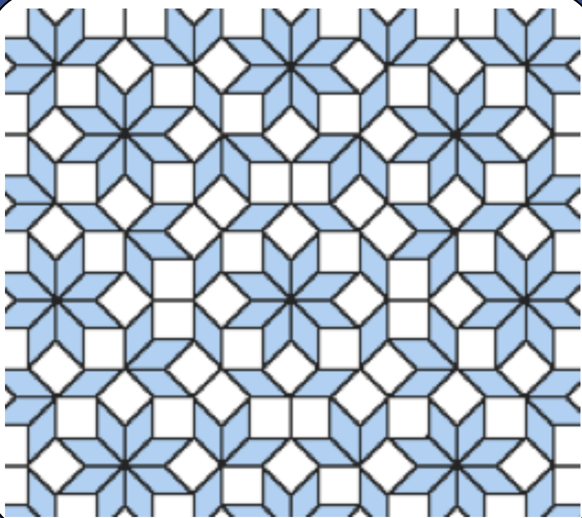
University of Leicester

Pure Mathematics

Representation theory (Baranov, Pirashvili, Snashall)

Representation theory is a vast area of algebra that has immediate connections to algebraic and differential geometry and theoretical physics. Much of the development of modern representation theory was motivated by representations of finite groups; this classical area is still important and continues to provide instructive examples. Another source of motivation is the classification theory of finite-dimensional representations of simple Lie algebras. The latter provides an extremely rare example of a situation when a complete as well as a highly nontrivial classification could be achieved.

Particularly relevant to the research interests of the group are representations of Lie algebras, finite-dimensional associative algebras, quivers and quantum groups. One of the ways to characterize an algebra is through its categories of modules, or representations. This leads to the notion of *Morita equivalence* of algebras – two algebras are Morita equivalent if they have equivalent categories of modules. Morita theory has recently found rich and manifold generalizations, touching upon such areas as stable homotopy theory, derived categories of algebraic varieties and A_∞-algebras and categories.



Noncommutative geometry (Hunton, Lazarev, Mudrov, Snashall)

This relatively new subject came to the fore in the middle of the 1980's, mainly thanks to the fundamental works of Alain Connes and Vladimir Drinfeld (Fields medals, 1982, 1986). The main idea of noncommutative geometry is to treat noncommutative algebras as if they were algebras of function of 'noncommutative spaces'. A surprising amount of classical geometric techniques and concepts (such as smoothness, homogeneous spaces, the De Rham complex, symplectic structures etc.) could be carried over in the framework of noncommutative geometry. The noncommutative analogue of a Lie group is called a *quantum group*; this purely mathematical notion has its roots in mathematical physics, in particular exactly soluble models of statistical mechanics. Quantum groups have various applications in other parts of mathematics, in particular they give rise to invariants of isotopy classes of knots and links. Noncommutative spaces naturally occur in problems which involve dealing with spaces having bad topology, such as moduli spaces of tilings, of leaves of foliations and various representation spaces. Noncommutative geometry also figures prominently in the study of operad algebras (such as A_∞, L_∞ or C_∞-algebra) which were introduced by topologists in order to study homotopy invariant structures of topological spaces.

Homological Algebra (Lazarev, Pirashvili, Snashall)

This area of mathematics provides a common language for experts in topology, algebraic geometry and pure algebra. Recently it was realized that homological algebra is also fundamental for theoretical physics, particularly those aspects related with conformal field theories and quantum gauge theories and deformation quantization. A large part of modern algebraic geometry is now formulated in terms of derived categories of coherent sheaves on complex and algebraic varieties which is the the language of homological algebra. Other topics of current interest in homological algebra include Hochschild and cyclic homology, differential-graded categories and operadic algebras.