

## **The Vaxjo QDT Program**

*1. What is the difference between quantum decision theory applied to social sciences and quantum mechanics applied to physics?*

Quantum mechanics was discovered at the beginning of the 20<sup>th</sup> century by a diverse group of brilliant scientists trying to understand paradoxical departures from classical physics. The list of contributors includes Planck, Einstein, De Broglie, Heisenberg, Schrödinger, Born, Bohr, and Dirac, and others. But its mathematical basis was not well understood until Dirac and Von Neumann provided axiomatic foundations, at which point they discovered that quantum principles entail a new type of logic (quantum logic), probability theory (quantum probability), and dynamics (quantum dynamics). These new ideas were needed to solve the problems that physicists faced, problems that arose from a dependence on classic logic, probability, and dynamics.

On more than one occasion, humans have been faced with radical departures in their beliefs about how the world works. It was initially difficult to realize that rational numbers were insufficient for representing objects in the world, and instead irrational or even imaginary numbers were necessary. It was even harder to realize that Euclid's axioms of geometry were not always appropriate, and that other geometries such as Minkowski or Riemannian geometries are sometimes required to represent the world.

Birkhoff and von Neumann logic broke the strong grip that classic (Bernoulli) logic had on our minds to reveal the existence and the need of alternative logics for understanding the world. Classic (Kolmogorov) probability theory is founded on classic logic, and departures of classic logic entail departures from classic probability theory.

Thus it was necessary to relax the constraints of classic probability theory and form generalized probability theories. Quantum probability is one such generalization, which is based on quantum logic rather than classic logic.

Let us consider some more analogies. Newton and Leibniz discovered the calculus and differential equations primarily for the purpose of modeling physical phenomena. Later, mathematicians such as Euler, Gauss, Laplace and others identified the essential principles underlying these earlier physical models. Once these ideas were released as general mathematical tools, they became actively used throughout the social sciences including psychology, sociology, and economics. A similar spread of ideas occurred with the mathematical field of stochastic processes. For example, Markov and diffusion models were originally developed to describe the motion of particles, but these mathematical forms eventually found their way into applications in the social and behavioral sciences too.

Von Neumann released the mathematical theory underlying quantum mechanics from the restrictive domain of physics and opened it up to all sciences. No longer is physics the sole owner or user of this new branch of mathematics. Just as social scientists learned to use differential equations and stochastic processes, they can learn to use quantum mathematics. The need to use quantum principles arises whenever social scientists encounter the same types of problems that physicists faced long ago. In fact, decision theorists are now facing paradoxes of human decision making that are difficult to explain using classic decision theory.

The distinction between quantum decision theory applied to social and behavioral sciences from quantum mechanics applied to physics and chemistry can be stated as

follows -- these two applications share the same mathematical tools, but only the mathematics tools. Quantum mechanics is concerned with applying these mathematical tools to physical phenomena such as photons, electrons, atoms, etc... Preparations such as forces applied to particles or particle filtering, and parameters such as Planck's constant or spin correlations, are only applicable to physical applications. Quantum decision theory is concerned with applying these mathematical tools to psychological and economic phenomena such as judgment, choice, decision, strategy selection, etc... New experimental factors need to be determined to form preparations of these new systems, and new parameters need to be determined for this new type of application. Perhaps one day we can come up with a financial/economics equivalent of the Planck constant.

*2. Why consider quantum probabilities rather classical probabilities for modeling human decision making?*

Social and behavioral scientists face the same basic problem that forced physicists to abandon classical theory. That is the problem of only being able to obtain partial information about a complex system at any point in time. Combining the partial information about a system into a coherent understanding of the entire system is the hallmark of quantum theory.

Physicists discovered that some of their observables were incompatible, such as position and momentum of a particle, or direction of spin of an electron, or the angle of polarization of a photon. For these incompatible observables, one cannot simultaneously observe all the properties of a system; measurement of one observable interferes with another; and the order of measurement changes the results. The famous Heisenberg

uncertainty principle follows from these basic facts, which is one of the main building blocks of quantum mechanics.

Incompatible observables are replete in the social sciences as well. In finance, researchers face the impossibility of simultaneously measuring prices and their derivatives. Preferences in particular are actively constructed on the fly in response to context, rather than passively waiting to be revealed. It is not possible to simultaneously observe preferences from different perspectives (e.g. making a medical decision from personal perspective versus a policy perspective); measurement from one perspective can distort or interfere with measurement from another perspective; and the order effects are a common nuisance in the social and behavioral sciences.

A problem arises when measuring incompatible observables. It is impossible to simultaneously measure incompatible observables – one measure must follow the other – and taking a measure of the first can disturb the measurement of the second observable. Thus we can only obtain partial information about a system with one measure or at one time point without disturbing these measures.

With incompatible observables, the probability of an outcome obtained by measuring one observable does not necessarily equal the sum across the joint probabilities of all outcome combinations produced by both observables.<sup>1</sup> Thus the classical law of total probability can be violated. Similarly, with dynamic observables – the probability of an outcome obtained by a measurement at one time point does not necessarily equal the sum across the joint probabilities of all combinations produced by

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<sup>1</sup>  $\mathbf{X}$  and  $\mathbf{Y}$  are incompatible observables with finite non degenerate spectrum.  $Pr(Y = y) = |\langle y | \psi \rangle|^2 = |\langle y | \mathbf{I} | \psi \rangle|^2 = |\langle y | \cdot \sum |x_i\rangle\langle x_i| \cdot | \psi \rangle|^2 = |\sum \langle y | x_i \rangle \langle x_i | \psi \rangle|^2 \neq \sum |\langle y | x_i \rangle \langle x_i | \psi \rangle|^2$

measurements at two time points.<sup>2</sup> Again this implies that the classic law of total probability can fail. These failures of classic probability theory force the need for a generalized probability theory.

It is important to point out that when all measures are compatible across observables or across time, so that the joint distribution of all the random variables is well defined, then quantum probabilities agree exactly with classic probabilities. Thus there is no difference between quantum and classical probability assignments when all the measures are assumed to be compatible. This is why quantum probability is arguably the most coherent mathematical basis for analyzing complex systems that sometimes involve incompatible observables – if we restrict our attention to the compatible measures, we recover the classical model.

### *3. Are quantum models more complex than classical models?*

It is true that classic models are usually restricted to be real valued systems while quantum systems are often complex valued. However, it is not true that quantum models are always more complex than classic models. For example, quantum dynamic systems obey a set of constraints imposed by the use of unitary operators; Markov dynamic systems obey a different set of constraints imposed by the use of transition operators. The constraints are different and the quantum class does not contain the Markov class. Furthermore, even though the unitary operator is usually complex, it can be generated from a real valued Hamiltonian.

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<sup>2</sup>  $\mathbf{X}$  is an observable with a finite non-degenerate spectrum.  $Pr(X = x' \text{ at time } t) = |\langle x' | \Psi_t \rangle|^2 = |\langle x' | \mathbf{U}_t | \Psi_0 \rangle|^2 = |\langle x' | \mathbf{U}_t \mathbf{I} | \Psi_0 \rangle|^2 = |\langle x' | \cdot \mathbf{U}_t \cdot \sum |x_i\rangle\langle x_i| \cdot | \Psi_0 \rangle|^2 = |\sum \langle x' | \mathbf{U}_t | x_i \rangle \langle x_i | \Psi_0 \rangle|^2 \neq \sum |\langle x' | \mathbf{U}_t | x_i \rangle \langle x_i | \Psi_0 \rangle|^2$ .

Rather than being more complicated, quantum models can often simplify matters that appear complicated from a classical point of view. When incompatible observables are involved, a classic model has to postulate additional states to account for the changes in probabilities across orders of measurement. The quantum model can accommodate these order effects without postulating any new additional states. In such cases, the quantum model is actually simpler than the classic model.

In short, there is no a priori reason for assuming that quantum models are more complex than classical models. This will depend on the specific application, and model complexity must be evaluated on a case-by-case analysis. Mathematical models of decision making should be evaluated and tested and compared on the same grounds whether they are constructed from quantum or classical principles.

#### *4. How could the brain implement quantum computations?*

Of course, social sciences are concerned with people whose actions are guided by physical brains. Who knows if the brain is quantum mechanical in a physical sense? We are not really interested in this question. The use of quantum mathematics for describing behavior does not necessarily require one to assume that an underlying quantum mechanical brain guides this behavior. On the one hand, quantum probability calculations can be computed on a classic computer (although generating observations is problematic). On the other hand, quantum mechanics in physics can be rewritten as a classical theory using the Bohmian interpretation.

Even if the brain is classical, there are compelling reasons, mentioned above, to analyze its behavior according to quantum mathematics. How the brain implements these

computations is a question that can be answered later. This is not such an unusual position to take. Bayesian models of decision-making face exactly the same issue. We don't know how the brain implements these computations either. Some day we hope to answer these questions, but all the questions do not need to be answered at once.

A related question is why would the brain evolve in such a manner that it would follow a quantum probability model? There are at least two good answers to this question. First, although most of the events that we observe in this world are classical (e.g., hitting a ball with a bat), personal and social interactions may not be. Second, the brain must deal with the common problem of partial measurements of complex systems, and we have argued that the quantum approach provides the most coherent system for understanding such systems.