$\lambda(0.871)$	IntMSE_{10^3}	68.41	5.450	1135	10.35	1364	1018
$\alpha = 10$	Int$Bias^2_{10^3}$	7.712	0.000	527.3	0.040	815.3	427.4
$\tau(0.833)$	IntVar_{10^3}	19.36	2.475	855.3	3.716	361.7	202.7
$\lambda(0.933)$	IntMSE_{10^3}	27.07	2.475	1383	3.756	1177	630.1
$\alpha = 12$	Int$Bias^2_{10^3}$	2.851	0.000	367.7	0.004	181.1	175.9
$\tau(0.857)$	IntVar_{10^3}	6.236	1.068	590.9	1.578	59.44	46.12
$\lambda(0.944)$	IntMSE_{10^3}	9.086	1.069	958.7	1.582	240.5	222.0

For each α, evaluation is based on the common support of 1000 MC simulated data. Reported integrated $Bias^2$, integrated Var and the integrated MSE are the true ones multiplied by 1000.

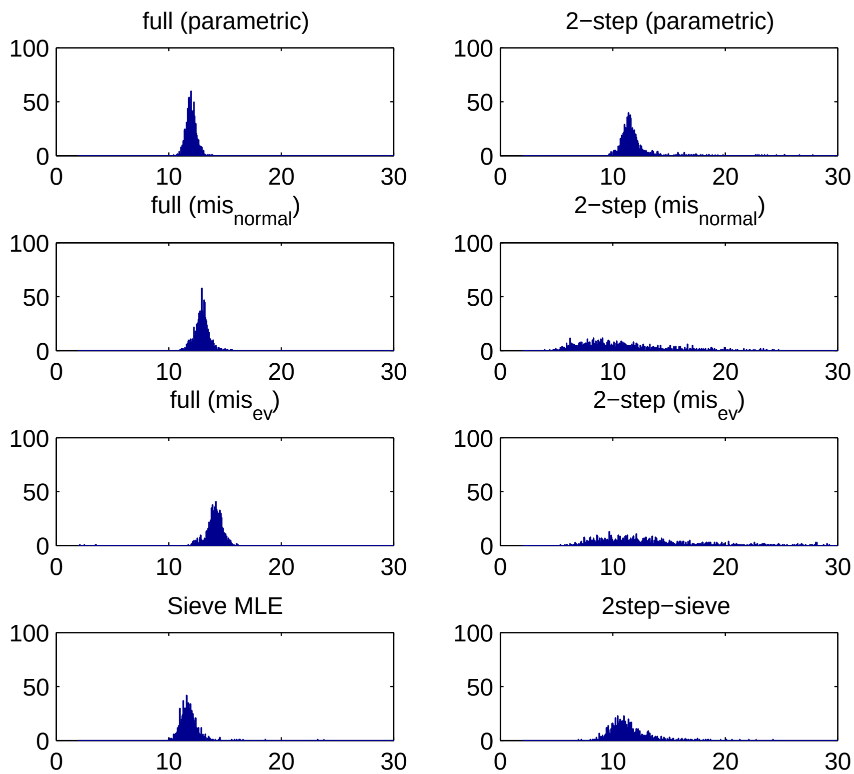


Figure 4: Clayton copula (true $\alpha = 12$, marginal $G = t_5$): Histograms of α estimates: 2-step v.s. full-Likelihood estimators

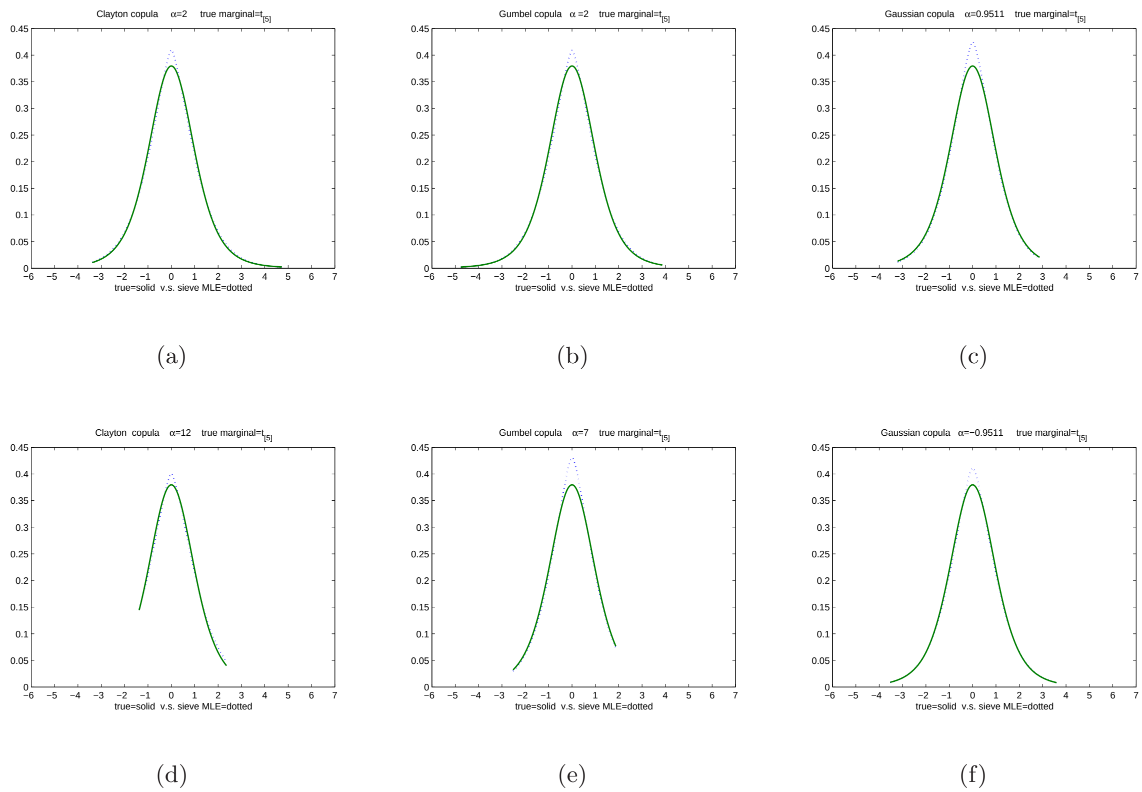


Figure 7: Sieve MLE of marginal density function (true marginal $G = t_5$); Clayton copula: (a) $\alpha = 2$, (d) $\alpha = 12$; Gumbel copula: (b) $\alpha = 2$, (e) $\alpha = 7$; Gaussian copula: (c) $\alpha = 0.9511$, (f) $\alpha = -0.9511$. True=solid, Sieve MLE=dashed. Evaluation is based on the common support of 1000 MC simulated data.

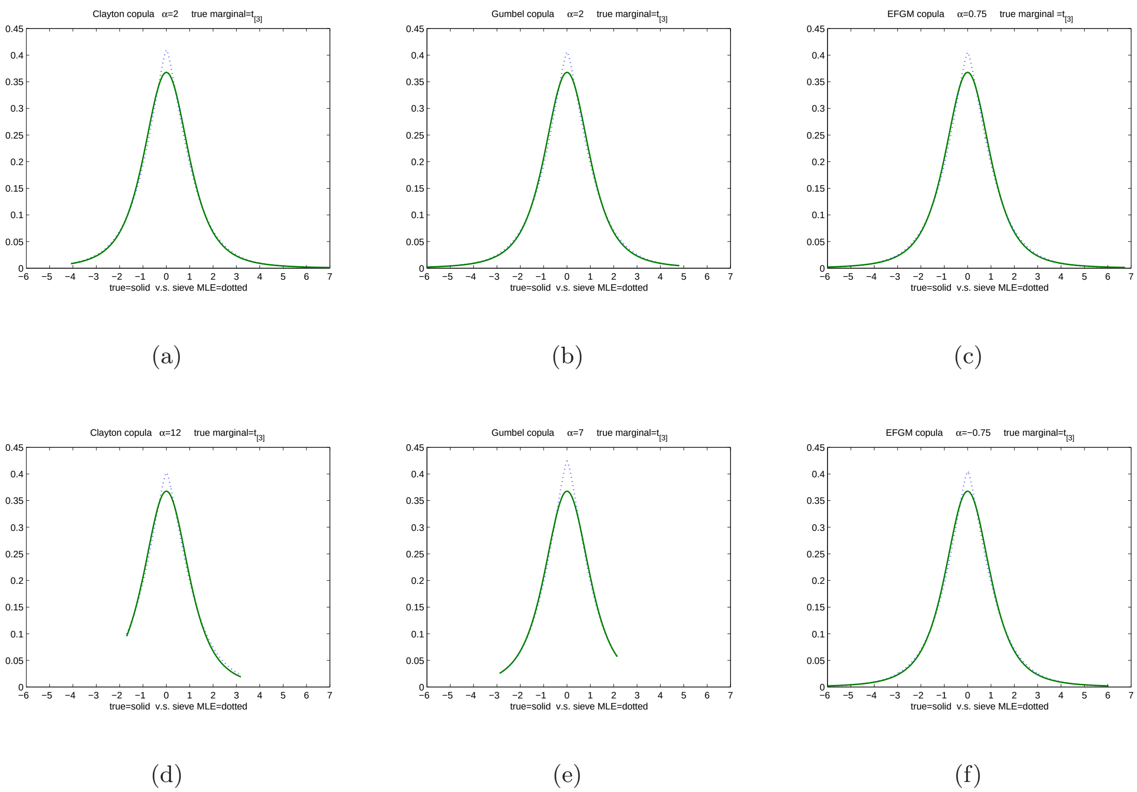


Figure 8: Sieve MLE of marginal density function (true marginal t_3); Clayton copula: (a) $\alpha = 2$, (d) $\alpha = 12$; Gumbel copula: (b) $\alpha = 2$, (e) $\alpha = 7$; EFGM copula: (c) $\alpha = 0.75$, (f) $\alpha = -0.75$. True=solid, Sieve MLE=dashed. Evaluation is based on the common support of 1000 MC simulated data.

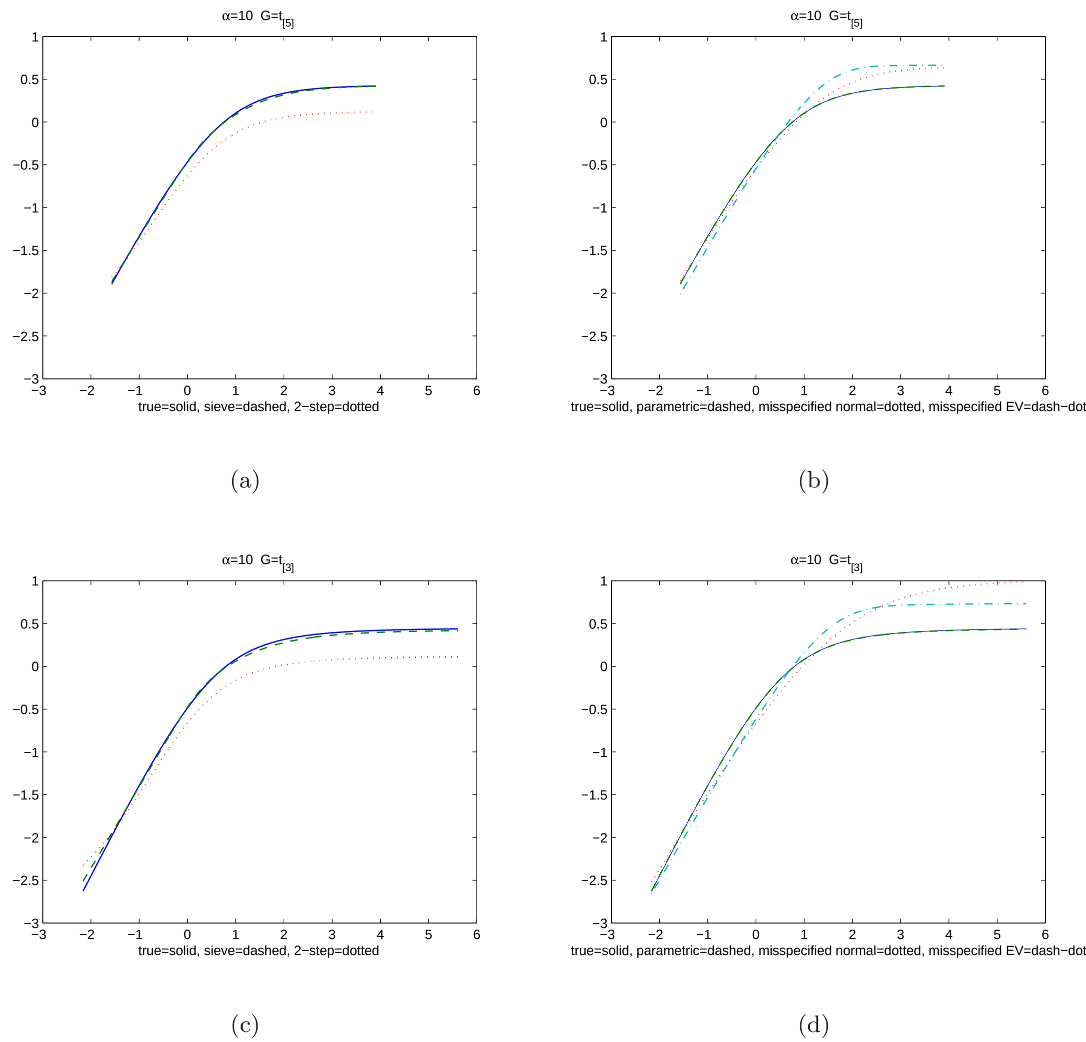


Figure 9: Clayton copula (true $\alpha = 10$, marginal $G = t_5$, t_3): estimation of 0.01 conditional quantile. Evaluation is based on the common support of 1000 MC simulated data.

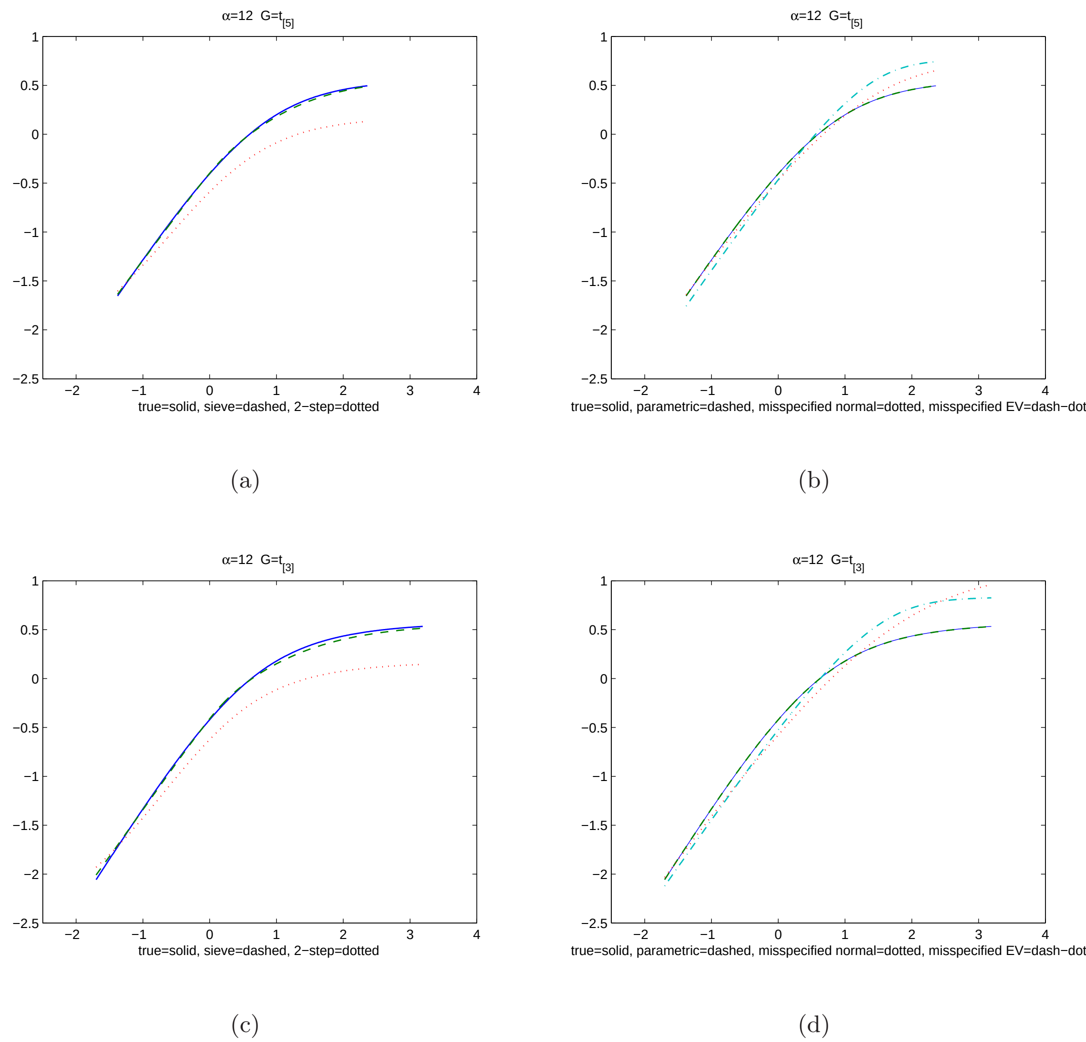


Figure 10: Clayton copula (true $\alpha = 12$, marginal $G = t_5$, t_3): estimation of 0.01 conditional quantile. Evaluation is based on the common support of 1000 MC simulated data.

Future work

- To improve the finite sample performance of 2-step estimators.
- To allow for time-varying copula parameter.
- To allow for copula-based Markov models of higher order.
- To allow for time series fitted residual.
- To study semiparametric extreme quantile autoregression (Chen, Koenker and Xiao (2009)).

Comments are welcome

Thanks !