ABSTRACTS

Spectral and Cubature Methods in Finance and Econometrics

An interdisciplinary international research workshop

University of Leicester, UK, June 18- 20, 2009

Supported by
AMAMEF, EPSRC, LMS, **Oxford-Man Institute** and **University of Leicester**

Organizers

Sergei Levendorskiĭ, Dept. of Mathematics, University of Leicester
Terry Lyons, Oxford-Man Institute, University of Oxford
Martijn Pistorius, Dept. of Mathematics, Imperial College, London

Scientific Committee

Claudio Albanese (King's College London)

Operator methods for analytically solvable and non-parametric models

It is convenient to use operator methods to derive classification schemes for analytically solvable models. However, numerical implementations may encounter difficulties due to the presence of pseudo-spectrum. A numerically robust methodology which applies to general non-parametric Markov processes is based on the combination of the algorithm of fast exponentiation along with block diagonalisations. Convergence theorems for diffusion processes in the continuous limit can be established also in the case of rough coefficients. Operator methods lead to the notion of Abelian processes as a natural and useful extension of stochastic integrals.

Mitya Boyarchenko* (University of Chicago)
Svetlana Boyarchenko (University of Texas at Austin)

Double Barrier Options in Regime-Switching Hyper-Exponential Jump-Diffusion Models

We present a fast and accurate algorithm for calculating prices of finite lived double barrier options with arbitrary terminal payoff functions under regime-switching hyper-exponential jump-diffusion models, which generalize Kou's model. Our method is based on Carr's randomization approximation, which is more efficient and more numerically stable than alternate methods. We also give a proof of convergence of Carr's randomization for double barrier options in regime-switching models.

Mitya Boyarchenko (University of Chicago)
Sergei Levendorskiĭ* (University of Leicester)

Prices and sensitivities of barrier, first touch digital and double barrier options in Lévy-driven models

We develop new FFT techniques (which we call "refined FFT" and "enhanced FFT"), and use them to calculate the prices and sensitivities of barrier options, first-touch digital options and double barrier options on stocks whose log-price follows a Levy process. The numerical results obtained via our approach are in good agreement with the results obtained using other (sometimes fundamentally different) approaches that exist in the literature. However, our method is computationally much faster (often, dozens of times faster). Moreover, our technique has the advantage that its application does not entail a detailed analysis of the underlying Levy process: one only needs an explicit analytic formula for its characteristic exponent. Thus our algorithm is very easy to implement in practice. Our method yields accurate results for a wide range of values of the initial spot price of the underlying, including those that are very close to the barrier(s), regardless of whether the maturity period of the option is long or short.

To solve the pricing problem for a (single or double) barrier option or a first-touch digital option, we apply Carr's randomization approximation. In the case of a double barrier option, each step in the resulting backward induction algorithm is solved using a simple iterative procedure that reduces the problem of pricing options with two barriers to pricing a sequence of certain contingent claims with first-touch single barrier features. This procedure admits a clear financial interpretation that can be formulated in the language of embedded options.

The prices of an option are computed at once for a fine grid of initial log-spot prices of the underlying, which allows one to calculate the sensitivities of the option using numerical differentiation.

Peter Carr* (Bloomberg and Courant Institute)
Roger Lee (University of Chicago)

Pricing and Hedging Volatility Derivatives on Time Changed Diffusions

Interest in volatility derivatives such as options on realized variance have become more liquid now that sector and index variance swaps have started to cap payoffs. We show how to hedge and value these derivatives relative to co-terminal options on the underlying of all strikes. We assume that the underlying price process is given by a specified diffusion running on an unspecified independent stochastic and continuous clock. We show how spectral methods can be used to read the law of the clock from the co-terminal smile. We then show how volatility derivatives can be valued and hedged.

Ning Cai (Hong Kong Univ. of Science and Technology) and
Steven Kou* (Columbia University)

Laplace Transforms and Integro-Differential Equations for Asian and Other Path-Dependent Options

In this talk we shall give some explicit connections between Laplace transforms and integro-differential equations, which enable us to reduce integro-differential equations to ordinary differential equations. This approach leads to give analytical solutions for Asian options, barrier options, lookback options under a hyper-exponential jump diffusion model. Moreover, the approach is more general and relatively simple, and is achieved without using more specialized identities such as Lamperti representation or Wiener-Hopf factorization. Numerically results suggest the solution can be easily implemented.

Xiaohong Chen (Yale University)

Estimation of option pricing and default pricing from the point of view of nonlinear ill-posed inverse problem

Dan Crisan (Imperial College London) and **Konstantinos Manolarakis**

Solving a Backward SDE with the Cubature method

By considering Backward Stochastic Differential Equations (BSDE) where the terminal condition is of the form *C*(X_T), where X is a diffusion, we are able to extend the well known Feynman-Kac formula to semi linear PDEs. Hence, probabilistic methods for the solution of BSDEs provide us with a new approach to the problem of approximating the solution of a semi-linear PDE. Utilizing the Markovian nature of these BSDE's we show how one may consider the problem of numerical solutions to BSDEs within the area of weak approximations of diffusions. To emphasize this point, we suggest an algorithm based on the Cubature method on Wiener space of Lyons and Victoir. When the function © is at least Lipschitz continuous, we are able to recover satisfactory error estimates. We present numerical experiments that validate the method in both linear and non linear setups.

John Crosby* (Glasgow University),
Aleksandar Mijatovic (Imperial College London)
Nolwenn Le Saux (Imperial College London)

Approximating Lévy processes by hyperexponential jump-diffusion processes with a view to option pricing

We examine how to approximate a Lévy process by a hyperexponential jump-diffusion (HEJD) process, composed of Brownian motion and of an arbitrary number of sums of compound Poisson processes with double exponentially distributed jumps. This approximation will facilitate the pricing of exotic options since HEJD processes have a degree of tractability that other Levy processes do not have. The idea behind this approximation has been applied to option pricing by Asmussen at al. (2007) and by Jeannin and Pistorius (2008). In this paper we introduce a more systematic methodology for constructing this approximation which allow us to compute the intensity rates, the mean jump sizes and the volatility of the approximating HEJD process (almost) analytically. Our methodology is very easy to implement. We compute vanilla option prices and barrier option prices using the approximating HEJD process and we compare our results to those obtained from other methodologies in the literature. We demonstrate that our methodology gives very accurate option prices and that these prices are much more accurate than those obtained from existing methodologies for approximating Levy processes by HEJD processes.

Michael Dempster (Centre for Mathematical Sciences University of Cambridge & Cambridge Systems Associates Limited)

Wavelet Optimized PDE Valuation of Derivatives

We introduce a simple but robust and efficient PDE method that uses interpolation wavelets for their advantages in compression and interpolation in order to define a sparse computational domain. It uses finite difference filters for approximate differentiation, which

provide us with a simple and sparse stiffnes matrix for the discrete system. Since the method only uses a nodal basis, the application of non-constant terms, boundary conditions and free-boundary conditions is straightforward. We give empirical results for financial products from the equity and fixed income markets in 1,2 and 3 dimensions and show speed up factors over alternatives between 2 and 4 with no reduction in precision.

Alexander Eydeland* (Morgan Stanley)
Daniel Mahoney (RBS-SEMPRA)

Fast Convolution Algorithm

The paper describes a general FFT-based option pricing model. Important model characteristics include

- High computational efficiency
- Flexible numerical set-up
- Wide range of applications

The model has been successfully used for many years as a general-purpose option pricing tool. We will summarize our experience with the method, present a variety of real-life examples, provide implementation suggestions, and discuss future developments.

Liming Feng* and **Xiong Lin** (University of Illinois at Urbana-Champaign)

Hilbert transform approach for pricing Bermudan options in Lévy models

We present a transform method for the pricing of Bermudan style vanilla, barrier and lookback options in Lévy process models. Our method involves a sequential evaluation of Hilbert transforms, which can be discretized with exponentially decaying errors. The resulting discrete approximation can be efficiently evaluated using the fast Fourier transform. The computational cost of our method is O(NMlog(M)), where N is the number of monitoring dates, and M is the number of discrete points used to compute the Hilbert transforms.

Bruno Feunou (Duke University)
Nour Meddahi* (University of Toulouse)

Generalized Affine Models

Affine models are very popular in modeling financial time series as they allow for analytical calculation of prices of financial derivatives like treasury bonds and options. The main property of affine models is that the conditional cumulant function, defined as the logarithmic of the conditional characteristic function, is affine in the state variable. Consequently, an affine model is Markovian, like an autoregressive process, which is an empirical limitation. The paper generalizes affine models by adding in the current conditional cumulant function the past conditional cumulant function. Hence, generalized affine models are non-Markovian, such as ARMA and GARCH processes, allowing one to disentangle the short term and long-run dynamics of the process. Importantly, the new model keeps the tractability of prices of financial derivatives. This paper studies the statistical properties of the new model, derives its conditional and unconditional moments, as well as the conditional cumulant function of future aggregated values of the state variable which is critical for pricing financial derivatives. It derives the analytical formulas of the term structure of interest rates and option prices. Different estimating methods are discussed (MLE, QML, GMM, and characteristic function based estimation methods). Empirical applications are developed.

Damir Filipovic (Vienna Institute of Finance)

Affine Transform Analysis and Asset Pricing

In this talk I review the key methods in affine transform analysis and asset pricing. Some new applications are given for top-down credit risk models. Moreover, I present some recent results on matrix-valued affine processes, which have been derived in a joint paper with Christa Cuchiero, Eberhard Mayerhofer, and Josef Teichmann.

José da Fonseca (Auckland University of Technology, ESILV Ecole Supérieure d'Ingénieurs Léonard de Vinci and Zeliade Systems)
Martino Grasselli* (Università di Padova and ESILV)
Florian Ielpo (Pictet & Cie, Genéve)

Riding on the Smiles I

This paper investigates the calibration performance of several multifactor stochastic volatility models. There is an empirical evidence that the dynamics of the implied volatility surface is driven by several factors, see e.g. Skiadopoulos, Hodges and Clewlow (1999). This leads to the extensions of the seminal Heston stochastic volatility model proposed by Da Fonseca, Grasselli and Tebaldi (2008) and Christoersen, Heston and Jacobs (2007). Using a data set of derivatives on the major indices we study the calibration properties of these models using the FFT as the pricing methodology (see Carr and Madan 1999). We also study if adding jumps improves significantly the calibration accuracy of the models. Then we focus on basket option pricing models and more precisely on the WASC model (Wishart Affine Stochastic Correlation) proposed by Da Fonseca, Grasselli and Tebaldi (2007). We analyse the calibration property of this model and compare it with other models such as the one proposed by Barndorff -Nielsen and Stelzer (2007). Finally, we provide some price approximations for vanilla options in the spirit of Benabid, Bensusan and El Karoui (2009) that speed up the pricing procedure thus leading to reasonable calibration time. Also we provide some results on Malliavin calculus allowing the computation of sensitivities for derivative products in our models.

P.Gruber, F. Trojani (University of Lugano)
C.Tebaldi* (Bocconi University Milano)

Estimation of Volatility Factor Models with Wishart Spectral Dynamics

We explore the pricing of index options in an analytically tractable model that decomposes market volatility and skewness into three independent components: two short/long term volatility factors and a separate component modeling stochastic skewness. The evolution of these factors is determined by an innovative spectral representation of the multifactor volatility model of Da Fonseca, Grasselli, Tebaldi (Quant. Fin. 8(6) 2008) We estimate the model using US index option data in order to quantify these risk components empirically. We found that they all play an important role in the accurate description of the dynamics of level, term structure and skew of option{implied volatilities. Short and long term volatility dynamics successfully capture the characteristics of level and term structure of the smile. However, the smile also depends on a non-negligible component linked to short term stochastic skewness, which is to a large extent independent of short and long term volatility movements. Thus, a separate specification of stochastic short term skewness, in addition to multi{factor volatility, is an important model ingredient for explaining both the dynamic and the cross sectional patterns of index option smiles.

Lech Grzelak* and **C. W. Oosterlee** (Delft University of Technology, Institute of Applied Mathematics)

The Heston model with stochastic interest rates under Fourier based pricing algorithms

In this presentation we discuss the Heston model with stochastic interest rates driven by Hull-White or Cox-Ingersoll-Ross processes. We define a so-called compensator which guarantees that the Heston hybrid model with non-zero correlation between the equity and interest rate processes is properly defined. Moreover, we present an approximation for the characteristic function, so that derivative pricing can be efficiently done using the Fourier Cosine expansion technique. We discuss the effect of the approximations on the instantaneous correlations, and check the influence of the correlation between stock and interest rate on the implied volatilities. Finally, we derive an approximation for the characteristic function of the Heston-Hull-White model with a full matrix of correlations.

Lars Peter Hansen* (University of Chicago)
José A. Scheinkman (Princeton University)

Pricing Growth-Rate Risk

We characterize the compensation demanded by investors in equilibrium for incremental exposure to growth-rate risk. Given an underlying Markov diffusion that governs the state variables in the economy, the economic model implies a stochastic discount factor process S and a reference stochastic growth process G for the macroeconomy. Both are modeled conveniently as multiplicative functionals of a multi-dimensional Brownian motion. To study pricing we consider the pricing implications of parameterized family of growth processes G^ϵ, with $G^0=G$, as ϵ is made small. This parameterization defines a direction of growth-rate risk exposure that is priced using the stochastic discount factor S. By changing the investment horizon we trace a *term structure* of risk prices that shows how the valuation of risky cash flows depends on the investment horizon. Using methods of Hansen and Scheinkman (Econometrica, 2009), we characterize the limiting behavior of the risk prices as the investment horizon is made arbitrarily long.

Alexey Kuznetsov (York University)

Wiener-Hopf factorization and distribution of extrema for a family of Lévy processes

In this talk we will introduce a ten-parameter family of Lévy processes for which we obtain Wiener-Hopf factors and distribution of the supremum process in semi-explicit form. This family allows an arbitrary behavior of small jumps and includes processes similar to the generalized tempered stable, KoBoL and CGMY processes. Analytically it is characterized by the property that the characteristic exponent is a meromorphic function, expressed in terms of beta and digamma functions. We prove that the Wiener-Hopf factors can be expressed as infinite products over roots of certain transcendental equation and the density of the supremum process can be computed as an exponentially converging infinite series. In several special cases when the roots can be found analytically we are able to identify Wiener-Hopf factors and distribution of the supremum in closed form. In general case we prove that all the roots are real and simple, we provide localization results and asymptotic formulas which allow an efficient numerical evaluation. We also derive a convergence acceleration algorithm for infinite products and a simple and efficient procedure to compute the Wiener-Hopf factors for complex values of parameters. As a numerical example we discuss computation of the density of the supremum process.

Andreas Kyprianou (University of Bath)

Scale functions for spectrally negative Lévy- processes and their appearance in economic models

We give an overview of the theory of scale functions for spectrally negative Levy processes and indicate how they have repeatedly surfaced in a number of studies concerning classical economic models including exotic option pricing, default models and controlled risk insurance processes

Peter Laurence (Universita di Roma 1)

Some asymptotic results for local volatility models

We describe theoretical developments of asymptotic expansions for option prices in local volatility models and illustrate these numerically. Based on joint work with
Jim Gatheral (Merrill Lynch and Courant Institute)
Elton Hsu (Northwestern University)
Cheng Ouyang (Purdue Unievrsity)
Tai-Ho Wang (Baruch College)

Roger Lee (University of Chicago)

Joint Transforms of Prices and Clocks, with Applications to Path-Dependent Options

Under time-changed Levy dynamics, we find the joint Laplace transform of the clock and the log price. We give applications to the pricing of path-dependent options, including the mileage option, a contract which can manage exposure to both price risk and variance risk.

Vadim Linetsky and **Lingfei Li*** (NorthWestern University)

Commodity Derivatives Models with Mean-Reverting Jumps and Stochastic Volatility: A Spectral Expansion Approach

We construct a novel class of pure jump and jump-diffusion commodity and energy models with state-dependent and mean-reverting jumps by applying time changes to the classical commodity models based on mean-reverting diffusions. The initial futures curve is an input into the model, and the dynamics of the futures curves over time exhibits mean-reverting jumps. We obtain analytical solutions for the pricing of commodity derivatives, such as options on futures, by applying the spectral expansion methodology. The models are flexible enough to capture a variety of implied volatility smile patterns observed in energy, metals, and agricultural commodities futures options.

Alex Lipton* (Bank of America Merrill Lynch & Imperial College London)
Artur Sepp (Bank of America Merrill Lynch)

Quantitative methods for Counterparty Risk: I. Analytical Formulation and Results

We present a multi-dimensional jump-diffusion version of a structural default model (SDM) and show how it can be used in order to value the credit value adjustment (CVA) for a credit default swap (CDS) and other derivative instruments. We develop robust analytical and asymptotic methods for model calibration via forward induction and instrument pricing via backward induction, which are applicable in one, two, and, potentially, three dimensions.

Alex Lipton (Bank of America Merrill Lynch & Imperial College London)
Artur Sepp* (Bank of America Merrill Lynch)

Quantitative methods for Counterparty Risk: II. Implementation Issues and Examples

We develop robust FFT- and PIDE-based numerical methods for solving the boundary value problems for the credit value adjustment (CVA) in a multi-dimensional jump-diffusion structural default model (SDM) with a special emphasis on the role of negative asset value jumps. We demonstrate how these methods can be applied for calibration and pricing. Using recent market data, we show that under realistic assumptions CVA greatly reduces the value of CDS sold by a risky counterparty compared to the one sold by a non-risky counterparty. We identify features having the biggest impact on CVA, namely, default correlation and spread volatility

Christian Litterer (Mathematical Institute, Oxford)
Terry Lyons* (Mathematical Institute and Oxford-Man Institute, Oxford University, Wales Institute of Mathematical and Computational Sciences, Swansea University)

Cubature and super-accurate Monte Carlo PDE solvers

Cubature is a powerful technique for achieving high order Markovian approximations to second order PDE's. However, it's use requires a great deal of care because, as time evolves, standard implementations result in an exponential explosion in the number of points considered. Even so these methods have proved effective in some intractable moderate dimensional problems.

Christian Litterer and the speaker have combined the cubature technique with a high-order recombination method which overcomes this exponential explosion and they have proved that these new methods are similtaneously high order and of moderate computational complexity.

Real world implementations of these methods are capable of producing extremely high accurate results in a tractable period of time (machine precision in a second or two) for simple PDE's of the kind one observes in mathematical finance.

The challenge seems to be to develop useable software that encodes the expert knowledge needed to exploit these algorithms.

Dilip Madan (University of Maryland)

Saddlepoint Pricing, Loan Spreads and Counterparty Credit Risk Evaluation

Saddlepoint methods are developed for option pricing and applied to determine loan spreads implied by option data for counterparties. The Merton compound option model is applied with Levy driven asset value processes to infer credit qualities as seen in option volatility surfaces.

Alexandar Mijatovic and Martjin Pistorius* (Imperial College London)

Barrier options for general jump-diffusions

In this talk we present an approximation algorithm to compute prices and Greeks if the underlying is modelled by a general jump-diffusion process. As illustration, we give several numerical examples

Alexandar Mijatovic* and Martjin Pistorius (Imperial College London)

Double-no-touch options and the volatility smile

In this talk we consider a Markov additive process, which incorporates both stochastic volatility and jumps, and describe analytically tractable results for double barrier option pricing. The formulae are explicit up to a Laplace transform for the double-no-touch options and a Laplace-Fourier transform for double barrier call or put options. The results rely on an explicit complex-matrix Wiener-Hopf identity and fluid embedding for the model in question. We present and discuss results of the simultaneous calibration of the model to the vanilla and double-no-touch option prices for three major currency pairs EUR-USD, GBP-USD and USD-JPY.

POSTER SESSION

Edward Hoyle (Imperial College London)

Lévy Random Bridges and the Modelling of Financial Information

The information-based asset-pricing framework of Brody, Hughston and Macrina (BHM) is extended to include a wider class of models for the market information filtration. In the BHM framework, each asset is associated with a collection of random cash flows. The price of the asset is the sum of the discounted conditional expectations of the cash flows. The conditional expectations are taken with respect to a filtration generated by a set of so-called information processes. The information processes carry noisy or imperfect market information about the cash flows. To model the flow of information, we introduce in this paper a class of processes which we term *Lévy random bridges* (LRBs).This class generalises the Brownian bridge and gamma bridge information processes considered by BHM. An LRB is defined over a finite time horizon. Conditioned on its terminal value, an LRB is identical in law to a Lévy bridge. We consider in detail the case where the asset generates a single cash flow X_T occurring at a fixed date T. The flow of market information about X_T is modelled by an LRB terminating at the date T with the property that the (random) terminal value of the LRB is equal to X_T. An explicit expression for the price process of such an asset is then found by working out the discounted conditional expectation of X_T with respect to the natural filtration of the LRB. The prices of European options on such an asset are also calculated.
This is a joint work with L.P.Hughston and Andrea Macrina.

Antoine Jacquier (Imperial College London)

The large maturity smile for the Heston model

Using the method of steepest descent, we derive an asymptotic formula for the price of a call option under the Heston model in the large-maturity limit, when the log-moneyness is also proportional to the maturity. This sharpens the asymptotic result in Forde and Jacquier (2009), where the leading order term is obtained using the Gartner-Ellis theorem from Large deviations theory. We also derive the leading order and correction terms for the asymptotic implied volatility, which is a more useful formula in practice. This shows that the smile does not flatten as the maturity increases, but rather it spreads out, and we need the large-time, largemoneyness regime to capture this effect. This can also be used to rigourize and generalize the result by acquier (2009) for the price of a call and its associated implied volatility in the large-time, fixed-strike regime. The asymptotic formulae require calculating the Legendre transform of a simple function to calculate the position of its saddlepoint on the imaginary axis, and are closely related to the Lugannani-Rice saddlepoint approximation formula. This is a joint work with Martin Forde and Aleksandar Mijatovic.

Zhao Bo (Cass Business School)

Inhomogeneous Geometric Brownian Motion

The CIR process and the Ornstein-Uhlenbeck process are very well-known mean reverting processes in modelling various asset classes. Another mean-reverting stochastic process called inhomogeneous geometric Brownian motion(IGBM for short) gains a lot of attention among academic researchers and practitioners. The main advantage for using the IGBM to describe the underlying assets lies in the fact that this process enjoys a very interesting feature, which is, if one unit of the underlying reverts to some mean value, then two units of the same underlying reverts to twice that mean meanwhile keeping the same speed of reversion towards the long run mean and the instantaneous volatility.

This paper studies main properties of the IGBM process. These include its boundary types, transition density functions, and hitting times. Applications in the field of real options, specifically, valuation of perpetual American call and perpetual American put options are investigated. It is interesting to notice that perpetual American call options when the underlying asset is characterized by an IGBM process always have minimum option value and minimum option delta.